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G. Dattoli, G. Matone and D. Prosperi: ELECTROMAGNETIC
PROPERTIES OF CHARMED HADRONS.

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1. - INTRODUCTION.

The discovery, a few years ago, of the $J(\psi)$ particles⁽¹⁾ followed, more recently, by the observation of charmed mesons⁽²⁾ and, perhaps, of charmed baryons⁽³⁾, seems to confirm the hypothesis that at least a further quark is to be added to the usual Gell-Mann and Zweig three quarks. This new "elementary building block", together with the previous ones, is assumed to belong to the fundamental representation of SU(4). The hadrons, thought as quark aggregates, belong to higher representations of SU(4), or SU(8), if the spin is taken into account. Moreover, the bulk of the available data (mass spectra transition probabilities), clearly suggests that SU(4) symmetry is badly broken, due to the intrinsic mass difference among the quarks and to their mutual interactions.

A number of authors have studied hadron mass formulae assuming SU(4) or SU(8) partially broken symmetries. Some of them have made explicit use of the quark idea, while used the group theory only as guide to the description of the transformation properties of the hadron mass operator. In the following we will devote our attention exclusively to the first kind of approach. A first group of papers neglects electromagnetic effects⁽⁴⁾. This means that all the particles within the same I-spin multiplet are assumed to have identical masses.

The most intuitive way to calculate e. m. mass splittings seems to assume that they arise from the difference in the quark effective masses besides the Coulomb and magnetic quark-quark interactions. This kind of approach has been extensively developed in the framework of SU(3) or SU(6)⁽⁵⁾; a preliminary extension including the charmed quark has been recently done by Lichtenberg⁽⁶⁾. For the sake of completeness let us also mention the existence of other calculations performed without explicit recourse to the quark concept⁽⁷⁾; more recently, e. m. mass splittings have been studied also in the framework of the MIT bag model⁽⁸⁾; finally, Elitzur and Harary showed how the e. m. hadron masses can be expressed in terms of measurable inelastic electron scattering cross sections⁽⁹⁾.

Another subject strictly related to the previous one is the study of magnetic moments. Among the most recent works let us mention those due to Choudhury and Yoshi^(10, 11) and to Lichtenberg⁽¹²⁾. In refs.(10) and (11) the magnetic moments of charmed baryons have been expressed in terms of those of uncharmed baryons, respectively assuming U(4) and U(8) symmetries; on the contrary, Lichtenberg assumed the quark magnetic moments to be proportional to their charge-to-mass ratios ($\mu_i \sim e_i/m_i$), introducing in a natural way symmetry breaking effects.

The aim of the present work is to systematically calculate both e. m. mass splittings and magnetic moments of charmed hadrons in the framework of a four quark model^(13, 14). Although mass splittings and magnetic moments of charmed hadrons cannot be measured at present, we produce our results now because they might be useful to other authors for comparison purposes; it is in fact particularly interesting to test how the provisions concerning the hadron e. m. properties depend on the

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model details. In addition, there may be other calculations more accessible to experimental tests (for example, radiative decay matrix elements of charmed vector mesons) to which our results are strictly related.

We work in a non relativistic framework and assume, following the gauge theory model of De Rujula, Georgi and Glashow⁽¹⁵⁾, $\mu_i \propto e_i/m_i$, as in ref. (12). In addition we neglect any contribution from the orbital angular momentum of the quarks. Labelling by u, d, s and c the quarks^(13, 14), we assume:

$$\frac{\mu_d}{\mu_u} = -2 \frac{m_u}{m_d}, \quad \frac{\mu_s}{\mu_u} = -2 \frac{m_u}{m_s}, \quad \frac{\mu_c}{\mu_u} = \frac{m_u}{m_c}, \quad (1)$$

with $m_u \sim m_d < m_s < m_c$. It is to be noted that this mass scale really fixes a hierarchy of the symmetry breaking effects, with the charm breaking the symmetry more than strangeness, and strangeness more than the third component of isospin.

Furthermore we need to assume something about the wave functions of SU(8). They are not unique unless it is specified the subgroup of SU(8) to which they belong; to eliminate any ambiguity, their SU(4) content was classified in terms of SU(3) \otimes U^c, U^c being the charm unitary group. Symmetry breakings in the wave functions likely introduce only second order effects in our calculations and hence have been neglected. Let us finally note that if one wishes to preserve Fermi statistics, one may include colour indices, but they do not affect our results, being the e. m. current a singlet of colour.

Section 2 will be devoted to a short review of the adopted hadron classification scheme, while theoretical calculations and numerical estimates concerning baryons and mesons will be given in sects. 3 and 4, respectively. Finally, sect. 5 will be devoted to some final remarks.

2. - SU(4) CLASSIFICATION OF HADRONS.

The SU(4) classification scheme requires four quarks, three (u, d, c) belonging to the fundamental representation of SU(3) and a fourth (c) carrying an additive quantum number (Charm)^(13, 14). Let us now introduce the notations:

$$Y = B + S, \quad \tilde{Y} = Y + C, \quad (2)$$

where the symbols Y, B and S denote, respectively, hypercharge, baryon number and strangeness. The well known relation $Q = I_3 + Y/2$, can be generalized as follows:

$$Q = I_3 + \tilde{Y}/2. \quad (3)$$

Moreover, we can again define the third U-spin component U₃ by the usual relation $U_3 = (Y - Q)/2$; it can be put also in the equivalent forms:

$$U_3 = (-4I_3 + 3Q - 2C)/2 = (3Y - 2I_3 - C)/4. \quad (4)$$

The usual tridimensional weight diagram of the basic representation of SU(4) is reported in Fig. (1a), while in Fig. (1b) is represented a planar diagram displaying the U-spin quark properties (the inclined dotted lines connect states having the same U-spin). Similar diagrams will be produced for hadrons in the continuation of the present paper.

Let us first consider baryons which, according to current ideas, we assume to belong to $(4 \otimes 4 \otimes 4)$. In terms of the irreducible representations of SU(4), we have:

$$4 \otimes 4 \otimes 4 = 4_A \oplus 20_M \oplus 20'_M \oplus 20_S, \quad (5)$$

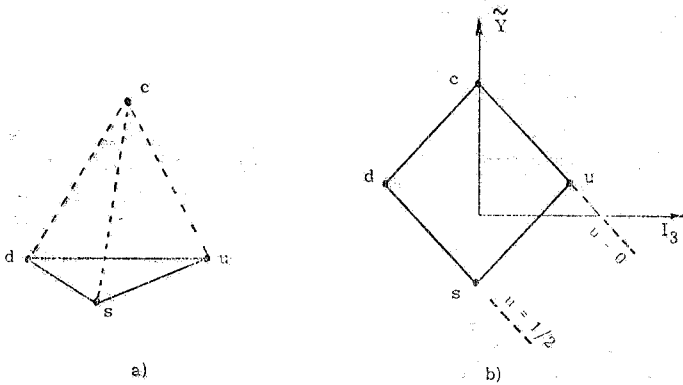


FIG. 1- a): standard weight diagram of the SU(4) fundamental representation; b): Planar weight diagram in the $\tilde{Y} - I_3$ plane. In a) the vertical axis is charm.

where the right subscripts S, M and A denote the symmetry of the given representation (S=symmetric, M=Mixed, A=Antisymmetric). On the other hand, if we express these multiplets in terms of $SU(3) \otimes U^C$, we find:

$$\begin{aligned} 4 &= (1, 0) \oplus (3, 1) \\ 20_A &= (8, 0) \oplus (6, 1) \oplus (\bar{3}, 1) \oplus (3, 2) \\ 20_M &= (10, 0) \oplus (6, 1) \oplus (3, 2) \oplus (1, 3); \end{aligned} \quad (6)$$

here we have assumed the notation (m, C) , where m labels the $SU(3)$ multiplets and C is the charm quantum number. Let us mention that $(1, 0)$, $(8, 0)$ and $(10, 0)$ are the usual uncharmed baryon multiplets.

In the present work we limit ourselves to study baryons classified within the 20_M and 20_S multiplets. As for $SU(8)$, they belong to the symmetric 120 representation having the following $SU(4)$ and spin content:

$$120 \supset^2 20_M \oplus^4 20_S, \quad (7)$$

where the left superscript denotes the spin multiplicity.

The situation is schematically displayed in Figs. 2 to 4. Figs. (2a) and (2b) show weight diagrams for 4_A in which we find the usual $SU(3)$ singlet as well as its charmed partners: a doublet ($\Xi_1^{0,+}$) and a \mathbf{T} -spin singlet (Λ_1^+). Figs. (3a) and (3b) concern 20_M : we have the usual $J^P(1/2)^+$ baryon octet besides a charm-one nonet and a charm-two triplet; the nonet splits off in an \mathbf{T} -spin triplet ($\Sigma_1^{0,+}$) two doublets ($\Xi_1^{0,+}$, $\Xi_1^{10,+}$) and two singlets (Ω_1^0 , Λ_1^+), while the triplet consists of a doublet ($\Xi_2^{+,+}$) and singlet (Ω_2^+). Finally, Figs. (4a) and (4b) concern 20_S : we have the usual $J^P(3/2)^+$ decuplet besides a charm-one sextet, a charm-two triplet and charm-three singlet; the sextet splits in an I -spin triplet ($\Sigma_2^{*,0,+}$) a doublet ($\Xi_2^{*,0,+}$) and a singlet (Ω_2^{*+}), while the triplet splits in an I -spin doublet ($\Xi_2^{*,++}$) and a singlet (Ω_2^{*+}). The nomenclature for the charmed baryons is that introduced in refs. (4, 12).

As for mesons, we have the following multiplets of $SU(3) \otimes U^C$:

$$4 \otimes \bar{4} = (10, 0) + (\bar{3}, 1) + (3, -1) \quad (8)$$

Moreover, mesons belong to a 64-dimensional representation of $SU(8)$:

$$64 \supset \left\{ {}^1(10, 0) \oplus {}^1(\bar{3}, 1) \oplus {}^1(3, -1) \right\} \oplus \left\{ {}^3(10, 0) \oplus {}^3(\bar{3}, 1) \oplus {}^3(3, -1) \right\} \quad (9)$$

Let us now fix our attention on vector mesons; the situation is displayed in Figs. (5a) and (5b). The charm-zero decuplet contains the ψ meson in addition to the $SU(3)$ nonet, while ${}^3(\bar{3}, 1)$ and ${}^3(3, -1)$ split in a doublet ($D^{*,0,+}$) and a singlet (S^{*+}) of charmed mesons, besides the corresponding antiparticles. The charmed scalar mesons are denoted by similar symbols ($D^{0,+}$, S^{*+}).

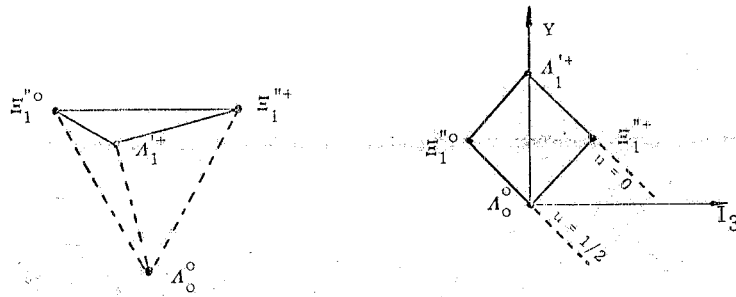


FIG. 2 - The $J^P = 1/2^+$ baryons belonging to the 4_A representation. See Fig. 1 for further details.

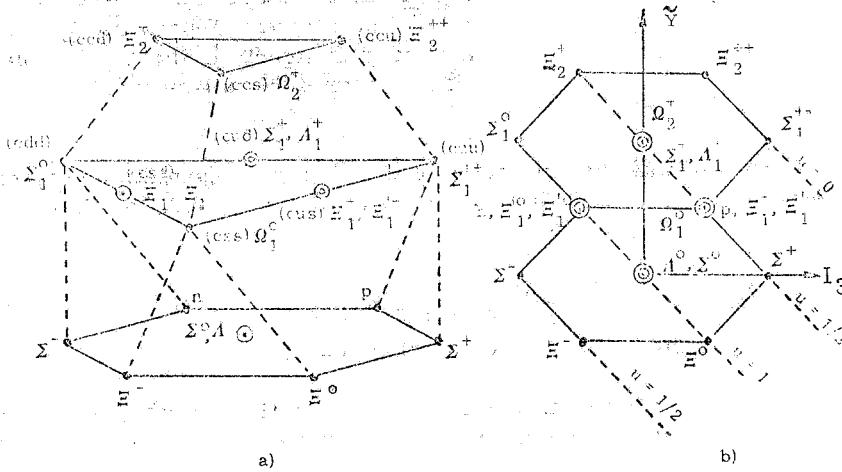


FIG. 3 - The $J^P = 1/2^+$ baryons belonging to the 20_M representation. Circled dots indicate positions where more than one state is located. See Fig. 1 for further comments.

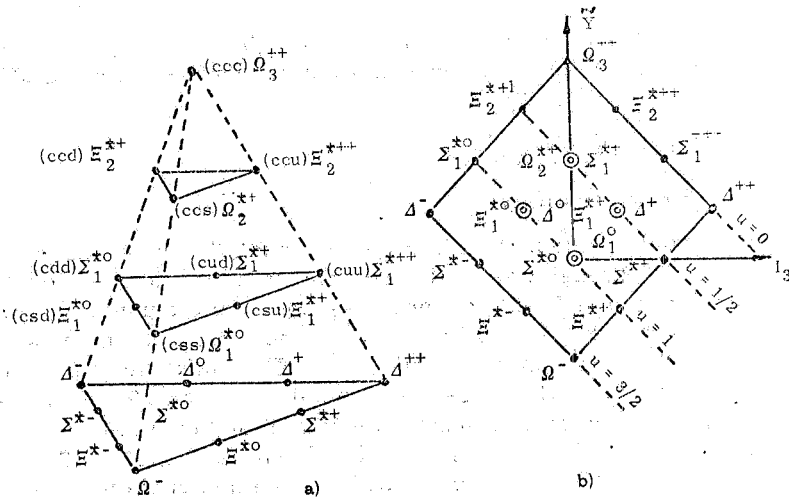


FIG. 4 - The $J^P = 3/2^+$ baryons belonging to the 20_S representation. See Fig. 1 for further comments.

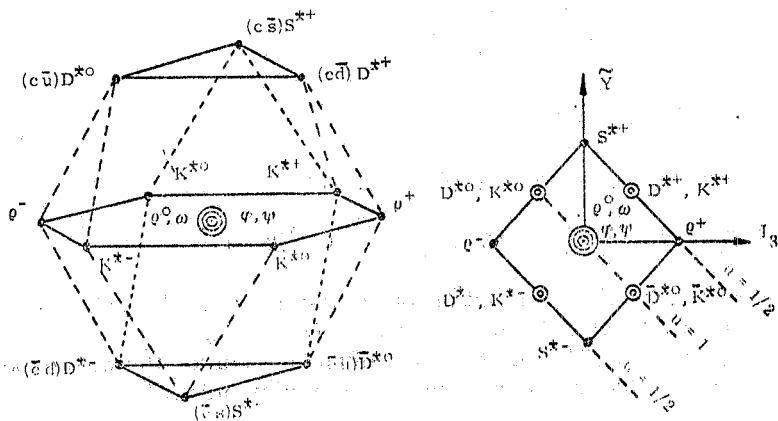


FIG. 5 - The 16. vector mesons. See Fig. 1 for further comments.

3. - MASS SPLITTING AND MAGNETIC MOMENTS OF CHARMED BARYONS.

We assume here the e. m. masses of hadrons to be the sum of three contributions: a term deriving from the e. m. self energy of the single quark (S), a pure Coulomb contribution (C) and a term containing both magnetic contributions and relativistic corrections (M). Therefore, we assume for the e. m. hamiltonian a form of the kind:

$$H_{e. m.} = H_S + H_C + H_M \quad (10)$$

Let us now denote by i any quark in the hadron, by \hat{e}_i its charge operator (in units of e), by $\vec{\mu}_i = \hat{e}_i (m_u/m_i) \vec{\sigma}_i$ its dipole magnetic operator in units of $\mu_u = e\hbar/2m_u c$ and finally, by $\vec{r}_{ij} = \vec{r}_i - \vec{r}_j$ the distance between the i -th and the j -th quark.

One may introduce the notations

$$x = \frac{m_u}{m_s}, \quad y = \frac{m_u}{m_c}$$

and since x and y are in general different from one, a well defined symmetry breaking is introduced.

We put :

$$H_S = \sum_i \hat{e}_i^2 F_i^{(S)}(j), \quad H_C = \sum_{i < j} (\hat{e}_i \hat{e}_j) F_i^{(C)}(r_{ij}). \quad (11)$$

We have tentatively assumed the e. m. self mass of the quark i to be proportional to \hat{e}_i^2 , denoting by $F_i^{(S)}(j)$ the proportionality factor; it would be noted that it can, in principle, depend on the presence of all other quarks in a complex way, but to avoid any unneeded heavy formalism, we assume $F_i^{(S)}(j)$ to be a constant factor $F^{(S)}$. We have also $F_i^{(C)}(r_{ij}) = e^2(1/r_{ij})$.

The calculation of the interaction term H_M seems to be rather model dependent. We assume here all the quark-quark pairs to be in a relative S-state. The most general expression containing terms of order $(e/m)^2$ and symmetric in the variable of quarks i and j is:

$$\begin{aligned} H_M = & \sum_{i < j} (\hat{e}_i \hat{e}_j) \frac{m_u^2}{m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) G^{(1)}(\vec{r}_{ij}) + \sum_{i < j} (\hat{e}_i \hat{e}_j) x \\ & x \frac{m_u^2}{m_i m_j} G^{(2)}(\vec{r}_{ij}) + \sum_{i < j} (\hat{e}_i \hat{e}_j) m_u^2 \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) (\vec{\sigma}_i \cdot \vec{\sigma}_j) x \\ & x G^{(3)}(\vec{r}_{ij}) + \sum_{i < j} (\hat{e}_i \hat{e}_j) m_u^2 \left(\frac{1}{m_i^2} + \frac{1}{m_j^2} \right) G^{(4)}(\vec{r}_{ij}). \end{aligned} \quad (12)$$

For illustrative purposes, let us fix our attention on a particular model. Let us suppose H_M to be of the kind introduced by Schwinger for two point like nucleons and used by Okamoto and Pask(20) to study the binding energy difference of ${}^3\text{H}$ and ${}^3\text{He}$; up to terms of order $(e/m)^2$ we have

$$\begin{aligned} G^{(1)}(\vec{r}_{ij}) &= -\mu_u^2 \frac{8}{3} \pi \delta(\vec{r}_{ij}) \\ G^{(2)}(\vec{r}_{ij}) &= \mu_u^2 \left(\vec{P} \frac{1}{r_{ij}} \vec{P} - 4\pi \delta(\vec{r}_{ij}) \right) \\ G^{(3)}(\vec{r}_{ij}) &= 0 \\ G^{(4)}(\vec{r}_{ij}) &= -\mu_u^2 (2\pi) \delta(\vec{r}_{ij}) \end{aligned} \quad (13)$$

where \vec{P} is the linear momentum operator of the particles in the c. m. system.

For two and three quark systems one could expect that also $L \neq 0$ states give a significative contribution to the hadron structure. In such a case one should add to expression (12) other terms like

$$-\mu_u^2 \sum_{i < j} (\hat{e}_i \hat{e}_j) \frac{m_u^2}{m_i m_j} \left\{ S_{ij} + 6 \vec{L}_{ij} \cdot \vec{S}_{ij} + 2(m_j - m_i) \vec{L}_{ij} \cdot \left(\frac{1}{m_i} \vec{S}_i - \frac{1}{m_j} \vec{S}_j \right) \right\} \frac{1}{r_{ij}^3} \quad (14)$$

where S_{ij} is the usual tensor force operator, while \vec{S}_{ij} and \vec{L}_{ij} are, respectively the total spin and the angular momentum of the i - j pair. However according to our previous assumption, these contributions will be neglected.

Since H_M contains both charge and spin, the magnetic energy critically depends on the symmetry of the hadron wave functions. For sake of simplicity, we assume here the spatial wave functions, R , to be totally symmetric for the interchanging of the spatial coordinates of any two quarks. This condition, which seems a-priori difficult to be fully justified, could be easily relaxed if required from experimental data. For any hadron state (h) we put:

$$\begin{aligned} M_{e.m.} &= \langle h | H_{e.m.} | h \rangle = \\ &= a(h) \langle \sum_i \hat{e}_i^2 \rangle_h + b(h) \langle \sum_{i < j} (\hat{e}_i \hat{e}_j) \rangle_h + \\ &+ c(h) \langle \sum_{i < j} (\hat{e}_i \hat{e}_j) \frac{m_u^2}{m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \rangle_h + d(h) \langle \sum_{i < j} (\hat{e}_i \hat{e}_j) \frac{m_u^2}{m_i m_j} \rangle_h + \\ &+ e(h) \langle \sum_{i < j} (\hat{e}_i \hat{e}_j) m_u^2 \left(\frac{1}{m_i} + \frac{1}{m_j} \right) \rangle_h. \end{aligned} \quad (15)$$

where:

$$\begin{aligned} a(h) &= \langle R_h | F^{(S)} | R_h \rangle, \quad b(h) = \langle R_h | F^{(C)}(\vec{r}_{ij}) | R_h \rangle, \\ c(h) &= \langle R_h | G^{(1)}(\vec{r}_{ij}) | R_h \rangle, \quad d(h) = \langle R_h | G^{(2)}(\vec{r}_{ij}) | R_h \rangle, \quad e(h) = \langle R_h | G^{(4)}(\vec{r}_{ij}) | R_h \rangle. \end{aligned} \quad (16)$$

With our assumptions all constants are at sight independent of "i" and "j"; moreover the symbol $\langle \rangle_h$ means an average on the SU(6) wave functions (in practice SU(6) \otimes UC). By analogy with the specific model we presented, we have neglected the term containing $\langle R_h | G^{(3)}(\vec{r}_{ij}) | R_h \rangle$.

Actually six mass differences between terms of different isomultiplets have been measured. We have (16)

$$\begin{aligned} \Delta_1 &= M(n) - M(P) = (1.29343 \pm 0.00004) \text{ MeV.} \\ \Delta_2 &= M(\Sigma^-) - M(\Sigma^+) = (7.98 \pm 0.08) \text{ MeV.} \\ \Delta_3 &= M(\Sigma^-) - M(\Sigma^0) = (4.88 \pm 0.06) \text{ MeV.} \\ \Delta_4 &= M(\Xi^-) - M(\Xi^0) = (6.4 \pm 0.6) \text{ MeV.} \\ \Delta_5 &= M(\Xi^{*-}) - M(\Xi^{*0}) = (3.3 \pm 0.7) \text{ MeV.} \\ \Delta_6 &= M(\Sigma^{*-}) - M(\Sigma^{*+}) = (4.1 \pm 1.3) \text{ MeV.} \end{aligned} \quad (17)$$

The adopted baryon wave functions are reported in the first columns of Table I and II. The constants of our mass formula, assumed to be independent on the particular hadron h , can be now obtained by the known mass differences and the relations:

$$\begin{aligned} 3 \Delta_1 &= -a - b - c - d - 2e, \\ 3 \Delta_2 &= -2a + b - (1+4x)c + (1-2x)d + 2x^2e, \\ 3 \Delta_3 &= -a + 2b + (1-2x)c - (1+x)d + (3-x^2)e, \\ 3 \Delta_4 &= -a + 2b - 4xc - 2xd + 2(1+x^2)e, \\ 3 \Delta_5 &= -a + 2b + 2xc - 2xd + 2(1+x^2)e, \\ 3 \Delta_6 &= -2a + b - (1-2x)c + (1-2x)d + 2x^2e. \end{aligned} \quad (18)$$

TABLE I - Results obtained for $J^P = 1/2^+$ charmed baryons belonging to the 20_M -plet of $SU(3) \otimes U^C$. The wave functions reported in the first column refer to $SU(6) \otimes U^C$.

Baryon wave functions	μ / μ_P	δM_S	δM_C	δM_M
$ \Xi_2^{++}\rangle = \frac{1}{\sqrt{18}} \{ cuc\rangle [\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow] + \text{perm.} \}$	$\frac{2}{9}(4y-1)$	4/3	4/3	$-4/9(-4y+y^2)$
$ \Xi_2^+\rangle = \frac{1}{\sqrt{18}} \{ cdc\rangle [\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow] + \text{perm.} \}$	$\frac{1}{9}(8y+1)$	1	0	$-4/9(y^2+2y)$
$ \Omega_2^+\rangle = \frac{1}{\sqrt{18}} \{ csc\rangle [\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow] + \text{perm.} \}$	$\frac{1}{9}(x+8y)$	1	0	$-4/9(y^2+2xy)$
$ \Sigma_1^{++}\rangle = \frac{1}{\sqrt{18}} \{ cuc\rangle [\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow] + \text{perm.} \}$	$\frac{2}{9}(4-y)$	4/3	4/3	$-4/9(-4y+1)$
$ \Sigma_1^+\rangle = \frac{1}{\sqrt{36}} \{ 2 d^{\uparrow}u^{\uparrow}c^{\downarrow}\rangle - d^{\downarrow}u^{\uparrow}c^{\uparrow}\rangle - d^{\uparrow}u^{\downarrow}c^{\uparrow}\rangle + \text{perm.} \}$	$\frac{2}{9}(1-y)$	1	0	$+2/9(2y+1)$
$ \Sigma_1^0\rangle = \frac{1}{\sqrt{18}} \{ dcd\rangle [\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow] + \text{perm.} \}$	$-\frac{2}{9}(2+y)$	2/3	-1/3	$-1/9(8y+1)$
$ \Lambda_1^+\rangle = \frac{1}{\sqrt{12}} \{ (udc\rangle - duc\rangle) - (\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow) + \text{perm.} \}$	$2/3 y$	1	0	-2/3
$ \Omega_0'\rangle = \frac{1}{\sqrt{18}} \{ scs\rangle (\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow) + \text{perm.} \}$	$-2/9(y+2x)$	2/3	-1/3	$-1/9(8xy+x^2)$
$ \Xi_1^{\prime 0}\rangle = \frac{1}{\sqrt{18}} \{ dsc\rangle (\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow) + \text{perm.} \}$	$2/3 y$	2/3	-1/3	$+1/3 x$
$ \Xi_1^{\prime +}\rangle = \frac{1}{\sqrt{12}} \{ usc\rangle (\uparrow\uparrow\uparrow - \downarrow\uparrow\uparrow) + \text{perm.} \}$	$2/3 y$	1	0	$-2/3 x$
$ \Xi_1^+\rangle = \frac{1}{\sqrt{36}} \{ 2 u^{\uparrow}s^{\uparrow}c^{\downarrow}\rangle - usc\rangle (\uparrow\uparrow\uparrow + \downarrow\uparrow\uparrow) + \text{perm.} \}$	$\frac{2}{9}(2-x-y)$	1	0	$-\frac{2}{9}(-4y-x+2xy)$
$ \Xi_1^0\rangle = \frac{1}{\sqrt{36}} \{ 2 d^{\uparrow}s^{\uparrow}c^{\downarrow}\rangle - dsc\rangle (\uparrow\uparrow\uparrow + \downarrow\uparrow\uparrow) \}$	$-\frac{2}{9}(1+x+y)$	2/3	-1/3	$-\frac{1}{9}(4y+x+4xy)$

$$\delta M_S = \left\langle \sum_i \hat{e}_i^2 \right\rangle_h, \quad \delta M_C = \left\langle \sum_{i<j} \hat{e}_i \cdot \hat{e}_j \right\rangle_h$$

$$\delta M_M = \left\langle \sum_{i<j} \hat{e}_i \cdot \hat{e}_j \frac{m_i^2}{m_i m_j} (\vec{\sigma}_i \cdot \vec{\sigma}_j) \right\rangle_h$$

TABLE II - Results obtained for $J^P = 3/2^+$ charmed baryons belonging to the 20_S -plet of $SU(3) \otimes U^C$. The wave functions reported in the first column refer to $SU(6) \otimes U^C$.

Baryon wave functions	μ/μ_p	δM_S	δM_C	δM_M
$ \Omega_3^{++}\rangle = c \uparrow c \uparrow c \uparrow\rangle$	$2y$	$4/3$	$4/3$	$-4/3 y^2$
$ \Omega_2^{*+}\rangle = \frac{1}{\sqrt{3}} [ccs\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$-\frac{1}{3}(x-4y)$	1	0	$-\frac{4}{9}(y^2-2xy)$
$ \Xi_2^{*++}\rangle = \frac{1}{\sqrt{3}} [ccu\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$\frac{2}{3}(1+2y)$	$4/3$	$4/3$	$-\frac{4}{9}(y^2+2y)$
$ \Xi_2^{*+}\rangle = \frac{1}{\sqrt{3}} [ccn\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$\frac{1}{3}(1-4y)$	1	0	$-\frac{4}{9}(y^2-y)$
$ \Omega_1^{*0}\rangle = \frac{1}{\sqrt{3}} [ssc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$-\frac{2}{3}(x-y)$	$2/3$	$-1/3$	$-\frac{1}{9}(x^2-4xy)$
$ \Xi_1^{*+}\rangle = \frac{1}{\sqrt{6}} [usc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$\frac{1}{3}(2-x+2y)$	1	0	$-\frac{2}{9}(-x+2y-xy)$
$ \Xi_1^{*0}\rangle = \frac{1}{\sqrt{6}} [dsc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$-\frac{1}{3}(1+x-2y)$	$2/3$	$-1/3$	$-\frac{1}{9}(-x-2y-2xy)$
$ \Sigma_1^{*++}\rangle = \frac{1}{\sqrt{3}} [uuc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$\frac{2}{3}(2+y)$	$4/3$	$4/3$	$-\frac{4}{9}(1+2y)$
$ \Sigma_1^{*+}\rangle = \frac{1}{\sqrt{6}} [udc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$\frac{1}{3}(1+2y)$	1	0	$-\frac{2}{9}(-1+y)$
$ \Sigma_1^{*0}\rangle = \frac{1}{\sqrt{3}} [uuc\rangle + \text{perm.}] \uparrow \uparrow \uparrow$	$-\frac{2}{3}(1-y)$	$2/3$	$-1/3$	$-\frac{1}{9}(1-4y)$

As it can be easily checked, these expressions critically depend on x ; it can be derived from the experimentally known magnetic moments of Λ^0 , Σ^+ and Ξ^- baryons⁽¹⁶⁾. Further, for y we assume the rough estimate by De Rujula, Georgi and Glashow⁽¹⁵⁾. We have:

$$x = 0.73 \pm 0.06, \quad y \approx 0.20.$$

From equations (18) we immediately extract the following relations:

$$\begin{aligned}
 & \left[M(\Xi^-) - M(\Xi^0) \right] - \left[M(\Xi^{*-}) - M(\Xi^{*0}) \right] = \left[M(\Sigma^-) - M(\Sigma^+) \right] - \\
 & \left[M(\Sigma^{*-}) - M(\Sigma^{*+}) \right], \quad \left[M(\Xi^-) - M(\Xi^0) \right] = \\
 & \left[M(\Sigma^-) - M(\Sigma^0) \right] - \left[M(n) - M(p) \right]
 \end{aligned} \tag{19}$$

The first one numerically gives $(3.1 \pm 0.9) \text{ MeV} = (3.9 \pm 0.7) \text{ MeV}$, while the second one gives $(6.4 \pm 0.6) \text{ MeV} = (6.7 \pm 0.08) \text{ MeV}$. Both conditions are fully satisfied. We are so left with a system of four equations, while we have five parameters to be determined. As far d and e, we can only fix the linear combinations $b(1+x)e$ which, by fit to the available data, were found to be completely undetermined ($\sim 1.3 \pm 2$) MeV owing to the fact that in our standard model, d and e are predicted probably to have the same sign ($d \sim 2e$ if we neglect the $\vec{P} \cdot \vec{P} / r$ term) we are forced to exactly put $d=e=0$. From the first three relations of the system (18), all valid in the 20_M representation, we finally obtain:

$$\begin{cases} a(20_M) = -(5.661 \pm 0.006) \text{ MeV} \\ b(20_M) = (3.98 \pm 0.12) \text{ MeV} \\ c(20_M) = -(2.21 \pm 0.11) \text{ MeV} \end{cases} \quad (20)$$

To test our results, we have then calculated other known mass differences (see Table III). The agreement is satisfactory in any case, both for baryons belonging to $20_M (\Xi^- - \Xi^0)$ and $20_S (\Xi^{*+} - \Xi^{*0}, \Sigma^{*-} - \Sigma^{*0})$. This means that, at least for uncharmed baryons, our mean assumptions (classification of wave functions according to $SU(6) \otimes U^C$, total spatial symmetry) are substantially correct. Furthermore, it appears that constants (20) do not sensibly vary within the 120-dimensional multiplet of $SU(6) \otimes U^C$, this confirming the consistency of our procedure.

Estimates for charmed baryons are also reported in Table III. Let us note that our specific model, at least in principle, allows to extract the charmed quark magnetic moment, or equivalently y , directly from experiments on charmed baryons.

We could finally combine our results to obtain other inequalities or linear relations connecting the baryon e.m. masses, in the spirit of refs. (4,6). It is beyond argument that this theoretical information handling is of considerable interest, but it would lead to excessively lengthen our exposition. As an example, let us note that, when $x, y=1$, we have:

$$M(\Sigma_1^+) - M(\Sigma_1^0) = M(\Xi_1^+) - M(\Xi_1^0),$$

and both these mass splittings can be expressed in terms of those given by expressions (18). However, in the most general case, we have the more complex relation

$$M(\Sigma_1^+) - M(\Sigma_1^0) = \left[M(\Xi_1^+) - M(\Xi_1^0) \right] + \left[M(\Xi_1^{*+}) - M(\Xi_1^{*0}) \right] + \left[M(\Xi_2^{*++}) - M(\Xi_2^{*+}) \right] - \left[M(\Sigma_1^{*++}) - M(\Sigma_1^{*+}) \right].$$

In addition, these mass splittings cannot be expressed only in terms of those of uncharmed baryons.

The charmed baryon magnetic moments are also reported in the second column of Tables I and II. They coincide with those found by Lichtenberg⁽¹²⁾ and reproduce the Choudhury and Joshi results⁽¹¹⁾ in the limit of exact $SU(8)$ symmetry ($x=y=1$).

Let us extend to our case the main results obtained in refs. (17) for the magnetic moments of $SU(3)$ baryons assuming an exact U-spin invariance. The $SU(4)$ extension is straightforward and gives e_i :

$$\begin{aligned} \mu(20_M) &= 2/3 \left[(Q/2 + I_3)^2 - U(U+1) \right] \mu_p, \quad (U \neq 0), \\ \mu(20_S) &= Q \mu_p \end{aligned} \quad (21)$$

It reproduces all the results we obtained when $x=y=1$, with the exception of those 20_M baryons which are not U-spin eigenstates (see sect. 2). In this limit we obtain equalities like:

TABLE III - Results obtained for uncharmed and charmed baryons.

Mass Splittings	Theoretical MeV	Experimental ⁽¹⁶⁾ MeV
$M(\Xi^-) - M(\Xi^0)$	6.69 ± 0.14	6.4 ± 0.6
$M(\Sigma^{*-}) - M(\Sigma^{*+})$	4.78 ± 0.10	4.1 ± 1.3
$M(\Xi^{*-}) - M(\Xi^{*0})$	3.46 ± 0.10	3.3 ± 0.7
$M(\Delta^0) - M(\Delta^{++})$	0.82 ± 0.27	1.3 ± 0.4
$M(\Delta^-) - M(\Delta^{++})$	4.36 ± 0.05	7.9 ± 6.8
$M(\Xi_2^{*++}) - M(\Xi_2^{*+})$	2.83 ± 0.02	-
$M(\Xi_1^{*+}) - M(\Xi_1^{*0})$	-0.67 ± 0.04	-
$M(\Sigma_1^{*++}) - M(\Sigma_1^{*+})$	1.63 ± 0.18	-
$M(\Sigma_1^{*+}) - M(\Sigma_1^{*0})$	-0.12 ± 0.05	-
$M(\Xi_2^{++}) - M(\Xi_2^+)$	4.60 ± 0.16	-
$M(\Xi_1^+) - M(\Xi_1^0)$	0.56 ± 0.07	-
$M(\Sigma_1^{++}) - M(\Sigma_1^+)$	2.53 ± 0.16	-
$M(\Sigma_1^+) - M(\Sigma_1^0)$	-1.88 ± 0.07	-
$M(\Xi_1^+) - M(\Xi_1^0)$	~ 0	-

$$\begin{aligned}
 \mu(\Omega_2^+) &= \mu(\Xi_2^+) = \mu_P \\
 \mu(\Xi_1^0) &= \mu(\Omega_1^0) = \mu(\Sigma_1^0) = \mu(n) = -\frac{2}{3} \mu_P, \\
 \mu(\Xi^-) &= \mu(\Sigma^-) = -\frac{1}{3} \mu_P
 \end{aligned} \tag{22}$$

for the 20_M representation, and

$$\begin{aligned}
 \mu(\Delta^{++}) &= \mu(\Sigma_1^{*++}) = \mu(\Xi_2^{*++}) = \mu(\Omega_3^{*++}) = 2\mu_P, \\
 \mu(\Sigma^{*+}) &= \mu(\Delta^+) = \mu(\Xi_1^{*+}) = \mu(\Sigma_1^{*+}) = \mu(\Omega_2^{*+}) = \mu(\Xi_2^{*+}) = \mu_P, \\
 \mu(\Xi^{*0}) &= \mu(\Omega_1^{*0}) = \mu(\Sigma^{*0}) = \mu(\Delta^0) = \mu(\Xi_1^{*0}) = \mu(\Sigma_1^{*0}) = 0, \\
 \mu(\Omega^-) &= \mu(\Xi^{*-}) = \mu(\Sigma^{*-}) = \mu(\Delta^-) = -\mu_P,
 \end{aligned} \tag{23}$$

for the 20_S representation. Otherwise, by eliminating x and y from the results of Tables I and II, we can obtain a set of linear combinations between baryon magnetic moments.

Finally, let us note that we could carry out similar calculations for uncharmed baryons in the framework of the Han-Nambu model, but we would find results identical to those obtained in SU(6) (see ref. (5)).

4. - ELECTROMAGNETIC PROPERTIES OF CHARMED MESONS.

Results obtained for e. m. properties of scalar and vector mesons are reported in Table IV. We assume in a first approach that the same approximations discussed for baryons are still valid.

As for the e. m. masses of mesons, we can attempt to calculate the physical constants a, b and c by the well established mass differences⁽¹⁶⁾:

$$\begin{aligned} M(\pi^-) - M(\pi^0) &= \frac{1}{2}(b-3c) = (4.6043 \pm 0.0037) \text{ MeV}, \\ M(K^+) - M(K^0) &= 1/3(a+b-3xc) = -(3.99 \pm 0.13) \text{ MeV}, \\ M(K^{*+}) - M(K^{*0}) &= 1/3(a+b+xc) = (-4.1 \pm 0.6) \text{ MeV}. \end{aligned} \quad (24)$$

We obtain the following estimates:

$$a = (-21.0 \pm 2.6) \text{ MeV}, \quad b = (8.9 \pm 1.9) \text{ MeV}, \quad c = (-0.11 \pm 0.62) \text{ MeV}. \quad (25)$$

It appears that the self-mass constant (a) is rather different from the baryon one: we have $a(\text{mesons}) \sim 4 a(\text{baryons})$. At first glance, this result could suggest the existence of non negligible many-body corrections. Further, we have $b(\text{mesons}) \sim 2b(\text{baryons})$. This last statement would indicate that the meson wave functions favour smaller values of $r_{1,2}$, but this conclusion conflicts with the smallness of c. However, as we will see in the following, (our results cannot be considered as conclusive), because even a little percentage of D-state in the vector meson wave function strongly depresses both |a| and b.

It could be also interesting to test if our estimates can be applied to other vector mesons. In this context an important role could be played by the $\rho^+ - \rho^0$ mass difference:

$$\left[M(\rho^+) - M(\rho^0) \right]_{\text{th.}} = \frac{1}{2}(b+c) = (4.4 \pm 1.0) \text{ MeV}. \quad (26)$$

Unfortunately, the present experimental situation is not clear at all: in the past, indications for both positive and negative mass splittings have been reported. In his recent work Lichtenberg^(4,6) accepted some results showing $M(\rho^+) - M(\rho^0)$ to be negative $(-4.3 \pm 2.4 \text{ MeV})$; if confirmed, this information would bring us to claim for a more complex approach in which some of the previous simplifying hypothesis are relaxed; for example, the assumption of symmetrical spatial wave functions or that of a pure L=0 two-body state. In this second case one has to take into account the tensor and spin-orbit magnetic interactions given in eq. (14) which add to the vector meson mass splitting a term of the form:

$$M_{\mu}^{\nu} \approx -e \langle \hat{e}_1 \hat{e}_2 \rangle \frac{m_u^2}{m_1 m_2} \left[\left(8 P_D (1 - P_D) - 10 P_D \right) \right], \quad (27)$$

where $e = 2\mu_P^2 \langle \frac{1}{r_{12}^3} \rangle$, $\langle \frac{1}{r_{12}^3} \rangle = \langle R_S | \frac{1}{r_{12}^3} | R_D \rangle \sim \langle R_D | \frac{1}{r_{12}^3} | R_D \rangle$ and, finally, P_D is the D-state percentage. If we assume $\langle \frac{1}{r_{12}^3} \rangle$ and P_D to be identical for $K^{*+,0}$ and $\rho^{+,0}$ mesons we obtain, for $[M(\rho^+) - M(\rho^0)] < 0$, a lowering of |a| of about a factor two besides a negative value of b that this is an unphysical conclusion. Therefore we cannot include the term (27) until a fourth reliable meson mass splitting e. g. $[M(\rho^+) - M(\rho^0)]$ is available. However, before arriving to definite conclusions we must take into account that the ρ^0 meson is not at present fully understood in terms of SU(8): this last conclusion is supported by the well known fact that the radiative width of the process $\rho^0 \rightarrow \pi^0 \gamma$ is not correctly reproduced by the model.

TABLE IV - Results obtained for vector and scalar mesons.

Meson wave functions	μ/μ_p	δM_S	δM_C	δM_M
$ \rho^+\rangle = u \bar{d}\rangle \uparrow\uparrow$	1	5/9	2/9	-2/9
$ \rho^0\rangle = (u \bar{u}\rangle - d \bar{d}\rangle) \uparrow\uparrow$	0	5/9	-5/18	5/18
$ K^{*+}\rangle = u \bar{s}\rangle \uparrow\uparrow$	$\frac{1}{3}(2+x)$	5/9	2/9	-2/9 x
$ K^{*0}\rangle = d \bar{s}\rangle \uparrow\uparrow$	$-\frac{1}{3}(1-x)$	2/9	-1/9	+1/9 x
$ \varphi_1\rangle = \frac{1}{\sqrt{3}} (u \bar{u}\rangle + d \bar{d}\rangle + s \bar{s}\rangle) \uparrow\uparrow$	0	4/9	-1/6	$\frac{1}{5}(5+4x^2)$
$ \omega_8\rangle = \frac{1}{\sqrt{6}} [u \bar{u}\rangle + d \bar{d}\rangle - 2 s \bar{s}\rangle] \uparrow\uparrow$	0	1/3	-5/2	$(\frac{5}{9} + \frac{4}{9}x^2)/6$
$ \varphi\rangle = - s \bar{s}\rangle \uparrow\uparrow$	0	2/9	-1/9	+1/9 x ²
$ \omega\rangle = \frac{1}{2} (u \bar{u}\rangle + d \bar{d}\rangle) \uparrow\uparrow$	0	5/9	-5/18	5/18
$ D^{*0}\rangle = c \bar{u}\rangle \uparrow\uparrow$	$\frac{2}{3}(y-1)$	8/9	-4/9	+4/9 y
$ D^{*+}\rangle = c \bar{d}\rangle \uparrow\uparrow$	$\frac{1}{3}(2y+1)$	5/9	2/9	-2/9 y
$ S^{*+}\rangle = c \bar{s}\rangle \uparrow\uparrow$	$\frac{1}{3}(2y+x)$	5/9	2/9	-2/9 y
$ \pi^+\rangle = \frac{1}{2} u \bar{d}\rangle (\uparrow\downarrow - \downarrow\uparrow)$	0	5/9	2/9	5/18
$ \pi^0\rangle = \frac{1}{2} (u \bar{u}\rangle - d \bar{d}\rangle) (\uparrow\downarrow - \downarrow\uparrow)$	0	5/9	-5/18	-5/6
$ K^+\rangle = \frac{ u \bar{s}\rangle}{2} (\uparrow\downarrow - \downarrow\uparrow)$	0	5/9	2/9	2/3 x
$ K^0\rangle = \frac{ u \bar{s}\rangle}{2} (\uparrow\downarrow - \downarrow\uparrow)$	0	2/9	-1/9	-1/3 x
$ \eta\rangle = \frac{1}{6} (u \bar{u}\rangle + d \bar{d}\rangle + s \bar{s}\rangle) (\uparrow\downarrow - \downarrow\uparrow)$	0	4/9	-1/9	-1/3
$ D^0\rangle = \frac{1}{2} (c \bar{u}\rangle (\uparrow\downarrow - \downarrow\uparrow))$	0	8/9	-4/9	-1/3 y
$ D^+\rangle = \frac{1}{2} c \bar{d}\rangle (\uparrow\downarrow - \downarrow\uparrow)$	0	5/9	2/9	+2/3 y
$ S^+\rangle = \frac{1}{2} c \bar{s}\rangle (\uparrow\downarrow - \downarrow\uparrow)$	0	5/9	2/9	2/3 xy

Let us finally note that our results satisfy all inequalities of ref. (6). In addition, we have:

$$M(D^+) - M(D^0) \sim M(D^{*+}) - M(D^{*0}) = (8.2 \pm 1.3) \text{ MeV.}$$

This estimate must be considered not too far from the recent experimental result given in ref. (21) for the first case $M(D^+) - M(D^0) = (5.0 \pm 0.8) \text{ MeV}$. However, we have a discrepancy not easily interpretable in the second case $(M(D^{*+}) - M(D^{*0})) = (2.6 \pm 1.8) \text{ MeV}$.

5. - FINAL REMARKS.

One could believe that the expressions we have derived in the previous sects. do not allow us to give quantitative predictions of the e. m. properties of charmed hadrons until suitable chosen experiments give more precise information on γ or, to be more precise, on the magnetic interaction of the charmed quark. However, a simple look at our results shows that this doubt is probably ungrounded for the e. m. splittings, being very small all the magnetic terms involving charmed really quarks.

Finally, to conclude, let us point out the following remarks:

- a) Being $\gamma \ll 1$, both mass differences and magnetic moments of charmed hadrons change quite drastically from the SU(8) results (6, 12);
- b) Any experimental measurement of e. m. splittings or magnetic moments of charmed hadrons carries important information on the charge of the quark c which is not yet definitively established⁽¹⁸⁾;
- c) As noted at the end of sect. 3, mass differences and magnetic moments of uncharmed hadrons cannot help us to decide between the two fundamental alternatives for the quark charges, the conventional one (Gell-Mann and Zweig fractionally charged quarks) and the Han-Nambu one (integrally charged quarks⁽¹⁹⁾). A choice could be made, in principle, on the basis of the properties of the "charmed" states which have, in the two models, \checkmark definitions but, at present, it is not possible to establish a biunivocal correspondence between the "charmed" states defined in the two models; it will be done only when we have a satisfactory experimental systematics. *v. different*

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