

To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-77/41(P)
5 Agosto 1977

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CONDENSATION AND TENSOR BINDING.

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ABSTRACT. -

In infinite nuclear matter with pion condensates or tensor binding both parity and isospin symmetries are broken. Finite nuclei with pion condensates or tensor binding, however, can have definite parity. They cannot have a definite value of isospin, whose average value is of the order of the number of nucleons.

Some years ago it has been suggested that nuclear matter beyond some critical density could be unstable against creation of pion condensates⁽¹⁾(πC), and independently that a tensor potential strong enough could give rise to bound states with novel structures, called tensor binding^(2, 3, 4)(TB).

It has been objected later that πC cannot be relevant for normal nuclei⁽⁵⁾, because in πC parity is broken while normal nuclei have a definite parity. As a consequence one should expect low lying states of opposite parity in finite nuclei, which are not observed. (See ref. (5))

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and Irvine, ref. (1)).

The same argument could apply to TB.

In the present note we will show that finite nuclei with π C or TB can have a definite parity, and that there is no reason to expect almost degenerate levels of opposite parity. Finite nuclei with π C or TB cannot have, however, a definite value of isospin. We will see, with one qualification to be discussed at the end, that nuclei with π C or TB have an average value of the isospin of the order of the number of nucleons. This qualifies such nuclei as abnormal, in the sense that their isospin structure is very much different from that of the nuclei we know, whose isospin is very close to the lowest possible value⁽⁶⁾.

These two features of good parity and high isospin qualify π C or TB as possible final states in ion-ion collisions, provided the Coulomb potential can produce sufficiently high isospin excitations.

We will first consider matter with TB or with a condensate of neutral pions (π^0 C), which are two aspects of the same thing⁽⁷⁾. For the onset of both π^0 C and TB is indeed essential the tensor part of the OPE potential, and both π^0 C and TB give rise to a laminated structure of nuclear matter. Their equivalence will result clearer in the following discussion of parity and isospin. We consider such general properties more reliable than conclusions drawn from detailed calculations. This is because short range effects are known to be very important, and to the extent that they are important they should be properly taken into account. This would require Jastrow correlation functions or Brueckner effective interactions anisotropic and spin-isospin dependent. Some of the short range repulsion is in fact accounted for by the spin-isospin dependent correlation in the z-direction⁽³⁾, so that different (on the average weaker) short range correlations are required with respect to the homogeneous case. Inclusion of such correla-

tions seems at present practically impossible:

As far as TB is concerned we will essentially follow ref. (3). In ref. (3) nuclear matter is described by a Slater determinant of single-particle wave functions

$$\psi_{\underline{k}\lambda} = V^{-1/2} e^{i\vec{k}\cdot\vec{r}} \chi(z - \delta_\lambda) a_\lambda, \quad (1)$$

where the index $\lambda = 1, 2, 3, 4$ stands for spin-up protons, spin-down protons, spin-up neutrons, spin-down neutrons and a_λ is the corresponding spin-isospin state. χ is a periodic real function which introduces a density modulation in the z-direction, taken as the axis of spin quantization. It is defined in such a way that the wave functions (1) are orthonormal. Besides the period 1, χ depends on a parameter α . For small α , χ is approximately a constant, while for large α , χ is a periodically sharply peaked function.

Six configurations of nuclear matter have been considered, differing by the shifts δ_λ assigned at each of them (see⁽⁹⁾ the Table). For

TABLE - Shifts δ_λ defining the 6 configurations of nuclear matter.

i	δ_1	δ_2	δ_3	δ_4
1	0	1/4	1/2	3/4
2	0	1/4	3/4	1/2
3	0	1/2	1/4	3/4
4	0	0	1/2	1/2
5	0	1/2	0	1/2
6	0	1/2	1/2	0

each configuration the function χ has been varied with respect to ℓ and α minimizing a nuclear Hamiltonian with the OBE potentials referred to in ref. (3) as UG1, UG2 and BS. Local minima of the energy per particle have been found in configurations 1, 2, 3, 4, 6 with the UG1 potential

and in configurations 1, 2, 3, 6 with the BS potential at normal nuclear density (no minima have been found with the UG2 potential). The best values of α are such that⁽⁸⁾ $\chi \sim a+b \cos kz$.

We will now evaluate the average value of the π^0 field in each of the six configurations, in the same spirit in which we calculate the potential of a system of electric charges whose distribution has been determined by taking into account only Coulomb forces, i. e. suppressing the degrees of freedom of the time-like photons.

In the nonrelativistic approximation for the nucleons the average value of the π^0 field in the state $|i\rangle$ obeys the equation

$$(\square^2 + m^2)\langle i|\pi^0|i\rangle = \frac{f}{m_\pi} \vec{\nabla} \cdot \langle i|\psi_p^* \vec{\sigma} \psi_p - \psi_n^* \vec{\sigma} \psi_n|i\rangle. \quad (2)$$

For any of the 6 states of nuclear matter considered the righthand side becomes $\frac{f}{m_\pi} \frac{\partial}{\partial z} h_i$, where

$$\begin{aligned} h_1 = h_3 &= \frac{1}{4} \bar{\rho} \left[\chi^2(z) + \chi^2(z - \frac{3}{4}\ell) - \chi^2(z - \frac{1}{4}\ell) - \chi^2(z - \frac{1}{2}\ell) \right] \neq 0, \text{ period } \ell \\ h_2 &= \frac{1}{4} \bar{\rho} \left[\chi^2(z) + \chi^2(z - \frac{\ell}{2}) - \chi^2(z - \frac{1}{4}\ell) - \chi^2(z - \frac{3}{4}\ell) \right] \neq 0, \text{ period } \frac{\ell}{2} \\ h_4 = h_5 &= 0 \\ h_6 &= 1/2 \bar{\rho} \left[\chi^2(z) - \chi^2(z - \frac{1}{2}\ell) \right] \neq 0, \text{ period } \ell \end{aligned} \quad (3)$$

where ℓ is the period of χ and $\bar{\rho}$ is the average nuclear density. The functions h_i are sketched in the Figure. Let us note that in the state 4 the contribution to the energy of the tensor part of the OPE potential vanishes identically, while in the state 5 it is very small⁽⁸⁾, so that these states turn out not to describe a TB, and accordingly they do not generate an average pion field. All the h_i are even or odd under inversions with respect to definite points on the z axis. According to eq.(2) the corresponding average values of the pion field are odd or even with

respect to the same points. The nuclear wave functions 1, 2 and 3, however, have no definite parity with respect to any point, while the nuclear wave function 6 is even with respect to the points $x=y=0$, $z = n \ell / 2$. The structure of the pion field does not contain all the information which is necessary to discuss parity, but direct knowledge of the nuclear wave function is necessary.

Summarizing, the states which are entitled to describe TB generate a π^0 C. On the other hand if a π^0 C is present, there must be a h function which fluctuates, and then different fluctuations of the density of nucleons of different spin-isospin. Note that the total density of matter can fluctuate very little, because the density fluctuations of nucleons of different spin-isospin can approximately compensate with each other. The density of matter is actually a constant in the state 6 with the single-particle wave functions of ref. (2).

π^0 C or TB break parity as a symmetry with respect to any arbitrary point, but can have definite parity with respect to some points, which is all we need for finite nuclei. For finite nuclei, in fact, inversion makes only sense with respect to their c. m. . So if there exist π^0 C or TB with points of symmetry with respect to parity, we can make finite nuclei by taking chunks of infinite nuclear matter with the c. m. coinciding with one of the symmetry points. Such nuclei have definite parity.

We conclude that π^0 C and TB are compatible with parity in finite nuclei provided infinite nuclear matter with π^0 C or TB has points of symmetry with respect to parity. The requirement that such points of symmetry should exist rules out the states 1 to 3 so that only the state 6 is left to describe TB for finite nuclei.

Let us now come to isospin. A condensate of π^0 has no definite iso-

spin (there is no way to couple to a definite isospin only π^0 's) and the average value of T^2 is of the order of the number of π^0 's, which is a finite fraction of the number of nucleons. This corresponds to the fact that the states of nuclear matter 1, 2, 3 and 6 which generate π^0 C, have no definite isospin, and the average value of T^2 is of the order of the number of nucleons, as can be checked from eq. (1). This qualifies these states as abnormal, in the sense specified at the beginning of the paper.

The above considerations can be easily extended to the case of π^\pm C. The qualification mentioned at the beginning is due to the possibility to obtain states of matter of definite and low isospin in presence of π C including nucleons and neutral and charged pions in a coherent state. The number of pions being completely unknown, their phases can in fact be completely determined⁽¹⁰⁾. Whether this can be done without substantially changing the theory is, as far as we know, an open problem.

We would like to thank F. Calogero for reading the manuscript. One of us (F. Palumbo) would also like to thank, Yu. A. Simonov for interesting conversations and A. B. Migdal for kindly explaining his point of view on pion condensation.

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