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F. Palumbo and Yu. A. Simonov: SUPPRESSION OF THE FORM FACTOR OF A RELATIVISTIC COMPOSITE SYSTEM DUE TO THE PAULI PRINCIPLE.

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F. Palumbo and Yu.A. Simonov[(x)]: SUPPRESSION OF THE FORM FACTOR OF A RELATIVISTIC COMPOSITE SYSTEM DUE TO THE PAULI PRINCIPLE.

ABSTRACT.

The effect of the Pauli principle on the asymptotics of the form factor of a relativistic composite system has been studied. It has been shown that, contrary to the nonrelativistic case, there is not power suppression factor, but a strong reduction of the coefficient of the leading power of the expansion. The transition from relativistic to nonrelativistic regime has been analyzed.

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In a recent paper the asymptotic behaviour of the elastic e.m. form factor (f. f.) of a many-body nonrelativistic system was investigated[1]. It was found that in the asymptotic regime of $q \longrightarrow \infty$ the Pauli principle brings about a suppression factor $q^{-2K_{min}}$, where K_{min} is the minimum degree in the homogeneous polynominals expansion of the properly antisymmetrized wave function[2]. This result holds true provided in the absence of Pauli principle the f.f. has a power fall off. In a subsequent paper[3] this result was used to predict consequences of the quark statistics for the elastic e.m.f.f. of nuclei, considered as composite systems of quarks. In that paper the whole analysis was done in the non-relativistic framework.

The suppression of the asymptotics of the f.f. in the nonrelativistic region has the same origin as the suppression of the f.f. of a two body system with nonzero angular momentum. The suppression due to angular momentum was suggested by L. Caneschi and M. Gell-Mann[4] in connection with the derivation of the asymptotics of f. f. by Brodsky and Farrar[5, 6]. In the framework of this derivation the coefficient of the leading power in the asymptotic expansion of the f. f. is proportional to the value of the wave function at the orgin $\psi(0)=\int d^4k\ \psi(k)$ which vanishes for zero angular momentum.

The point raised in ref. (3) is that due to Pauli principle nonzero orbital angular momenta must necessarily be introduced to obtain fully antisymmetric wave functions when the number of constituents is large enough. We would like to emphasize that even the Slater determinant containing only radial excitations, that is only the moduli of single particle coordinates or momenta, necessarily contains nonzero orbital angular momenta of pairs of particles.

In the present paper we repeat the analysis given in[1] in a relativistic framework, using the Bethe-Salpeter equation. Our conclusion is that there is not power suppression factor in spin-averaged asymptotic f. f. neither from the orbital angular momentum non from the Pauli principle. There is, however, due to the Pauli principle, a strong reduction of the coefficient of the leading power in the asymptotic expansion of the f. f.

Let us first consider a spin zero bound system of N spinless particles for which the f. f. has the form:

$$F(q)(2p+q)_\mu = \int d^4k_1 \ldots d^4k_N \, \delta(\sum_i k_i - P) \cdot \varphi_P(k_1 \ldots k_N) \; x$$
$$x \prod_{i=1}^{N} G_o(k_i)(2k_1+q)_\mu \quad G_o(k_1+q) \quad \varphi_{P+q}(k_1+q, k_2, \ldots k_N) \tag{1}$$

The asymptotics of the f. f. depends on that of $\varphi_{P+q}(k_1+q, k_2 \ldots k_N)$ for $-q^2 \equiv +Q^2 \longrightarrow +\infty$. We can use the Lorentz transformation to put all the q-dependence in the arguments of the vertex function:

$$\varphi_{P+q}(k_1+q, k_2, \ldots k_N) = \varphi_P(\Lambda_q(k_1+q), \Lambda_q k_2, \ldots \Lambda_q k_N). \tag{2}$$

Here $\Lambda_q(p_o, p_1, p_2, p_3) = (\gamma(p_o - vp_3), p_1, p_2, \gamma(p_3 - vp_o))$ and

$$\gamma = 1 + \frac{Q^2}{2M^2} \quad , \qquad v = \frac{\sqrt{(\gamma^2 - 1)}}{\gamma} . \tag{3}$$

We assume $\vec{q}$ to be along z axis. Note that

$$q_z^2 = \frac{Q^2}{2M}\left(\frac{Q^2}{2M} + 2M\right), \quad q_o = \frac{Q^2}{2M} . \tag{4}$$

In order to obtain the asymptotics of F(q) we iterate N-1 times the Bethe-Salpeter equation for $\varphi_P(\Lambda_q(k_1+q), \Lambda_q k_2 \ldots)$ getting the following expression for F(q):

$$F(q)(2p+q)_\mu = \int d^4k_1 \ldots d^4k_N \, \delta(\sum_{i=1}^{N} k_i - P) \, \varphi_P(k_1 \ldots k_N)$$
$$\cdot \prod_{i=1}^{N} G_o(k_i)(2k_1+q)_\mu \quad G_o(k_1+q) \mathscr{K}(\Lambda(k_1+q), \Lambda k_2, \ldots \Lambda k_N, p_1, \ldots p_N)$$
$$\cdot \prod_{i=1}^{N} G_o(p_i) \quad \overset{+}{\varphi}_P(p_1 \ldots p_N) d^4 p_1 \ldots d^4 p_N \, \delta(\Sigma p_i - P). \tag{5}$$

The connected kernel $\mathcal{K}$ has the following structure:

$$\mathcal{K}(k_1 \ldots k_N, p_1 \ldots p_N) = \prod_{\nu=1}^{N-1} v\left(\sum_i (\alpha_i^\nu k_i - \beta_i^\nu p_i)\right) \cdot$$

$$\cdot \prod_{\nu=1}^{N-2} G_o\left(\sum_i (a_i^\nu k_i - b_i^\nu p_i)\right) \qquad (6)$$

Here v(k) is a two-body interaction kernel which is supposed to be Lorentz invariant and therefore depends only on k^2. Hence the kernel in (5) depends on q in the following way:

$$\mathcal{K}(\Lambda(k_1+q), \Lambda k_2, \ldots \Lambda k_N, p_1, \ldots p_N) \sim$$

$$\sim f\left((k_1+q)^2, k_2^2, \ldots p_1^2, p_N^2, p_i \cdot \Lambda(k_1+q), \ldots, p_j \cdot \Lambda k_j\right). \qquad (7)$$

If Q^2 is much greater than the average single-particle masses and momenta squared[7] i.e. $(\frac{Q}{N})^2 \gg \langle k_i^2 \rangle, m^2$, but $Q^2 \ll 2M^2$, so that $\Lambda k \sim k$,

$$\mathcal{K}(\Lambda(k_1+q), \Lambda k_2, \ldots, \Lambda k_N, p_1 \ldots p_N) \sim Q^{-2\gamma}, \qquad (8)$$

assuming here and in the following power fall off.

The asymptotics of the form factor is then given by

$$F(q) \sim \left(\int \varphi_P(k_1 \ldots k_N) d^4k_1 \ldots d^4k_N \delta(k_1 + \cdots + k_N - P)\right)^2 (Q^{-2\gamma})^2. \qquad (9)$$

The coefficient of $Q^{-2\gamma}$ vanisches if i) the total orbital angular momentum L is different from zero ii) φ_P is antisymmetric due to Pauli principle, that amounts to have internal angular momenta.

We then have a suppression of the asymptotics of the f.f. which is q^{-2L} in case i)[6] and $q^{-2K_{min}}$ in case ii)[8), (1].

However, if $Q^2 \gg 2M^2$

$$\mathscr{K}(\Lambda(k_1+q),\Lambda k_2,..\Lambda k_N,p_1..p_N) \sim Q^{-2\gamma} f(k_{iz}p_{jz},k_{io}p_{jo}), \qquad (10)$$

so that the asymptotic f. f. no longer factorises as in eq. (9)

$$F(Q)=Q^{-2\gamma}\int d^4k_1..d^4k_N\,\delta(k_1+\cdots+k_N-P)\int d^4p_1..d^4p_N\,\delta(p_1+\cdots p_N-P)$$
$$\varphi_P(k_1..k_N)\,\varphi_P(p_1..p_N)\,f\,(k_{iz}p_{jz},k_{io}p_{jo}). \qquad (11)$$

Now both in case i) and ii) the coefficient of $Q^{-2\gamma}$, eq. (11), does not necessarily vanish. In case ii) it does not vanish if the Pauli principle is enforced by means of antisymmetry in the k_{iz} components on in k_i^2. The second possibility is always realized even for zero total orbital angular momentum due to radial excitations. In such a case we do not get an additional power suppression of the asymptotics of the f. f. The coefficient of the leading terms $Q^{-2\gamma}$, however, comes from the radial excitation admixture in the wave function which is usually very small. It follows according to the analysis of ref. (3), that when antisymmetry in the spatial part of the wave function is needed in order to enforce the Pauli principle, the coefficient of the leading term in the asymptotics is strongly reduced. Such a reduction in the coefficient is however, dependent on the dynamics and only in the framework of a specific model a quantitative prediction can be made. Coming now to point i) we notice that there is not suppression in the asymptotics of the spin averaged f. f., the leading contribution coming from the state with zero projection of the arbital angular momentum on the $\underline{q}$ direction (z axis in our notation).

Although this result agrees with eqs. (36) of ref. (9), there seems to be some confusion in the literature, where the statement is made that power suppression in the form factor should arise for states with non-zero angular momenta. Indeed Amati et al. (9) got a suppression factor for the so called invariant form factors in the Scadron expansion [(10)], but the matrix element of the e. m. current between nonzero angular momentum states, which enters the unpolarized cross section is not altogether suppressed and also the f.f. used in the discussion of the quark counting rule is not suppressed, in contrast with the assumption of ref. (6).

The above analysis can be repeated for the case of spin 1/2 constituents without modification as far as orbital angular momentum and Pauli principle are concerned.[*]

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(*) After the present work was completed we were informed that two papers had appeared dealing with the effect of orbital angular momenta in the asymptotics of f. f. : V. L. Chernyak and A. R. Zhitnitzkii, JEPT Letters 55, 544 (1977) and A. I. Vainstein and V. I. Zakharov, Preprint ITEP 21 (1977).

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