

To be submitted to
Nuovo Cimento

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-77/27(P)
6 Giugno 1977

C. Guaraldo, R. Scrimaglio, F. Balestra, R. Garfagnini, R. Mach,
G. Piragino and M. G. Sapozhnikov: THE PION-INDUCED
KNOCK-OUT REACTIONS ON ^4He .

C. Guaraldo, R. Scrimaglio, F. Balestra^(x), R. Garfagnini^(x), R. Mach^{(+)(o)}, G. Piragino^(x) and M. G. Sapozhnikov⁽⁺⁾; THE PION-INDUCED KNOCK-OUT REACTIONS ON ^4He .

ABSTRACT. - New experimental values of the ratio $\sigma(^4\text{He}(\pi^+, \pi^+ p)^3\text{He}) / (\sigma(^4\text{He}(\pi^+, \pi^+ n)^3\text{He}) + \sigma[^4\text{He}(\pi^+, \pi^0 p)^3\text{He}])$ are given at $E_{\pi}^{\text{lab}} = 110$ and 160 MeV. This ratio as well as the total and differential cross sections of pion-induced knock-out reactions on ^4He are discussed in terms of simple theoretical models.

1. - INTRODUCTION.

The study of the $(\pi, \pi N)$ reactions in the region of Δ_{33} -resonance has a long history. Experimentally best known are the excitation functions for the reactions $^{12}\text{C}(\pi^{\pm}, \pi N)^{11}\text{C}_{\text{bound}}$, new measurements of which⁽¹⁾ invoked a considerable theoretical activity^(2,3). It is well established, that the ratio

$$R_C = \frac{\sigma(^{12}\text{C}(\pi^-, \pi^- n)^{11}\text{C})}{\sigma(^{12}\text{C}(\pi^+, \pi^+ n)^{11}\text{C}) + \sigma(^{12}\text{C}(\pi^+, \pi^0 p)^{11}\text{C})} \quad (1)$$

depends rather strongly on the energy, reaching the value $R_C \approx 1.5$ near the Δ_{33} -resonance position, whereas a value near 3 is expected on the basis of the plane wave impulse approximation (PWIA). This apparent conflict indicates a remarkable role of the initial and/or final state interaction in pion-induced knock-out reactions on carbon. Using a semiclassical model, the authors of refs. (2, 3) succeeded to show that, first of all, the final state interaction of the outgoing nucleon is responsible for the deviations of the ratio R_C from the PWIA value.

Recently, new experimental data have been published⁽⁴⁾ on differential and total cross sections

(x) Istituto di Fisica dell'Università di Torino, and Istituto Nazionale di Fisica Nucleare, Sezione di Torino.

(+) Joint Institute for Nuclear Research, Dubna (USSR).

(o) On leave from Institute of Nuclear Physics, Rež-Prague, Czechoslovakia.

of the reactions ${}^4\text{He}(\pi^+ \pi^+ p) {}^3\text{H}$, ${}^4\text{He}(\pi^+ \pi^+ n) {}^3\text{He}$ and ${}^4\text{He}(\pi^+ \pi^0 p) {}^3\text{He}$ at $E_\pi^{\text{lab}} = 110$ and 160 MeV. In this paper, it will be shown that, even in the case of such a light nucleus as ${}^4\text{He}$, PWIA provides poor description of the knock-out reactions. Both the shape of the experimental cross sections of (π , π N) reactions and the ratio

$$R_{\text{He}} = \frac{\sigma({}^4\text{He}(\pi^+ \pi^+ p) {}^3\text{He})}{\sigma({}^4\text{He}(\pi^+ \pi^+ n) {}^3\text{He}) + \sigma({}^4\text{He}(\pi^+ \pi^0 p) {}^3\text{He})} \quad (2)$$

turn out to be quite different than those predicted by PWIA.

We discuss a simple theoretical model which incorporates the three-nucleon-exchange mechanism in the plane wave impulse approximation, in addition to the nucleon one. By other words, we use an antisymmetrical (in nucleon variables) final state wave function in evaluating the differential and total cross sections of the knock-out reactions. The model, similar in spirit to that of refs. (11, 12), enables us to explain qualitatively the main features of the knock-out reactions on ${}^4\text{He}$.

The paper is organized as follows. In Sect. 2, the experimental values of the ratio R_{He} at $E_\pi^{\text{lab}} = 110$ and 160 MeV are given. The experimental procedure and the method of obtaining R_{He} are described in some details. Sect. 3 is devoted to the discussion of the theoretical basis of the exchange model, that have been used in the present analysis. Comparison of the experimental and theoretical results, as well as the discussion of various aspects of the PWIA calculations, are given in Sect. 4. The main conclusions are summarized in Sect. 5.

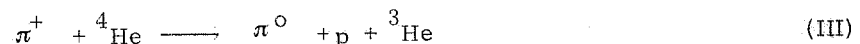
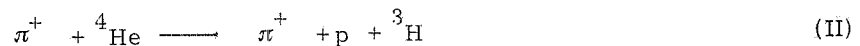
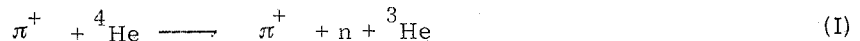
2. - EXPERIMENTAL PROCEDURE AND THE RATIO R_{He}

Since the present experimental results for the ratio of pion-induced nucleon knock-out cross sections are the first one concerning few nucleon systems, we will describe the experimental procedure in some details.

We have already presented in two previous papers⁽⁴⁾ the results of the π^+ inelastic interaction and absorption on ${}^4\text{He}$, at 110 and 160 MeV, in the angular range $10^\circ + 180^\circ$. The elastic scattering component of the data has been presented at the Los Angeles Conference⁽⁵⁾ and the elastic cross sections are in agreement with the results of the Dubna-Torino Collaboration^(6, 7).

The experimental apparatus consisted in a diffusion cloud chamber filled with helium at 15 atm and placed in a magnetic field. The apparatus and the experimental technique have been extensively described elsewhere⁽⁸⁾. In the experiment has been used the LEALE positive pion beam of the Laboratori Nazionali di Frascati⁽⁹⁾, whose energy had a large spread around the mean value of 140 MeV. The experimental results have therefore been grouped in the two energy intervals 110 ± 12 MeV and 160 ± 18 MeV. Determination of the contamination of the pion beam with other particles was accomplished by means of differential and integral range-energy measurements. The ($e^+ + \mu^+$) contamination was found to be about 12% .

In this paper the three π^+ inelastic interactions:



are investigated in detail.

The reactions (I) and (III) appear as two-prongs events. The reaction (II) manifests itself as a three-prongs star. A double scanning of some of the films has been performed in order to measure the scanning efficiency. As far as concerns two-prongs events, the efficiency turned out to be about 97% and for three-prongs events was about unity.

The reactions have been assigned to the above inelastic channels according to the ionization of the tracks and verifying, on the ground of kinematical considerations, the assignment of the

reconstructed events. The accuracy on the measurement of the scattering angle turned out to be better than 1° . The accuracy on the measurement of the curvature radius depends upon the value of the radius itself and upon length and spatial direction of the track. In our case, the resulting accuracy on the energy was about 3% for tracks longer than 20 cm and about 5% for tracks 10 to 20 cm long. The accuracy on the measurement of range turned out to be better than 1mm.

The separation between the events of the inelastic channel (I) and the events of the elastic channel



has been achieved by calculating, with the kinematics of the elastic reaction, the scattering angle and the energy of the recoiled α -particle, on the basis of the measured values for the incoming pion energy and the scattered pion angle. The events have been assigned to the inelastic channel (I) whenever the angle and the range of the heavy particle were different from the calculated elastic values by more than $\pm 5^\circ$ and ± 5 mm, respectively.

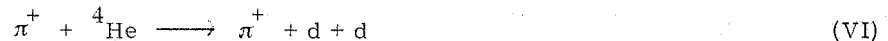
Moreover, ionization criteria have been used to discriminate reaction (I) from the reaction



and reaction (I) against reaction (III).

However, it is not possible to select, with our apparatus, reaction (III) from reaction (V).

The identification of reaction (II) has been made particularly easy by the characteristic ionization of the outgoing prongs. Three-prongs reactions of the type



might be confused with reactions (II). However, on account of the small binding energy of the deuteron, we can assume the probability of occurrence of these last reactions to be negligible.

The total cross section for single reactions has been evaluated by counting the events and the total π -meson track length within a rectangular fiducial region, where the conditions for detecting particles were most favourable. The total cross sections have been computed with the aid of the following formula⁽¹⁰⁾

$$\sigma_{\text{tot}} = N\tau / L n_{\text{He}} (1 - q) s \quad (3)$$

where N is the number of occurrences of a given reaction in the prefixed region; L is the total track length of the incoming pions; n_{He} is the number of ${}^4\text{He}$ nuclei/cm³ in the sensitive volume of the chamber (calculated at the effective pressure of 15 atm and at the middle plane of the sensitive volume temperature of -55°C); q is the total ($e^+ + \mu^+$) contamination of the pion beam; s is the scanning efficiency; τ is a coefficient which takes into account scanning losses for particular angles of the prongs ($\tau=1$ in our experiment).

The total cross sections relative to the three considered inelastic processes, at the two energies of the experiment, are reported in Table I.

TABLE I - Experimental total inelastic cross sections and R_{He} ratios.

T_π (MeV)	$\sigma_{\pi p}^+ {}^3\text{He}$ (mb)	$\sigma_{\pi n}^+ {}^3\text{He}$ (mb)	$\sigma_{\pi p}^0 {}^3\text{He}$ (mb)	R
110 ± 12	84.6 ± 7.3	38.8 ± 3.0	22.3 ± 3.7	1.4 ± 0.2
160 ± 18	102.3 ± 7.6	52.7 ± 5.0	48.6 ± 5.3	1.0 ± 0.1

The error quoted are statistical only. In the same table, are also reported the corresponding R_{He} ratios, defined by equation (2). In the table, we denoted, for instance,

$$\sigma_{\pi^+ p}^+ {}^3H \equiv \sigma \left[{}^4He (\pi^+, \pi^+ p) {}^3H \right] \quad (4)$$

In computing total cross sections, a lower tail at 10 degrees has been considered for all reactions. However, as far as concerns the R_{He} ratio for quasi-elastic knock-out reactions, it can be shown that its value is quite insensitive to the choice of the lower limit of integration, both in the plane wave impulse approximation and in the PWIA plus the three-nucleon-exchange mechanism corrections.

3. - THE MODEL OF THE KNOCK-OUT REACTION

As can be seen from Table I, simple PWIA fails to reproduce the experimental values of the ratio R_{He} . Moreover, the following ratios hold for the cross sections:

$$\sigma_{\pi^+ p}^+ {}^3H : \sigma_{\pi^+ n}^+ {}^3He : \sigma_{\pi^0 p} {}^3He = 9 : 1 : 2, \quad (5)$$

if the P_{33} dominance is assumed in PWIA. Therefore, the total cross section of the reaction (II) is predicted to be one half of $\sigma_{\pi^0 p} {}^3He$, whereas the experiment tells us that $\sigma_{\pi^0 p} {}^3He < \sigma_{\pi^+ n}^+ {}^3He$, in the resonance region. It is evident that some step beyond PWIA must be made, in order to solve the mentioned discrepancies.

In derivation of our very simple model, a fully antisymmetrized wave function of the final state was used and the effects connected with Pauli principle were studied. As in PWIA, the distortion of the incoming and outgoing waves was completely neglected. We used the approximate expressions for the pion-nucleus reaction matrix T

$$\langle f | T | i \rangle (2\pi)^3 \delta(\vec{P}_f - \vec{P}_i) = 4 \langle f | t_{\pi N}(E) | i \rangle. \quad (6)$$

Introducing the relative coordinates

$$\begin{aligned} \vec{\xi}_1 &= \vec{r}_4 - \vec{r}_3 & \vec{\xi}_3 &= \frac{\vec{r}_4 + \vec{r}_3 + \vec{r}_2}{3} - \vec{r}_1 \\ \vec{\xi}_2 &= \frac{\vec{r}_4 + \vec{r}_3}{2} - \vec{r}_2 & \vec{\xi}_4 &= \frac{\vec{r}_4 + \vec{r}_3 + \vec{r}_2 + \vec{r}_1}{4} - \vec{r}_0 \\ \vec{R}_f &= \frac{M(\vec{r}_4 + \vec{r}_3 + \vec{r}_2 + \vec{r}_1) + m \vec{r}_0}{m + 4M}, \end{aligned} \quad (7)$$

we can write

$$f = e^{i\vec{P}_f \cdot \vec{R}_f} e^{i\vec{Q} \cdot \vec{\xi}_4} \frac{A}{2} \left\{ e^{i\vec{z} \cdot \vec{\xi}_3} \psi^{(3)}(\vec{\xi}_1, \vec{\xi}_2) \psi^{(1)} \right\}. \quad (8)$$

Being canonically conjugated with coordinates \vec{R}_f , $\vec{\xi}_4$ and $\vec{\xi}_3$, the quantities \vec{P}_f , \vec{Q} and \vec{z} denoted the total momentum, the relative momentum of pion and system of all the nucleons and the relative momentum of outgoing nucleon and 3He (or 3H), respectively, in the final state. The functions $\psi^{(1)}$ and $\psi^{(3)}$ are the nuclear wave functions of outgoing nucleon and 3He (or 3H), respectively. Further, the antisymmetrization operator A can be expressed in terms of transposition operators $P(ij)$ of the nucleon pair (ij) as

$$A = 1 - P(12) - P(13) - P(14). \quad (9)$$

Pion and nucleons masses are denoted as m and M , respectively. In an analogical fashion, the initial state will be expressed by

$$|i\rangle = e^{i\vec{P}_1 \cdot \vec{R}_1} e^{i\vec{Q} \cdot \vec{\xi}'_4} \psi^{(4)}(\vec{\xi}'_1, \vec{\xi}'_2, \vec{\xi}'_3) \quad (10)$$

where $\psi^{(4)}$ is the nuclear wave function of the initial state system of four nucleons (${}^4\text{He}$). The spin and isospin variables are not written explicitly, if not stated otherwise. The terms obtained using eqs. (8) and (10) in evaluating the matrix element $\langle f | T | i \rangle$ can be splitted into two parts

$$\langle f | T | i \rangle = T_d + T_e \quad (11)$$

the first of which is identical to that obtained with usual PWIA. The term T_e describes a process in which pion is scattered by a nucleon that remains bound in ${}^3\text{He}$ (or ${}^3\text{H}$) and another nucleon is emitted. Eq. (11) can be visualized by the graphs of Fig. 1.

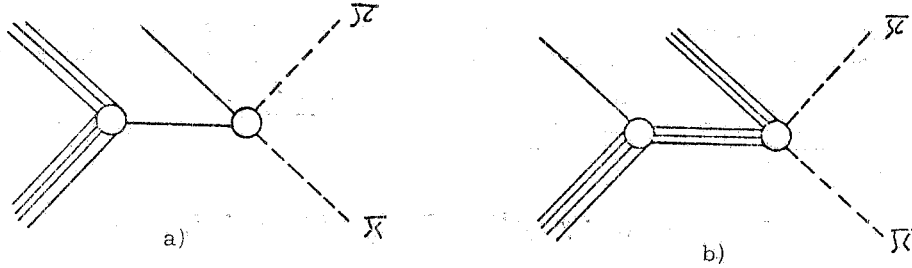


FIG. 1 - Direct a) and exchange b) processes in pion-induced knock-out reactions on ${}^4\text{He}$.

Using the relation

$$\begin{aligned} \langle \vec{r}'_1 \vec{r}'_2 \vec{r}'_3 \vec{r}'_4 \vec{r}'_0 | t_{\pi N}(E) | \vec{r}'_0 \vec{r}'_1 \vec{r}'_2 \vec{r}'_3 \vec{r}'_4 \rangle &= \delta(\vec{r}'_4 - \vec{r}'_4) \delta(\vec{r}'_3 - \vec{r}'_3) \delta(\vec{r}'_2 - \vec{r}'_2) \times \\ &\times \delta\left(\frac{m\vec{r}'_0 + M\vec{r}'_1}{m+M} - \frac{m\vec{r}'_0 + M\vec{r}'_1}{m+M}\right) \langle \vec{r}'_1 - \vec{r}'_0 | t_{\pi N}(\bar{E}) | \vec{r}'_1 - \vec{r}'_0 \rangle \quad (12) \\ \bar{E} &= E - \frac{(\vec{\Delta}\vec{r}'_0 + \vec{\Delta}\vec{r}'_1)^2}{2(m+M)} \end{aligned}$$

it is easy to evaluate the nucleon-exchange part T_d of the amplitude

$$T_d = \langle \vec{q}'_d \sigma \tau t'_z | t_{\pi N}(E_d) | t_z - T_z - J_z \vec{q}'_d \rangle F\left(\vec{z} + \frac{3}{4}(\vec{Q}' - \vec{Q})\right) \quad (13)$$

In deriving eq. (13), it has been assumed tacitly that the nuclear wave functions $\psi^{(3)}(\vec{\xi}'_1, \vec{\xi}'_2)$ and $\psi^{(4)}(\vec{\xi}'_1, \vec{\xi}'_2, \vec{\xi}'_3)$ are products of a symmetrical in nucleon coordinates part and of an antisymmetrical spin-isospin component. The coordinate wave functions were constructed starting from the one-particle harmonic-oscillator functions

$$\psi(\vec{r}) = \frac{1}{(\pi a_o^2)^{3/4}} e^{-1/2(r/a_o)^2} \quad (14)$$

The values $(a_o)_{{}^4\text{He}} \equiv a = 1.234$ fm and $(a_o)_{{}^3\text{He}} \equiv b = 1.38$ fm can be deduced⁽¹³⁾ from electron scattering experiments on ${}^4\text{He}$ and ${}^3\text{He}$ nuclei, respectively. Assuming this, the overlap function has the form

$$F(\vec{X}) = \left(\frac{2ab}{a+b}\right)^3 (4a\sqrt{\frac{\pi}{3}})^{3/2} e^{-2/3(a^2 X^2)} \quad (15)$$

It is well known that the amplitude T_d has a factorized form, without making any additional assumptions (fixed scatterer approximation), unlike the situation which occurs, e. g., in the elastic scattering calculations by deriving the optical potential. In fact, the motion of target nucleons has been taken into account, as indicated by eqs. (7-12) and reflected by terms proportional to M/M in the expressions

$$\begin{aligned} \vec{q}'_d &= \frac{\mu}{M} \left\{ \vec{Q} - \frac{M}{M} \left[\vec{z} + \frac{3}{4} (\vec{Q}' - \vec{Q}) \right] \right\} \\ \vec{q}'_d &= \frac{\mu}{M} \left\{ \vec{Q}' - \frac{M}{M} \vec{z} \right\}, \quad E_d = \frac{1}{4\mu} (q_d^2 + q'_d{}^2) \end{aligned} \quad (16)$$

Here, the pion-nucleon and pion-nucleus reduced masses μ and M are defined as

$$\mu = \frac{mM}{m+M}, \quad M = \frac{4Mm}{4M+m}. \quad (17)$$

Finally, the isospin (spin) projections of the incoming and outgoing pion, nucleon and three-nucleon system are denoted as $t_z, t'_z, \tau(\sigma)$ and $T_z(J_z)$, respectively.

The derivation of the amplitude T_e is somewhat more complicated, since the factorization of the former type does not occur automatically. Using eqs. (8) - (12), we have, after some manipulations,

$$\begin{aligned} T_e &= \frac{6}{(2\pi)^3} \int e^{-i\vec{\xi}_3 \cdot (z + \frac{\vec{Q}-\vec{Q}'}{4})} e^{i\vec{k} \cdot (\vec{\xi}_2 - \vec{\xi}'_2)} e^{i\frac{\vec{\xi}_2 + \vec{\xi}'_2}{3} \cdot (\vec{Q} - \vec{Q}')} \psi^{*(3)}(\vec{\xi}_1, \vec{\xi}_2) \times \\ &\times \langle q'(\vec{k}) | t_{\pi N}(E(\vec{k})) | q(\vec{k}) \rangle \psi^{(4)}(\vec{\xi}_1, \vec{\xi}'_2, \vec{\xi}_3) d^3\xi_1 d^3k \prod_{i=1}^3 d^3\xi_i. \end{aligned} \quad (18)$$

If we neglect, now, the dependence of the matrix element $\langle q' | t_{\pi N} | q \rangle$ on the momentum \vec{k} in π -nucleus c. m. system, the amplitude T_e is obtained in the factorized form

$$\begin{aligned} T_e &= -\sqrt{3} \sum_{\mu_1 \mu'_1 \tau_1 \tau'_1} \sum_{\xi_1 \xi_2 \xi_3} (-1)^{y-\eta+x-\xi} C^{1/2 J_z} C^{y-\eta} C^{1/2 T_z} C^{x-\xi} \\ &\quad \mu_1 \mu'_1 \tau_1 \tau'_1 \quad y \eta \frac{1}{2} \mu'_1 \quad \frac{1}{2} \mu_1 \frac{1}{2} \sigma \quad x \xi \frac{1}{2} \tau'_1 \quad \frac{1}{2} \tau_1 \frac{1}{2} \tau \quad \delta_{y_1} |x-1| \quad x \\ &\times \langle \vec{q}'_e | \mu'_1 \tau'_1 | t_{\pi N}(E_e) | \vec{q}_e \mu_1 \tau_1 \rangle G(\vec{z} + \frac{\vec{Q}-\vec{Q}'}{4}, \vec{Q} - \vec{Q}'), \end{aligned} \quad (19)$$

where

$$\begin{aligned} \vec{q}'_e &= \frac{\mu}{M} \left\{ \vec{Q} + \frac{M}{3M} \left[\vec{z} - \frac{5}{4} (\vec{Q}' - \vec{Q}) \right] \right\}, \quad \vec{q}_e = \frac{\mu}{M} \left\{ \vec{Q}' + \frac{M}{3M} \left[\vec{z} + (\vec{Q}' - \vec{Q}) \right] \right\}, \\ E_e &= \frac{1}{4\mu} (q_e^2 + q'_e{}^2). \end{aligned} \quad (20)$$

in the nuclear model adopted, we have

$$G(\vec{X}, \vec{Y}) = \left(\frac{2ab}{a^2 + b^2} \right)^3 \left(4a \sqrt{\frac{\pi}{3}} \right)^{3/2} e^{-\frac{2}{3} a^2 X^2} e^{-\frac{1}{3} \frac{(ab)^2}{a^2 + b^2} Y^2}. \quad (21)$$

Let us comment briefly the factorization procedure used in eq. (18). Instead of integrating over d^3k , the matrix element $\langle | t_{\pi N} | \rangle$ was evaluated for the fixed effective momenta of the scattered nucleon in the initial and final state

$$\vec{k}_i = \frac{2}{3} (\vec{Q}' - \vec{Q}) + \frac{\vec{x}}{3}, \quad \vec{k}_f = \frac{\vec{x}}{3} - \frac{1}{3} (\vec{Q}' - \vec{Q}). \quad (22)$$

The momentum of outgoing ${}^3\text{He}$ or ${}^3\text{H}$ was denoted as \vec{x} . The choice of the effective momenta

(22) is optimal in that sense, that the expression (19) is correct up to terms linear in \mathcal{M}/M ^(*). It is important to note that the \mathcal{M}/M terms are different in the expressions (13) and (19). Due to this fact, the cross section of the reaction (III) will be influenced strongly by the \mathcal{M}/M terms, since a large cancellation occurs between T_d and T_e in this case.

It is convenient to express the scattering matrix $t_{\pi N}$ as follows

$$t_{\pi N} = P_{1/2} f(1) + P_{3/2} f(3) + i \vec{\sigma} \cdot \vec{v} \left[P_{1/2} g(1) + P_{3/2} g(3) \right], \quad (23)$$

where $P_{1/2}$ and $P_{3/2}$ are projection operators, projecting onto the states characterized by the full pion-nucleon isospin $T=1/2$ and $3/2$, respectively. We denoted the vector normal to the scattering plane as \vec{v} . After summation over spin projections, we obtain the following expressions for the squared scattering matrix of the reactions (I), (II) and (III)

$$\begin{aligned} |T_I|^2 &= \frac{2}{9} \left\{ \left[(f_d(3) + 2f_d(1)) F + (7f_e(3) + 2f_e(1)) G \right]^2 + \right. \\ &\quad \left. + \left[(g_d(3) + 2g_d(1)) \vec{v}_d F - (g_e(3) + 2g_e(1)) \vec{v}_e G \right]^2 \right\} \\ |T_{II}|^2 &= \frac{2}{9} \left\{ \left[3f_d(3) F + (5f_e(3) + 4f_e(1)) G \right]^2 + \left[3g_d(3) \vec{v}_d F - 3g_e(3) \vec{v}_e G \right]^2 \right\} \\ |T_{III}|^2 &= \frac{4}{9} \left\{ \left[(f_d(3) - f_d(1)) F - (f_e(3) - f_e(1)) G \right]^2 + \left[(g_d(3) - g_d(1)) \vec{v}_d F - \right. \right. \\ &\quad \left. \left. - (g_e(3) - g_e(1)) \vec{v}_e G \right]^2 \right\}. \end{aligned} \quad (24)$$

Arguments of the functions F and G , being the same as those in eqs. (13) and (19), were not written explicitly. The notations $\vec{v}_i = \vec{q}_i \times \vec{q}'_i / |\vec{q}_i \times \vec{q}'_i|$ and

$$f_i(3) = \langle \vec{q}'_i | f_i(3, E_i) | \vec{q}_i \rangle \text{ etc.} \quad (25)$$

where used in eqs. (24) for $i=d$ or e .

Before the cross sections of the reactions (I), (II) and (III) were calculated with the aid of eqs. (24), the amplitudes (25) were decomposed into partial waves. The corresponding pion-nucleon phase-shifts were calculated according to the prescriptions of ref. (16). Throughout the calculations, the the relativistic transformation of $t_{\pi N}$ matrix from π -N centre-of-mass system to lab system⁽¹⁷⁾ was performed and the relativistic expression for \vec{q}_i , \vec{q}'_i and E_i , for $i = d$ or e , were used.

4. - DISCUSSION AND CONCLUSIONS

From the inspection of formulae (24), it can be concluded that the three-nucleon-exchange terms (which arise if an antisymmetrized final state wave function is used) will interfere destructively with the PWIA amplitude, in the case of reaction (III), and constructively otherwise. This statement is valid, of course, in the region of the Δ_{33} -resonance, where the amplitude $f_i(1)$ ($i=d, e$) may be neglected, in first approximations. Spin-flip part of the matrix $t_{\pi N}$ gives very small contribution to the cross sections of all the reactions considered, because of the large cancellation between g_d and g_e in eqs. (24). It is instructive to compare in detail the predictions of our model with the existing experimental data.

(*) More detailed derivation of the effective momenta can be found in refs. (14, 15), where the first order optical potential was constructed. Notice that the effective momenta for the elastic and knock-out processes turn out to be different.

4. 1. - TOTAL CROSS SECTIONS AND THE RATIO R_{He} .

Since the distortion of the incoming and outgoing waves was not taken into account, it is not surprising that the calculated total cross sections are substantially bigger than the experimental ones. As far as concerns the reactions (I) and (II), our model overestimates the total cross sections, typically 3-5 times; whereas, the measured and calculated cross sections of the reaction (III) differ by a factor 2. One usually assumes^(2, 3), that the distortion of the pionic waves represents a nearly constant factor. For this reason, it is more meaningful to compare calculated and measured ratios of the type of eq. (5), than the total cross sections alone. Such a comparison is performed in Table II.

TABLE II - Measured and calculated partial ratios between total cross sections in knock-out processes on 4He .

$E^{lab} (MeV)$	$\sigma_{\pi^+ p} {}^3H$	$\sigma_{\pi^+ n} {}^3He$	$\sigma_{\pi^0 p} {}^3He$	Remarks
	9	: 4.13	: 2.37	exper. values
110	9	: 1.22	: 2.32	PWIA
	9	: 10.32	: 0.72	PWIA Pauli terms
	9	: 4.64	: 4.28	exper. values
160	9	: 1.09	: 2.04	PWIA
	9	: 9.49	: 0.65	PWIA Pauli terms

The experimental values in Table II were deduced from Table I. The PWIA ratios were obtained neglecting the amplitude T_e in eq. (11), and the remaining rows of Table II contain the results of the complete calculation. As was stated previously, PWIA fails to reproduce the experimentally observed inequality $\sigma_{\pi^+ n} {}^3He > \sigma_{\pi^0 p} {}^3He$. It can be seen from Table II that the inclusion of Pauli principle term T_e leads to the increasing of $\sigma_{\pi^+ n} {}^3He$ and to the decreasing of $\sigma_{\pi^0 p} {}^3He$. By this way, the aforementioned inequality will be fulfilled. However, the contribution to the total amplitude provided by T_e is too strong - so strong, that even $\sigma_{\pi^+ n} {}^3He \gtrsim \sigma_{\pi^+ p} {}^3H$ holds. This discrepancy may be caused by an unrealistic description of the overlap function $G(\vec{X}, \vec{Y})$, or by the oversimplified model adopted for the final state interaction, or by both reasons.

It is interesting to note that the contributions provided by T_e to $\sigma_{\pi^+ n} {}^3He$ and to $\sigma_{\pi^0 p} {}^3He$ will cancel to a large extent in the expression (1) for R_{He} . For that reason, the calculated values of R_{He} lie rather close to the experimental ones (see Fig. 2) - much closer than the PWIA values.

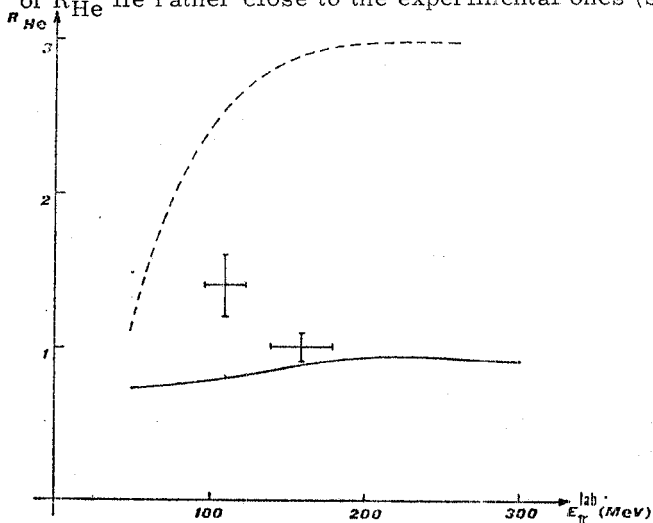


FIG. 2 - Dependence on energy of the ratio R_{He} . Experimental points: this work. The dashed line describes the PWIA result. The full line is obtained if the three-nucleon-exchange mechanism is taken into account.

We can conclude that the often measured and calculated ratio R is not very sensitive to the details of the final state interaction, considerably less sensitive than the ratios given in Table II.

In concluding this subsection, we make some comments on the most puzzling feature of the energy dependence of the ratio R_{He} , as results from Fig. 2. The calculated ratio R_{He} , as well as the measured⁽¹⁾ ratio R_C , are increasing functions of energy, for $E_{\pi}^{lab} \lesssim 220$ MeV. On the other hand, the experimental R_{He} values are decreasing in the same energy interval. We have no real explanation for this fact. We shall limit ourselves to the following remarks. The energy dependences of R_{He} and R_C need not to be necessarily similar. There are at least two factors, which might cause different behaviours of R_{He} and R_C .

- (a) The isospin-dependent part of the optical potential, which describes the distortion of the outgoing pion, is much stronger for the 4He reaction than the ${}^{12}C$ case.
- (b) The knock-out reactions on the 4He leads to the ground state of 3He (or 3H), whereas the existing expressions on ${}^{12}C(\pi^{\pm}, \pi N)$ ${}^{11}C$ reactions (all performed by activation techniques) can not distinguish between the various particle stable states of the residual nucleus.

A more complete experimental determination of the energy dependence of R_{He} would be of great importance.

4. 2. - DIFFERENTIAL CROSS SECTIONS

In Figg. 3 and 4, the measured differential cross sections of the reactions (I) and (II), versus the scattered pion angle, at $E_{\pi}^{lab} = 110$ MeV and 160 MeV, are compared with present calculations. Moreover, in Fig. 3, the same comparison is performed with the experimental angular distribution of the emitted protons in reaction (III), at $E_{\pi}^{lab} = 110$ MeV. Finally, in Fig. 4 calculated differential cross sections for the single charge exchange reaction (III), versus the π^0 emission angle, at $E_{\pi}^{lab} = 160$ MeV, are also reported. Calculated cross sections of the reactions (I) and (II) have been scaled by the factor

$$\underline{c} = \sigma_{tot} (exp) / \sigma_{tot} (theor), \quad (26)$$

while the results obtained for the reaction (III) are shown without any scaling. PWIA results are shown by dashed lines (the term T_e was neglected in eq. (11)). If the three-nucleon-exchange term is taken into account, the solid lines is obtained. In some cases, the differential cross sections obtained by neglecting the terms of order \mathcal{M}/M in eqs. (24) are also displayed. These dotted curves, indicating the role of the Fermi motion, are scaled by the same constant factor \underline{c} . The following conclusions can be drawn from Fig. 3 and 4.

- (a) The three-nucleon-exchange model describes the shape of differential cross sections of the reactions (I) and (II) much better than PWIA does. Unlike PWIA, our model predicts correctly the position of the dip in the cross section, as well as it explains the existence of the secondary maximum. Let us remind that the three-nucleon-exchange mechanism provides a similar build up in the backward angular region for the elastic p- 4He scattering at medium energies⁽¹⁸⁾. However, the agreement between our calculations and the experimental results is only qualitative.
- (b) Not only $\sigma_{\pi^0 p} {}^3He$, but also the cross sections of the other reactions are influenced by the Fermi motion terms (linear in \mathcal{M}/M), to a surprisingly large extent. Similarly to the elastic scattering⁽¹⁹⁾, these terms increase the cross sections of reactions (I) and (II) in the region of large scattering angles.

5. - SUMMARY

The plane wave impulse approximation -with or without the three-nucleon-exchange mechanism - fails to reproduce the experimental values of the ratios between the total cross sections of the reactions (π^+, π^+p) , (π^+, π^+n) and (π^+, π^0p) on 4He . Our results indicate that the initial and/or final state interaction plays a remarkable role in the pion-induced knock-out reactions on such a light nucleus as 4He . The three-nucleon-exchange mechanism gives an important contribution to the back-

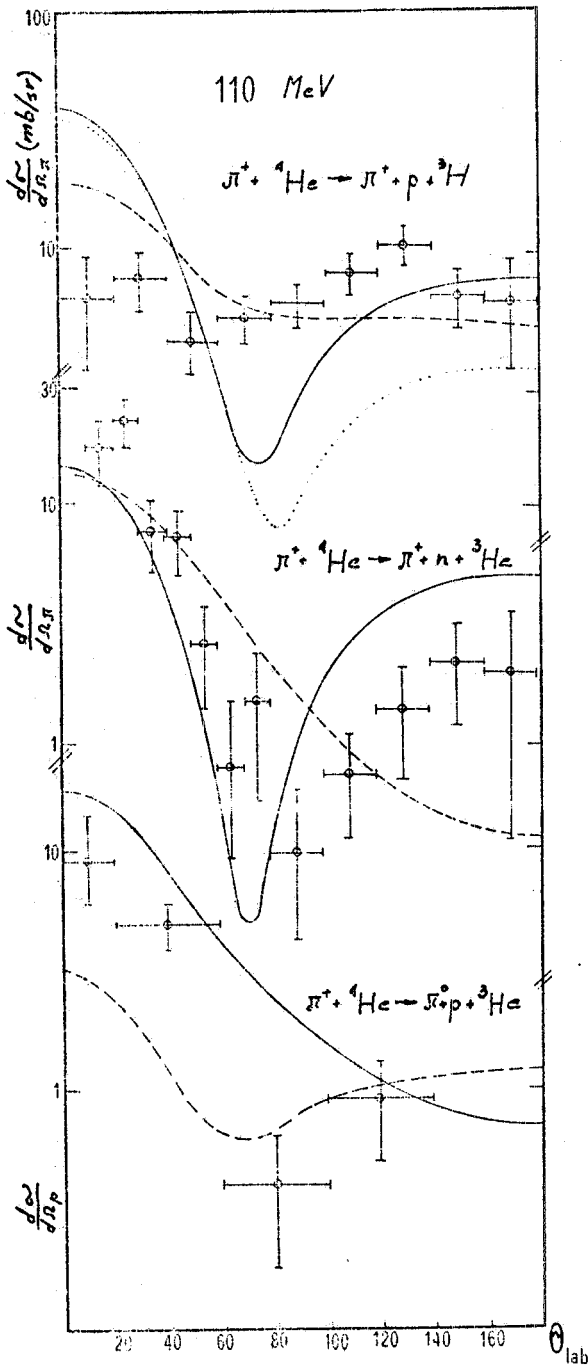


FIG. 3-Differential cross sections of the knock-out reactions on ${}^4\text{He}$ at $E_{\pi}^{\text{lab}}=110$ MeV. The dashed lines (PWIA) are obtained by neglecting the amplitude T_e . The solid lines represent the results of complete calculation. The dotted lines represent the case in which the Fermi motion is not taken into account. Experimental data are from ref. (4).

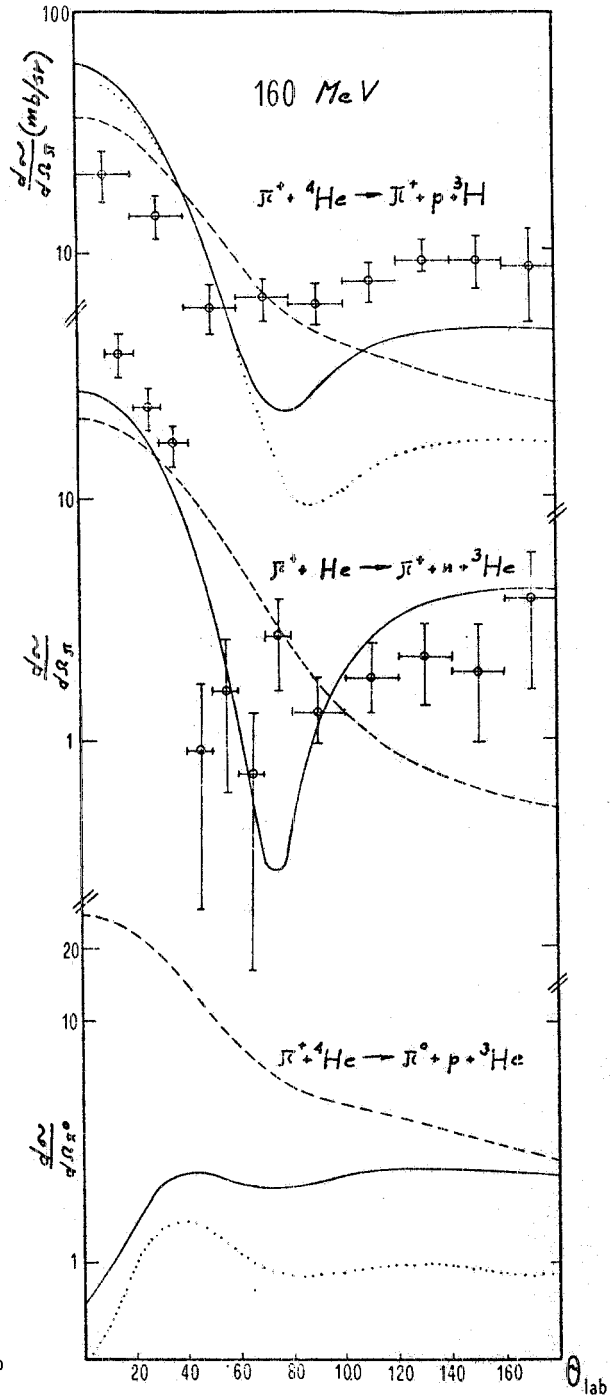


FIG. 4 - Differential cross sections of the knock-out reactions on ${}^4\text{He}$ at $E_{\pi}^{\text{lab}}=160$ MeV. The meaning of the curves is the same as in Fig. 3. Experimental data are from ref. (4).

ward scattering in the (π^+ , π^+p) and (π^+ , π^+n) reactions. This contribution explains the main feature of the measured differential cross sections at large scattering angles. As far as concerns the puzzling situation of the energy dependence of R_{He} ratio, more complete experimental determinations of R_{He} are urgently needed.

The authors would like to thank Prof. Yu.A. Shcherbakov for interesting discussions and helpful suggestions.

REFERENCES

- (1) - B. Dropesky, G. Butler, C. Orth, R. Williams, G. Friedlander, M. Iates and S. Kaufman, Phys. Rev. Letters, 34, 821 (1975); L. H. Batist, V. D. Vitman, V. P. Koptev, M. M. Makarov, A. A. Noberezhnov, V. V. Nelyubin, G. Z. Obrant, V. V. Sarantsev and G. V. Scherbakov, Nuclear Phys., A254, 480 (1975).
- (2) - M. Sternheim and R. Silbar, Phys. Rev. Letters, 34, 824 (1975).
- (3) - Y. Hüfner, H. Pirner and M. Thies Phys. Letters, B59, 215 (1975).
- (4) - F. Balestra, L. Busso, R. Garfagnini, G. Piragino, R. Barbini, C. Guaraldo and R. Scrimaglio, Lett. Nuovo Cimento 15, 535 (1976); 15, 542 (1976).
- (5) - R. Barbini, L. Busso, S. Costa, R. Garfagnini, C. Guaraldo, G. Piragino and R. Scrimaglio, Few Particle Problems in the Nuclear Structure (Amsterdam, 1972), pag. 886; Frascati Report LNF-72/62 (1972).
- (6) - I. V. Falomkin, M. M. Kulyukin, V. I. Lyashenko, F. Nichitiu, G. Piragino, G. B. Pontecorvo and Yu. A. Shcherbakov, Lett. Nuovo Cimento, 3, 461 (1972); 5, 1121 (1972).
- (7) - Yu. A. Shcherbakov, T. Angeluscu, I. V. Falomkin, M. M. Kulyukin, V. I. Lyashenko, R. Mach, A. Mihul, N. M. Kao, F. Nichitiu, G. B. Pontecorvo, V. K. Sarychieva, M. G. Sapozhnikov, M. Semerojeva, T. N. Troshchev, N. I. Troshcheva, F. Balestra, L. Busso, R. Garfagnini and G. Piragino, Nuovo Cimento, 31A, 249 (1976).
- (8) - R. Barbini, L. Busso, S. Costa, R. Garfagnini, C. Guaraldo, G. Piragino and R. Scrimaglio, Nuclear Instr. and Meth., 102, 1 (1972).
- (9) - R. Barbini, C. Guaraldo, P. Picozza, C. Schaerf and R. Scrimaglio, Nuclear Instr. and Meth., 105, 515 (1972).
- (10) - Yu. A. Budagov, P. F. Ermolov, E. A. Kushnirenko and V. I. Moskalev, Sov. Phys. JETP, 15, 824 (1962).
- (11) - V. M. Kolybasov, Phys. Letters, 27B, 3 (1968).
- (12) - A. Pak, L. Pocs and A. Tarasov, Yad FIZ, 21, 950 (1975).
- (13) - Y. Mc Carthy, I. Sick, R. Whitney and M. Yearin, Phys. Rev. Letters, 25, 884 (1970); R. F. Frosch, Y. Mc Carthy, R. Rand and M. Yearin, Phys. Rev., 160, 874 (1967).
- (14) - R. Mach, Proceedings of the Conference on Meson-Nuclear Physics, Pittsburg, 1976 (eds. P. D. Barnes et al.), p. 638.
- (15) - R. H. Landau, Ann. of Phys. 92, 205 (1975).
- (16) - A. W. Thomas, Nuclear Phys. A258, 417 (1976).
- (17) - L. Heller, G. E. Bohannon and F. Tabahin, Phys. Rev. C13, 742 (1976).
- (18) - H. Lesniak, L. Lesniak and Tekon, Nuclear Phys. A267, 503 (1976).
- (19) - R. Mach, Nuclear Phys. A205, 56 (1973).