To be submitted to
Nuovo Cimento

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-77/39(P)
20 Luglio 1977

M. Greco: RADIATIVE DECAYS OF 2^{++} AND 1^{++} MESONS. -

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ABSTRACT. -

Radiative decays of tensor and axial mesons are considered in the framework of a previously proposed scheme, where the basic currents of SU(4) are dominated by vector mesons and exhibit a quark structure asymptotically. Agreement is found with previous SU(3) estimates. The decay rates for $\psi' \to \gamma \chi_{1,2}$ also agree with experiments and non-relativistic estimates. Conversely the $\chi_{1,2} \to \gamma J/\psi$ widths have been found smaller by about a factor of two than those obtained in the n.r. bound state picture. Our results are also compared with those previously obtained for the χ_0 meson.

1. - INTRODUCTION. -

The radiative transitions from J/ψ and ψ' reveal(1) five C = +1 states, which is just the spectrum expected for the $c\bar{c}$ bound system, namely two pseudoscalars η_c and η'_c and three L = 1 states, $J^P = 0^+, 1^+, 2^+$ (hereafter referred as the χ states). However the comparison of experimental data with theoretical estimates for the radiative and hadronic widths in charmonium(2) causes various difficulties(3). The identification of the X(2.83) and $P_c(3.45)$ as η_c and η'_c is particularly troublesome for the very suppressed M1 transition $J/\psi \to \eta_c \gamma$ ($\Gamma_{exp}(J/\psi \to \eta_c \gamma)^{(4)} / \Gamma_{th}(J/\psi \to \eta_c \gamma) <$ < 0.1), the two photon annihilation $\eta_c \to \gamma\gamma$ ($Br_{exp}(\eta_c \to \gamma\gamma)^{(2)} / Br_{th}(\eta_c \to \gamma\gamma) > 5 \pm 2.5$) and the short distance Q.C.D. prediction for $\Gamma(\eta'_c \to \text{all})$. For the ^3P_J states the situation is rather good for the radiative transitions(4) $\psi' \to \gamma \chi_J$, but less satisfactory for the hadronic widths(5), whose absolute magnitudes however cannot be deduced directly from the experiments but have to be extracted from the poorly known branching ratios(6) $B(\psi' \to \gamma \chi_J \to \gamma\gamma J/\psi)$ and rely on the theoretical estimates for the transitions $\chi_J \to \gamma J/\psi$. Very recent data(7) however suggest very large fractions ($\sim 50\%$) for the decays $\chi_J \to \gamma \psi$, in disagreement with charmonium expectations.

In a previous series of papers(8, 9, 10) we have proposed a new scheme for the radiative decays of the old and new mesons, where the basic currents of SU(4) are dominated by vector mesons with appropriate quantum numbers and exhibit a quark structure asymptotically. A dual type

B_5 function is used as a realistic vertex, with suitable analiticity properties in q_1^2 and q_2^2, to extrapolate smoothly the asymptotics of the quark model down to the point $q_{1,2}^2 = 0$, where it is further constrained by low energy theorems. The model gives very good predictions for SU(3) pseudoscalar meson decays[8], including those for which the quark model by itself fails. In addition the suppression mechanism operating in the decays $\varphi \to \eta\gamma$ and $k^* \to k\gamma$ is of the right strength to produce for the η_c meson considerably smaller widths than obtained in the non-relativistic bound-state picture. The results for SU(3) scalar mesons decays[9] also agree with previous estimates based on FESR, whereas the radiative widths for the $\chi_0(3.41)$ meson are predicted about a factor of two smaller than non relativistic calculations, in closer agreement with experiments.

In the present paper we apply the same scheme to study the radiative decays of tensor and axial-vector mesons, namely $T \to \gamma\gamma$, $V \to T(A)\gamma$ and $T(A) \to V\gamma$, having particularly in mind, at the light of the above discussion, the case $T \equiv \chi_2$ and $A \equiv \chi_1$. In absence of low energy theorems, as in the previous treatment of scalar and pseudoscalar mesons, we use various sum rules for both real and virtual photons for constraining the vertex functions and implementing the appropriate large q^2 behaviour implied by the quark current algebra.

The results for SU(3) decays $T \to \gamma\gamma$ agree with previous estimates based on FESR[11], as for the scalar case, and with Renner's results[12] in the framework of tensor meson dominance. On the other hand the radiative transitions involving the $\chi_{1,2}$ mesons agree with non-relativistic calculations[4] and with experiments as far as the decays $\psi' \to \gamma\chi_{1,2}$ are concerned. Conversely our estimates for $\chi_{1,2} \to \gamma J/\psi$ are smaller by about a factor of two than those found in the n. r. bound state picture. The relative ratios of $\Gamma(\chi_J \to \gamma\psi)$ (J = 0, 1, 2) however are consistent with the simple E1 transition rule $\Gamma(\chi_J \to \gamma\psi) \propto k_J^3$.

The paper is organized as follows. In section 2 we define the vertex functions and fix our notations. The complete analysis of both SU(3) and χ states is worked out in sections 3 and 4 for tensors and axial-vectors respectively. Section 5 contains our conclusions.

2. - KINEMATICS AND NOTATIONS. -

The vertex function $T_M^{\mu\nu}(q_1, q_2)$ for a (virtual) photon pair going into a moson M of positive charge conjugation is given by

$$T_M^{\mu\nu}(q_1, q_2) = i\int d^4x\, e^{iQx} \langle M(P) \left| T\left[J^{\mu}(\tfrac{x}{2})\, J^{\nu}(-\tfrac{x}{2})\right] \right| 0 \rangle , \qquad (2.1)$$

where $Q = \frac{1}{2}(q_2 - q_1)$ and $P = (q_1 + q_2)$. Other kinematical variables used are $\nu = PQ$ and $\xi = \nu/Q^2$. The various helicity amplitudes are then defined as

$$T_M^{\lambda_1\lambda_2}(q_1, q_2) = T_M^{\mu\nu}(q_1, q_2)\, \varepsilon_{\mu}^{\lambda_1}(q_1)\, \varepsilon_{\nu}^{\lambda_2}(q_2),\ \lambda_{1,2} = \pm, 0 , \qquad (2.2)$$

and the known invariance principles can be used to reduce the number of the independent ones. For later purpose we also define the absorptive part of the forward ($q_1 = q_3$, $q_2 = q_4$) current-cur rent scattering amplitude as

$$W^{\mu\nu\lambda\sigma}(q_1, q_2) = \frac{1}{2} \sum_n (2\pi)^4 \delta^4(q_1+q_2-P_n)\, T_n^{\mu\nu}(q_1, q_2)\, T_n^{\lambda\sigma *}(q_1, q_2) = \frac{1}{2}\int d^4x\, d^4y\, d^4z \,\cdot$$

$$\cdot\; e^{-\frac{i}{2}\left[(q_2-q_1)(x-y)+(q_2+q_1)z\right]} \left\langle 0 \left| \bar{T}\left[J^\lambda(\tfrac{x}{2})J^\sigma(-\tfrac{x}{2})\right] T\left[J^\mu(\tfrac{y}{2}+z)J^\nu(-\tfrac{y}{2}+z)\right] \right| 0 \right\rangle . \qquad (2.3)$$

As already stated in the introduction the large Q^2 behaviour implied by the quark current algebra will be used to constrain our dual-type veritices. Then the light cone expansion of the time order product of two electromagnetic currents is[13]

$$T\left[J_\mu(\tfrac{x}{2})J_\nu(-\tfrac{x}{2})\right] = -\Big\{ s_{\mu\varrho\nu\sigma}\left[J^\sigma_{Q^2}(\tfrac{x}{2}, -\tfrac{x}{2}) - J^\sigma_{Q^2}(-\tfrac{x}{2}, \tfrac{x}{2})\right] -$$

$$- i\varepsilon_{\mu\varrho\nu\sigma}\left[J^{5\sigma}_{Q^2}(\tfrac{x}{2}, -\tfrac{x}{2}) + J^{5\sigma}_{Q^2}(-\tfrac{x}{2}, \tfrac{x}{2})\right]\Big\} \frac{\partial}{\partial x_\varrho} D_F(x) \;, \qquad (2.4)$$

where $s_{\mu\varrho\nu\sigma} = g_{\mu\varrho}g_{\nu\sigma} + g_{\mu\sigma}g_{\varrho\nu} - g_{\mu\nu}g_{\varrho\sigma}$ and $\frac{\partial}{\partial x_\varrho} D_F(x) = \frac{i}{2\pi^2 i} \times \frac{x_\varrho}{(x^2 - i\varepsilon)^2}$. $J^{(5)\sigma}_{Q^2}(\frac{x}{2}, -\frac{x}{2})$ are bilocal vector and axial-vector currents given in the free quark model by

$$J^{(5)\sigma}_{Q^2}(\tfrac{x}{2}, -\tfrac{x}{2}) = \bar{q}(\tfrac{x}{2})\gamma^\sigma(\gamma_5)Q^2 q(-\tfrac{x}{2}) \;,$$

and the usual currents are the local limits of the bilocal operators. The insertion of (2.4) in (2.1) will then lead to the asymptotic behaviour of the resonance form factors, as discussed below for the various cases of interest.

For a meson M of mass m and spin J decaying into two photons we use the notation $\Gamma(M \to \gamma\gamma) = \frac{1}{(2J+1)} \frac{e^4}{16\pi m} \left(|T^M_{++}|^2 + |T^M_{+-}|^2\right)$, $(T^P_{+-} = T^S_{+-} = 0)$. We also define the coupling constants as $T^P_{++} \equiv m_P^2 g_{P\gamma\gamma}/2$, $T^S_{++} \equiv g_{S\gamma\gamma} m_S^2$, $T^T_{+\pm} \equiv g_{T\gamma_+\gamma_\pm} m_T^2$ for scalar, pseudoscalar and tensor mesons respectively.

3. - TENSOR MESONS. -

There are five independent helicity amplitudes describing the vertex $\gamma(q_1^2)\gamma(q_2^2)T$. The complete kinematics is worked out in detail in ref. (14)(*). In terms of the form factors $F_i(q_1^2, q_2^2)$ (i = = 1, . . , 5) we have :

(*) - We use the same set of invariant amplitudes $B_i^{\mu\nu}$(i=1, . . , 5). The factor $(Q^\alpha Q^\beta E^*_{\alpha\beta})$ in $B_1^{\mu\nu}$ is substituted by $(q_1^\alpha, q_2^\beta E^*_{\alpha\beta})/[(q_1P)(q_2P)]$. After correcting some trivial misprints in the Appendix of ref. (14), the final representation of the helicity amplitudes is given in our eq. (3.1).

$$\frac{T^{++}}{i} = \frac{2}{\sqrt{6}}\frac{1}{m^2}\frac{(\nu^2 - m^2Q^2)^2}{\nu^2 - \frac{1}{4}m^4}F_1 - \frac{1}{\sqrt{6}}(\nu^2 - m^2Q^2)F_2 + \frac{2}{\sqrt{6}}(\frac{1}{4} - \frac{Q^2}{m^2})(\nu^2 - m^2Q^2)F_3 +$$

$$+\frac{1}{\sqrt{6}}\left[\nu^2(\frac{1}{4}m^2+Q^2) - 2m^2Q^4\right]F_4 + \frac{1}{\sqrt{6}}\left[2\nu^2 - m^2(\frac{1}{4}m^2+Q^2)\right]F_5 \quad ,$$

$$\frac{T^{+-}}{i} = (\nu^2 - m^2Q^2)F_2 + (\frac{1}{4}m^2 - Q^2)(\nu^2F_4 - m^2F_5) \quad , \tag{3.1}$$

$$\frac{T^{+o}}{i} = -\frac{1}{\sqrt{2}}\frac{\sqrt{m^2}}{\sqrt{-q_2^2}}(-Q^2 - \nu - \frac{1}{4}m^2)\left[(\frac{1}{2}\nu - Q^2)\nu F_4 + (\nu - \frac{1}{2}m^2)F_5\right] ,$$

$$\frac{T^{o+}}{i} = +\frac{1}{\sqrt{2}}\frac{\sqrt{m^2}}{\sqrt{-q_1^2}}(-Q^2 + \nu - \frac{1}{4}m^2)\left[-(\frac{1}{2}\nu + Q^2)\nu F_4 + (\nu + \frac{1}{2}m^2)F_5\right] ,$$

$$\frac{T^{oo}}{i} = \frac{2}{\sqrt{6}}\frac{1}{\sqrt{q_1^2q_2^2}}\left[(Q^2 + \frac{1}{4}m^2)^2 - \nu^2\right]\left[(Q^2 - \frac{\nu^2}{m^2})F_3 + \nu^2F_4 - m^2F_5\right] .$$

In the case of real photons the two surviving amplitudes T^{++} and T^{+-} have been shown[11, 15] from various sum rules to satisfy the inequality (for SU(3) mesons)

$$|T^{+-}|^2 \gg |T^{++}|^2 \quad , \tag{3.2}$$

namely tensor mesons couple mainly to two photons of opposite helicity. This can be simply tested by looking at the π angular distribution in the process $\gamma\gamma \to T \to \pi\pi$.

Furthermore, the couplings of tensor mesons to two photons can be related to the photonic decays of all other scalar and pseudoscalar mesons. More explicitly the following sum rule has been derived by various authors[16, 17] for $\gamma\gamma \to$ hadrons

$$\int_0^\infty \frac{d\nu}{\nu}\left[\sigma_{++}(\nu) - \sigma_{+-}(\nu)\right] = 0 \quad . \tag{3.3}$$

The saturation of eq. (3.3) with low lying SU(3) resonances of $J^P = 0^-, 0^+$ and 2^+ leads then to the relation

$$\sum_P{}_i \frac{\Gamma_i(P\to\gamma\gamma)}{m_i^3} + \sum_S{}_i \frac{\Gamma_i(S\to\gamma\gamma)}{m_i^3} + 5\sum_T \frac{\Gamma_i(T\to\gamma_+\gamma_+)}{m_i^3} \simeq 5\sum_T \frac{\Gamma_i(T\to\gamma_+\gamma_-)}{m_i^3} \quad . \tag{3.4}$$

Using the quark model relations $\sum_P g^2_{P\gamma\gamma} = 4g^2_{\pi\gamma\gamma}$, $\sum_S g^2_{S\gamma\gamma} = 3g^2_{\sigma\gamma\gamma}/2$ and $\sum_T g^2_{T\gamma\gamma} = 3g^2_{f_o\gamma\gamma}/2 =$

$= \frac{36}{25} g^2_{f\gamma\gamma}$, where σ and f_o are the scalar and tensor SU(3) singlets respectively, and f is the physical f-meson, eq. (3.4) can be rewritten

$$4\left|g_{\pi\gamma\gamma}\right|^2 + 6\left|g_{\sigma\gamma\gamma}\right|^2 + 6\left|g_{f_o\gamma_+\gamma_+}\right|^2 \simeq 6\left|g_{f_o\gamma_+\gamma_-}\right|^2 \quad , \tag{3.5}$$

which also reinforces eq. (3.2). The chiral result $g_{\pi\gamma\gamma} \simeq g_{\sigma\gamma\gamma}$, obtained in ref. (9), leads finally to

$$10\left|g_{\pi\gamma\gamma}\right|^2 + 6\left|g_{f_o\gamma_+\gamma_+}\right|^2 \simeq 6\left|g_{f_o\gamma_+\gamma_-}\right|^2 \quad . \tag{3.6}$$

In contrast to the previously discussed[8, 9] cases of scalar and pseudoscalar mesons, where the canonical trace anomaly of the energy momentum tensor and the PCAC triangle anomaly respectively, were used to constrain the vertex function at $q_1^2 = q_2^2 = 0$, there isn't a similar low energy theorem for tensor mesons. We will then use eq. (3.6) as normalization condition for mass shell photons. As discussed below, our final solution for the various form factors appearing in eqs. (3.1) will also satisfy a duality relation for large virtual photon masses (see eq. (3.21)).

Let us consider now the large Q^2 behaviour of the helicity amplitudes (3.1). Inserting the light cone expansion (2.4) in eq. (2.1), with $M = f_o$, the leading contributions come from the $s_{\mu\varrho\nu\sigma}$ part. The expansion of the bilocals near $x = 0$ gives

$$\langle f_o(P)\left|J^\sigma_{Q^2}\left(\tfrac{x}{2}, -\tfrac{x}{2}\right) - J^\sigma_{Q^2}\left(-\tfrac{x}{2}, \tfrac{x}{2}\right)\right|0\rangle \simeq 2\,i\,x_\varrho \langle f_o(P)\left|\theta^{\varrho\sigma}_{Q^2}(0)\right|0\rangle \quad , \tag{3.7}$$

where in the quark model:

$$\theta^{\varrho\sigma}_{Q^2}(x) = \sum_q \frac{i}{2}\,\bar{q}(x)\,\gamma^\varrho \overleftrightarrow{\partial}^\sigma Q^2 q(x) \quad . \tag{3.8}$$

Extracting from $\theta^{\varrho\sigma}_{Q^2}(x)$ the singlet piece ($\theta^{\varrho\sigma}_{Q^2} \sim \frac{2}{9}\,\theta^{\varrho\sigma}$) and defining $\langle f_o|\theta_{\varrho\sigma}(0)|0\rangle \equiv i\,m^3 e^*_{\varrho\sigma}/\gamma_{f_o}$ we get from (3.7)

$$\langle f_o\left|J^\sigma_{Q^2}\left(\tfrac{x}{2}, -\tfrac{x}{2}\right) - J^\sigma_{Q^2}\left(-\tfrac{x}{2}, \tfrac{x}{2}\right)\right|0\rangle \simeq -\frac{4}{3}\,\frac{m^3}{\gamma_{f_o}}\,x_\alpha e^{*\sigma\alpha} \quad . \tag{3.9}$$

A factor of three has been included to account for colour. Then a rather long calculation leads, in the large Q^2 limit, to

$$T^{\mu\nu}_{f_o}(q_1, q_2) \longrightarrow \frac{8\,i}{3}\,\frac{m^3}{\gamma_{f_o}}\left\{\frac{2}{\sqrt{6}}\,\xi^2\,\frac{g^{\mu\nu}}{m^2} + \frac{e^{*\mu\nu}}{Q^2}\right\} . \tag{3.10}$$

Notice that the $x \simeq 0$ expansion corresponds to the limit $\xi = \nu/Q^2$ fixed and then $\xi \rightarrow 0$. By projecting out eq. (3.10) on the various helicity states we finally obtain for the SU(3) singlet f_o :

$$T^{++} \rightarrow \frac{16\,i}{3\sqrt{6}}\ \xi^2\ \frac{m}{\gamma_{f_o}} \quad , \tag{3.11a}$$

$$T^{+-} \rightarrow \frac{8\,i}{3}\ \frac{m^3}{\gamma_{f_o} Q^2} \quad , \tag{3.11b}$$

$$T^{+o} \rightarrow \frac{8\,i}{3\sqrt{2}}\ \xi\ \frac{m^2}{\gamma_{f_o}\sqrt{-q_2^2}} \quad , \tag{3.11c}$$

$$T^{o+} \rightarrow \frac{-8\,i}{3\sqrt{2}}\ \xi\ \frac{m^2}{\gamma_{f_o}\sqrt{-q_1^2}} \quad , \tag{3.11d}$$

$$T^{oo} \rightarrow -\frac{16\,i}{3\sqrt{6}}\ \frac{m^3}{\gamma_{f_o}\sqrt{q_1^2 q_2^2}} \quad . \tag{3.11e}$$

This result shows explicitly that only the T^{++} amplitude survive in the scaling limit, as also found(14) for the scalar, pseudoscalar and axial mesons (see section 4). Furthermore, the same results holds(18) in the quark parton model (box diagram), suggesting a deeper connection between these two approaches. Indeed, a duality sum rule connecting in the scaling limit the resonance terms to the quark charges has been recently considered in ref. (17).

From eqs. (3.2) and (3.11) it follows the completely opposite rôle played by the T^{++} and T^{+-} amplitudes in the case of real and virtual $\gamma\gamma$ scattering, as already emphasized in ref. (14). Comparing eqs. (3.11) with the asymptotic behaviour of eqs. (3.1) in the same limit ($\xi = \nu/Q^2$ fixed, then $\xi \rightarrow 0$) the following relation is found among the various form factors

$$F_1 \sim \frac{m^2}{4Q^2} F \ , \quad F_2 \sim \frac{F}{m^2} \ , \quad F_3 \sim \frac{F}{Q^2} \ , \quad F_4 \sim \frac{1}{m^2 Q^2} F \ , \quad F_5 \sim -\frac{1}{4Q^2} F \ , \tag{3.12}$$

with

$$F \rightarrow -\frac{8}{3}\ \frac{m^3}{\gamma}\ \frac{1}{Q^4} \quad . \tag{3.13}$$

Furthermore the solution (3.12), assumed valid for all $q^2_{1,2}$, is consistent with the inequality (3.2) and leads also to the right k^3 dependence for the radiative transitions $1^- \rightleftarrows 2^+\gamma$. In fact, from eqs. (3.1) and (3.12) we get at $q^2_{1,2} = 0$

$$T^{++}_{\gamma\gamma} = \frac{1}{\sqrt{6}}\ Q^2 F \ , \qquad T^{+-}_{\gamma\gamma} = Q^2 F \ , \tag{3.14}$$

to be compared with (3.2).

As a specific form for $F(q_1^2, q_2^2)$ we will use in the following the dual vertex introduced in the previous treatments of pseudoscalar and scalar meson decays, which describes very succesfully the radiative decays of both the old and new mesons. We have

$$F(q_1^2, q_2^2) = kB_5\left[1-\alpha(q_1^2), \gamma, \gamma, 1-\alpha(q_2^2), \beta\right] = kB\left[1-\alpha(q_1^2), \gamma\right]B\left[1-\alpha(q_2^2), \gamma\right] \cdot$$
$$\cdot\ {}_3F_2\left[1-\alpha(q_1^2), 1-\alpha(q_2^2), 2\gamma-\beta; 1+\gamma-\alpha(q_1^2), 1+\gamma-\alpha(q_2^2); 1\right], \qquad (3.15)$$

where $\alpha(q_{1,2}^2)$ are the vector meson Regge trajectories, β and γ are two fixed parameters and k is a normalization constant. In more detail β controls the leading power behaviour in the large Q^2 limit, i.e. $F(q_1^2, q_2^2) \to (1/Q^2)^\beta$, while γ gives the q^2 dependence of the transition form factors, namely $(q_1^2 - m_n^2)F(q_1^2, q_2^2) \to (1/q_2^2)^\gamma$. In the case of pseudoscalar and scalar mesons we found[8, 9] $\beta = 1$, $\gamma = 3/2$ and $\beta = 2$, $\gamma = 5/2$ respectively.

From eq. (3.13) it follows: $\beta = 2$. Furthermore the value $\gamma = 5/2$, in exact analogy with the scalar case, leads to good agreement with previous estimates of $\Gamma(f \to \gamma\gamma)$, and gives as well a nice description of the transitions involving the new χ_2 mesons. From the large Q^2 behaviour of (3.15) and comparing with (3.13) we get[(*)]

$$k = -\frac{3}{2\pi}\left(\frac{8}{3}\right)^3 \alpha'^2 m^2 \left(\frac{m}{\gamma_{f_o}}\right), \qquad (3.16)$$

and $(\alpha' m^2 \simeq 3/2)$

$$T^{++}_{\gamma\gamma} = \frac{1}{\sqrt{6}}\, T^{+-}_{\gamma\gamma} = 8\,(m/\gamma_{f_o}). \qquad (3.17)$$

Inserting (3.17) in (3.6) we finally obtain

$$\frac{\gamma_{f_o}^2}{4\pi} = \frac{8}{m^2\pi}\,\frac{1}{g^2_{\pi_\sigma\gamma\gamma}} \simeq \frac{4 f_\varrho^2}{4\pi}, \qquad (3.18)$$

which, for the physical f meson $(g^2_{f\gamma\gamma} = \frac{25}{24}\, g^2_{f_o\gamma\gamma})$, leads to

$$\Gamma(f \to \gamma\gamma) = \frac{70}{36}\left(\frac{m_f}{m_\pi}\right)^3 \Gamma(\pi \to \gamma\gamma) \simeq 12.8 \text{ keV}. \qquad (3.19)$$

The results (3.18-3.19) agree with those found by Renner[12] in the framework of tensor meson dominance. Eq. (3.19) is also consistent with previous estimates[11] based on FESR.

A rather interesting consequence of eq. (3.18) is obtained comparing the scaling limit of the leading helicity amplitudes for tensor and scalar mesons. In the latter case one obtains[9, 10]

(*) - See ref. (9) for a discussion on the limiting procedure for (3.15).

$$T_{\sigma}^{++} \to \frac{8}{3} f_{\varepsilon}\, i\, \xi^{2} \ , \qquad (3.20)$$

which compared with eqs. (3.11a - 3.18), gives ($f_{\varepsilon} \simeq f_{\pi}$[9])

$$T_{\sigma}^{++} \simeq T_{f_o}^{++} \ . \qquad (3.21)$$

This result, together with the previous observation that s-channel resonance contributions to virtual photon-photon scattering survive in the scaling limit for those helicity amplitudes which also scale in quark parton model (box diagram), strongly suggest a kind of duality between the light cone algebra and resonance saturation. This has been discussed in detail in ref. (17) for the case of scalar and pseudoscalar mesons. Eq. (3.21) and the following (4.4) suggest the duality relation to work at a rather local level, the various resonances averaging the two scaling functions of the parton model which, as well known, are proportional to the fourth power of the quark charges.

We would like to discuss now the radiative transitions involving the new $c\bar{c}$ $\chi_2(3.55)$ meson. In particular we will be concerned with the $\psi' \to \gamma\chi_2$ and $\chi_2 \to \gamma J/\psi$ decays. The knowledge of the absolute magnitude is quite important, as already said in the introduction, for extracting the total χ_2 width from the cascade decays $\psi' \to \gamma\chi_2 \to \gamma\gamma J/\psi$ and the comparison with charmonium predictions. In the case of the scalar $\chi_0(3.41)$ we have found[9] decay rates smaller by a factor of about three than those obtained in the non-relativistic calculations of Eichter et al.[4]. In the present case, as well as for the $\chi_1(3.51)$ discussed below, we find agreement for $\psi' \to \gamma\chi_J$, whereas a reduction factor is obtained for $\chi_J \to \gamma\psi$, similar to the one occuring for the χ_o meson.

In complete analogy with the SU(3) case, the various form factors are related by eqs. (3.12-3.13), with $F(q_1^2, q_2^2)$ (eq. 3.15) dominated now by the ψ family ($\alpha_{\psi}(0) \simeq 3/2$, $1/\alpha'_{\psi} \simeq 4\ \mathrm{GeV}^2$). Using the same values $\beta = 2$ and $\gamma = 5/2$ the overall normalization is fixed by imposing in the scaling limit

$$T_{\chi_2}^{++} = T_{\chi_o}^{++} \ , \qquad (3.22)$$

as found in eq. (3.21) for the SU(3) case. Because of the rather small contribution of the charm sector to the sum rule (3.3) there is no reason for having eq. (3.4) satisfied by $c\bar{c}$ states alone. Therefore eq. (3.22) appears to be the only way to fix the relative normalization within the $c\bar{c}$ family. From the results of ref. (9),

$$T_{\chi_o}^{++}(q_1^2, q_2^2) = i\left\{\frac{\nu^2}{m_{\chi_o}^2} - 2Q^2 + \frac{1}{4} m_{\chi_o}^2\right\} k_o B_5(1-\alpha(q_1^2), \gamma, \gamma, 1-\alpha(q_2^2), \beta) \ , \qquad (3.23)$$

where $k_o = \frac{1}{\pi}(\frac{2}{3})^3 \alpha'^2_{\psi} m_{\chi}^2 f_{\chi}$, we find for the normalization constant $k \equiv k_2$ appearing in (3.15)

$$k_2 = \frac{\sqrt{6}}{2}\left(\frac{m_{\chi_2}}{m_{\chi_o}}\right)^2 k_o \ . \qquad (3.24)$$

For the two photon annihilation of the χ_2 meson we obtain

$$\left| T^{+-}_{\chi_2}(0,0)\right| = \sqrt{6}\left|T^{++}_{\chi_2}(0,0)\right| = \frac{1}{\sqrt{6}}\left(\frac{m_{\chi_2}}{m_{\chi_o}}\right)^4 \left|T^{++}_{\chi_o}(0,0)\right| , \qquad (3.25)$$

and therefore

$$\Gamma(\chi_2 \rightarrow \gamma\gamma) = \frac{1}{5}\ \frac{7}{36}\left(\frac{m_{\chi_2}}{m_{\chi_o}}\right)^7 \Gamma(\chi_o \rightarrow \gamma\gamma) \simeq 0.1 \text{ keV} , \qquad (3.26)$$

where the previous estimate[9] $\Gamma(\chi_o \rightarrow \gamma\gamma) \simeq 2.1$ keV has been used. Notice that, as in the case of the f meson, the main contribution comes from the helicity amplitude $T^{+-}_{\chi_2}$, in contrast to the χ_o meson.

As far as the $\psi' \rightarrow \gamma\chi_2$ transition is concerned the set of relations (3.12) is such to account for the right dependence on the photon energy k_γ. We find

$$\sqrt{6}\left|T^{++}_{\chi_2}\right| \simeq \sqrt{2}\left|T^{+o}_{\chi_2}\right| \simeq \left|T^{--}_{\chi_2}\right| \simeq (1+m^4_{\psi'}/m^4_{\chi_2}) \frac{(m^2_{\psi'} - m^2_{\chi_2})}{(m^2_{\psi'} - m^2_{\chi_o})}\left|T^{++}_{\chi_o}\right|\left(\frac{k_2}{k_o}\right) , \qquad (3.27)$$

where next to leading terms in $(k_{\gamma_i}/m_{\chi_i})$ $(i = 0, 2)$ have been neglected. Finally

$$\Gamma(\psi' \rightarrow \gamma\chi_2) \simeq 10\left(\frac{k_{\gamma_2}}{k_{\gamma_o}}\right)^3 \Gamma(\psi' \rightarrow \gamma\chi_o) , \qquad (3.28)$$

and,using $\Gamma(\psi' \rightarrow \gamma\chi_o) \simeq 11$ keV[9],

$$\Gamma(\psi' \rightarrow \gamma\chi_2) \simeq 16 \text{ keV} . \qquad (3.29)$$

This result leads to $Br(\psi' \rightarrow \gamma\chi_2) \simeq 0.07$, in excellent agreement with the experimental value[6,7] (0.08 ± 0.03). For comparison the non relativistic calculations by Eichten et al.[4] give $Br(\psi' \rightarrow \gamma\chi_2) \simeq 0.045$. Furthermore eq. (3.28) differs by only a factor of two from the simple E1 transitions formula[19] for $\psi' \rightarrow \gamma\chi_J$

$$\Gamma_J(E1) \propto (2J+1)\, k^3_{\gamma_J} . \qquad (3.30)$$

The transition rate for $\chi_2 \rightarrow \gamma J/\psi$ is obtained similarly. One gets eq. (3.27) again $(m_{\psi'} \rightarrow m_\psi)$ and finally

$$\Gamma(\chi_2 \rightarrow \gamma J/\psi) \simeq 1.5\left(\frac{k_{\gamma_2}}{k_{\gamma_o}}\right)^3 \Gamma(\chi_o \rightarrow \gamma J/\psi) , \qquad (3.31)$$

and using the result[9] $\Gamma(\chi_o \rightarrow \gamma J/\psi) \simeq 30$ keV,

$$(\chi_2 \to \gamma J/\psi) \simeq 130 \text{ keV} . \tag{3.32}$$

For comparison the E1 transition formula for $\chi_J \to \gamma\psi$ reads

$$\Gamma_J (E1) \propto k_J^3 , \tag{3.33}$$

in rather good agreement with (3.31). The absolute rate however differs from the value $\Gamma(\chi_2 \to \gamma J/\psi) \simeq 300$ keV found in the non-relativistic calculation of ref. (4).

To summarize, our analysis of tensor meson decays gives a fairly good description of both SU(3) and $c\bar{c}$ mesons in agreement with earlier results for $f \to \gamma\gamma$ and consistently with the simple E1 transition formulae for $\psi'(\psi) \rightleftarrows \chi_J\gamma$.

4. - AXIAL-VECTOR MESONS. -

Then are three independent helicity amplitudes in the coupling of 1^{++} mesons to two virtual photons. Defining the invariant amplitude as(14)

$$T^{\mu\nu} = i\frac{m}{2}\left(\frac{q_1\varepsilon^*}{q_1P} + \frac{q_2\varepsilon^*}{q_2P}\right)\varepsilon^{\mu\nu\varrho\sigma}q_{1\varrho}\, q_{2\sigma}\, F_1(q_1^2, q_2^2) + i\left\{(Pq_2)Q^\nu - (Qq_2)P^\nu\right\} \cdot$$
$$\cdot \varepsilon^{\mu\varrho\sigma\tau}P_\varrho Q_\sigma e^*_\tau F_2(q_1^2, q_2^2) + i\left\{(Pq_1)Q^\mu - (Qq_1)P^\mu\right\}\varepsilon^{\nu\varrho\sigma\tau}P_\varrho Q_\sigma e^*_\tau F_3(q_1^2, q_2^2) , \tag{4.1}$$

the helicity amplitudes are then given by

$$T^{++} = \frac{\nu^2 - m^2Q^2}{\nu^2 - m^4/4}\ \nu F_1(q_1^2, q_2^2) , \tag{4.2a}$$

$$T^{+o} = \frac{1}{\sqrt{-q_2^2}}\left(\nu + Q^2 + \frac{m^2}{4}\right)(\nu^2 - m^2Q^2)\, F_2(q_1^2, q_2^2) , \tag{4.2b}$$

$$T^{o+} = \frac{1}{\sqrt{-q_1^2}}\left(-\nu + Q^2 + \frac{m^2}{4}\right)(\nu^2 - m^2Q^2)\, F_3(q_1^2, q_2^2) . \tag{4.2c}$$

The amplitude T^{++} correctly vanishes for $q_1^2 = q_2^2 = 0$, and is related to the analogous amplitude for pseudoscalars into two photons. The scaling properties are also smaller, namely T^{++}_{Ax} and T^{++}_{PS} both scale, the leading common behaviour coming from the $\varepsilon_{\mu\varrho\nu\sigma}$ part of the light cone expansion (2.4). The same techniques used previously in the tensor case give in fact the A_1 meson

$$T^{++} \to 2\xi\frac{m_A}{f_A} , \tag{4.3}$$

,

where f_A is defined as $\langle A_1 | J_{5\sigma}(0) | 0 \rangle = e^*_\sigma m^2_A / f_A$. Recalling the similar result one obtains[10] for the pion, i.e. $T^{++}_\pi \rightarrow 2\xi f_\pi$, the combined Weinberg sum rule $m^2_A/f^2_A + f^2_\pi = m^2_\varrho / f^2_\varrho$ and KSFR relation $m^2_\varrho = 2 f^2_\pi f^2_\varrho$ lead to

$$T^{++}_\pi = T^{++}_{A_1} \quad , \tag{4.4}$$

in the scaling limit. This result is quite similar to eq. (3.21) and, as discussed in the previous section, supports a duality connection between light cone algebra and resonance saturation for virtual γ-γ scattering.

From eq. (4.4) one is naturally led to identify the form factor $F_1(q^2_1, q^2_2)$ appearing in (4.2a) with $F_\pi(q^2_1, q^2_2)$ as given in ref. (8), with the obvious substitution $f_\pi \rightarrow m_A/f_A$. Similarly, in the case of the $\chi_1(3.51)$ meson, the amplitude T^{++} is related to the corresponding one for the η_c meson.

The other two amplitudes T^{+o} and T^{o+} have been studied similarly in the light cone limit. The situation however is slightly more complicated for the fact that T^{+o} and T^{o+} get contributions also from the $s_{\mu\varrho\nu\sigma}$ part of the light cone expansion of T(JJ) (eq. 2.4). One finds in fact

$$T^{o+} \rightarrow -\frac{1}{2\sqrt{-q^2_2}} \frac{m_A}{f_A} - \frac{1}{\sqrt{-q^2_2}} \xi^2 g_A(0) \ , \tag{4.5}$$

where $g_A(0)$ is defined as $\langle A(p) | \left[J_{\sigma Q^2}(\frac{x}{2}/-\frac{x}{2}) - J_{\sigma Q^2}(-\frac{x}{2}/\frac{x}{2}) \right] | 0 \rangle \underset{x\sim 0}{\simeq} \varepsilon_{\sigma\alpha\beta\gamma} p^\alpha x^\beta e^\gamma g_A(0)$, and similarly for T^{o+}.

The above results lead to a two-component structure for the form factors $F_{2,3}(q^2_1, q^2_2)$ in eqs. (4.2) correspondingly to the $\varepsilon_{\mu\varrho\nu\sigma}$ and $s_{\mu\varrho\nu\sigma}$ components in the asymptotic expansion (2.4). The former can be related then to $F_1(q^2_1, q^2_2)$ through eqs. (4.3) and (4.5), with the usual smoothness assumption in $q^2_{1,2}$, while the unknown quantity $g_A(0)$ on the contrary makes the normalization of the latter component undertermined.

In practice for the only case of interest, namely the radiative transitions involving the $\chi_1(3.51)$ state, an estimate can be obtained in the spirit of local duality by demanding asymptotically:

$$(T^{+o})_{1^{++}} \underset{\xi^2=1}{\simeq} (T^{+o})_{2^{++}} \ . \tag{4.6}$$

Then we obtain for the decays involving the χ_1 meson

$$\frac{|T^{+o}(\psi' \rightarrow \gamma\chi_1)|}{|T^{++}(\psi' \rightarrow \gamma\chi_o)|} \simeq \frac{2(m^2_{\psi'} - m^2_{\chi_1})}{(m^2_{\psi'} - m^2_{\chi_o})} \ , \tag{4.7}$$

and similarly

$$\frac{|T^{+o}(\chi_1 \to \gamma\psi)|}{|T^{++}(\chi_o \to \gamma\psi)|} \simeq 2 \frac{(m^2_{\chi_1} - m^2_{\psi})}{(m^2_{\chi_o} - m^2_{\psi})} . \quad (4.8)$$

These results finally lead to

$$\Gamma(\psi' \to \gamma\chi_1) \simeq 4(\frac{k_{\gamma_1}}{k_{\gamma_o}})^3 \, \Gamma(\psi' \to \gamma\chi_o) , \quad (4.9)$$

and

$$\Gamma(\chi_1 \to \gamma\psi) \simeq 1.4 \, (\frac{k_{\gamma_1}}{k_{\gamma_o}})^3 \, \Gamma(\chi_o \to \gamma\psi) , \quad (4.10)$$

in fair agreement with the non relativistic E1 transition formulae (3.30) and (3.33). The absolute magnitude of (4.10) is however a factor of two smaller than the non relativistic calculation(4), as for the case of the χ_2 meson, when one previous estimate for the χ_o decay is taken into account. We obtain in fact from eqs. (4.9 - 4.10) and from ref. (9)

$$B(\psi' \to \gamma\chi_1) \simeq 0.06 , \quad \Gamma(\chi_1 \to \gamma\psi) \simeq 100 \text{ keV} , \quad (4.11)$$

to be compared with the experimental ratio(6, 7) $B(\psi' \to \gamma\chi_1) = (0.09 \pm 0.03)$ and the non-relativistic results by Eichten et al.(4) $B(\psi' \to \gamma\chi_1)$ 0.05 and $\Gamma(\chi_1 \to \gamma\psi)$ · 230 keV.

Notice that both T^{++} and the "$\varepsilon^{\mu\varrho\nu\sigma}$ component" of T^{+o} give negligeable contributions to the above transitions. This comes about for to the correlation between $T^{++}_{\chi_2}$ and $T^{++}_{\eta_c}$ and the corresponding weak decay $\psi' \to \gamma\eta_c$(8), together with the much smaller phase space available in the reaction $\psi' \to \gamma\chi_1$ compared with $\psi' \to \gamma\eta_c$.

5. - CONCLUSIONS. -

We have considered in the present paper the radiative decays of tensor and axial mesons, along the same lines of a previous treatment of pseudoscalar and scalar meson decays. The asymptotics of the quark current algebra is implemented by various sum rules for real and virtual photons to constrain the dual meson-photon-photon vertices. Our result for $\Gamma(f \to \gamma\gamma)$ agrees with previous FESR estimates. The transition rates obtained for $\psi' \to \gamma\chi_{1,2}$ agree with experiments and with non relativistic estimates, while those for $\chi_{1,2} \to \gamma J/\psi$ are smaller by a factor of two. The relative ratios $\Gamma(\chi_J \to \gamma\psi)$ however, are consistent with the simple E1 transition rule.

Considering also our previous results for pseudoscalar and scalar particles, the present scheme gives a unified relativistic description of radiative decays of the old and new mesons, with a particularly simple mechanism to account for the deviations from exact symmetry. In this respect, and because of the phenomenological success of the model, it is really remarkable the level of accuracy of the dual vertex in extrapolating the asymptotics of the quark current algebra to the small q^2 region. Finally our results give strong support to the idea of a duality relation between the light cone algebra and resonance saturation in virtual photon-photon scattering.

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