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Sum Rules and Non-Regge Terms in Photon-Photon Scattering (*).

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Summary. — We discuss whether resonances are dual to the Regge terms in both real and virtual photon-photon scatterings. Various sum rule considerations lead us to suggest that this is not the case.

1. – Introduction.

It was realized some time ago (1-3) that a measurement of the reaction $\gamma\gamma \to$ hadrons is feasible with high-energy e^+e^- colliding beams. In fact, some preliminary, albeit statistically poor, data already exist (4) from Adone in Frascati. Much more is expected to arrive shortly from other laboratories.

Theoretically, if one assumes Regge-asymptotic behaviour for $\gamma\gamma$ scattering, various finite-energy and superconvergence-type sum rules can be

(*) To speed up publication, the authors of this paper have agreed to not receive the proofs for correction.

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(1) S. BRODSKY, T. KINOSHITA and H. TERAZAWA: *Phys. Rev. Lett.*, **25**, 972 (1970).

(2) V. BUDNEV and I. GINZBURG: *XV International Conference on High-Energy Physics* (Kiev, 1970).

(3) A. JACCARINI, N. ARTEGA-ROMERO, J. PARISI and P. KESSLER: *Lett. Nuovo Cimento*, **4**, 933 (1970).

(4) S. ORITO, M. L. FERRER, L. PAOLUZI and R. SANTONICO: *Phys. Lett.*, **48** B, 380 (1974); L. PAOLUZI, F. CERADINI, H. L. FERRER, R. SANTONICO, G. BARBIELLINI, S. ORITO and T. TSURU: *Lett. Nuovo Cimento*, **10**, 435 (1974).

derived. One may then try to saturate these sum rules with low-lying resonances to obtain information on the various couplings to two photons. Similarly, much of the estimates ([5]) of the cross-sections for high-energy e^+e^- experiments are based on the extrapolation to low energies of pure Regge terms, obtained through factorization.

However, a natural question arises: are resonances dual to the Regge terms in $\gamma\gamma$ scattering? We would like to argue below that this is *not* the case.

Certainly, for Compton scattering off nucleons, by now there is a considerable body of theoretical and experimental evidence ([6]) for the existence of a $J = 0$ fixed pole, invalidating the resonance $\leftrightarrow$ Regge duality. Moreover, strong arguments have been presented ([7]) that both a $J = 0$ fixed pole and a Krönecker-delta singularity are present in pion Compton scattering as well.

It is well to stress that the absorptive part of a two-current amplitude, *i.e.* a current scattering off a hadron, is supposed to be Regge behaved even though its real part may have fixed poles and Krönecker deltas. In current algebra with charged photons on hadrons such fixed singularities, in the real part, are required by the Ward identities: Dashen–Fubini–Gell-Mann sum rules are celebrated examples of it. We have considered the scattering of four charged (conserved) currents and found that, through the Ward identities, the absorptive parts themselves pick up extra singularities in the J-plane. Heuristically this may be seen simply as follows. The elastic current-hadron amplitude is a T-product and hence its divergence is not zero, whereas its absorptive part is a current product and hence is divergenceless. Thus the real part of such an amplitude has a fixed pole (and/or a Krönecker delta), but the absorptive part has not. In contrast, the absorptive part of the current $\times$ current elastic amplitude is a product of two T-products and so its various divergences do not vanish (see the appendix). This is the genesis of fixed singularities for the four-current absorptive part, much in the same vein as the two-current real part.

Of course, the above argument from current algebra leaves the question for O_1 currents (*i.e.* the photon, whether real or virtual) moot. For photons, then, we turn to a phenomenological approach. For massless photons, we can write down certain sum rules, which have been shown by several people to be « above suspicion » ([8-11]), and obtain through them scalar, pseudoscalar and tensor coupling relationships. This allows us to estimate the low-energy

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([5]) S. Brodsky: *J. Physique*, C-2, *Suppl.*, **3**, 69 (1974).
([6]) M. Creutz, S. Drell and E. Paschos: *Phys. Rev.*, **173**, 2300 (1969); M. Damashek and F. Gilman: *Phys. Rev. D*, **1**, 1319 (1970).
([7]) G. Pancheri-Srivastava and Y. Srivastava: *Lett. Nuovo Cimento*, **13**, 221 (1975).
([8]) P. Roy: *Phys. Rev. D*, **9**, 2631 (1974).
([9]) V. Budnev, I. Ginzburg and V. Serbo: *Lett. Nuovo Cimento*, **7**, 13 (1974).
([10]) S. Gerasimov and J. Moulin: *Nucl. Phys.*, **98** B, 349 (1975).
([11]) P. Grassberger and R. Kögerler: *Nucl. Phys.*, **106** B, 451 (1976).

resonance contribution to the total $\gamma\gamma$ cross-section and compare it with the Regge contribution to the same. Approximately a factor-of-three discrepancy is found.

As explicitly shown in the appendix, the «box diagram» (one-fermion loop) is a source of non-Regge terms which consistently satisfies the Ward identities for the absorptive parts of charged SU_2 currents. Under the hypothesis that it provides the non-Regge part of the absorptive amplitudes also in the case of photons and, therefore, it accounts for the above discrepancy, we estimate its contribution to the low-energy $\gamma\gamma$ cross-section and find it improves considerably the comparison with the pure resonant contributions.

Next we consider the virtual-photon case in a particular kinematical limit with high energy and large masses for the photons. Here it is possible to isolate certain helicity amplitudes for which a striking duality may be inferred between the resonance terms and the «box» diagram (quark loops). This further strenghtens our belief that resonances are not dual to the Regge terms. In fact, a simple conjecture which seems to be consistent with all our analysis is that resonances are dual to the asymptotic behaviour of the amplitude, which in turn is generally made up of Regge and non-Regge terms.

The paper is organized as follows. Section **2** deals with kinematics and notations. In sect. **3**, the amplitudes for real photons are defined and sum rules are considered for them. Estimates for $\gamma\gamma$ total cross-sections are then discussed. The virtual-photon case is considered in sect. **4**, and our concluding remarks are contained in sect. **5**. The appendix deals with Ward identities for the absorptive parts of SU_2 charged currents and their satisfaction in the one-loop approximation.

2. – Kinematics and notation.

The absorptive part of the forward $(q_1 = q_3,\ q_2 = q_4)$ current$\times$current scattering is defined as

$$W^{\mu\nu\lambda\sigma}(q_1, q_2) = \frac{1}{2}\sum_n (2\pi)^4\delta^4(q_1+q_2-p_n)\,T_n^{\mu\nu}(q_1, q_2)\,T_n^{\lambda\sigma *}T(q_1, q_2) = \tag{1}$$
$$= \frac{1}{2}\int \mathrm{d}^4x\,\mathrm{d}^4y\,\mathrm{d}^4z\,\exp\left[-\frac{i}{2}\{(q_2-q_1)(x-y)+(q_2+q_1)z\}\right]\cdot$$
$$\cdot\langle 0|\overline{T}\left(j^\lambda\left(\frac{x}{2}\right)j^\sigma\left(-\frac{x}{2}\right)\right)T\left(j^\mu\left(\frac{y}{2}+z\right)j^\nu\left(-\frac{y}{2}+z\right)\right)|0\rangle\,,$$

where the «vertex function» $T_n^{\mu\nu}(q_1, q_2)$ is given by

$$T_n^{\mu\nu}(q_1, q_2) = i\int \mathrm{d}^4x\,\exp\left[-\frac{1}{2}(q_1-q_2)x\right]\langle n|T\left(j^\mu\left(\frac{x}{2}\right)j^\nu\left(-\frac{x}{2}\right)\right)|0\rangle\,. \tag{2}$$

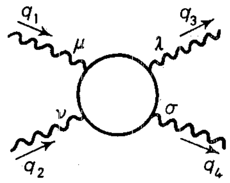

Fig. 1. – Current × current amplitude.

The decomposition of $W^{\mu\nu\lambda\sigma}$ for virtual photons in terms of the eight helicity amplitudes may be found in ref. [12]. We use the following kinematic invariants and scaling variables:

$$s = (q_1 + q_2)^2\,, \qquad t = (q_1 - q_2)^2\,, \qquad u = (q_1 - q_4)^2\,, \qquad \omega = 1 - \frac{s}{q_1^2 + q_2^2}$$

and

$$\xi = \frac{q_2^2 - q_1^2}{q_2^2 + q_1^2}\,.$$

For later purposes we give below the complete helicity decomposition of the scattering amplitude for *real* photons. Define the amplitudes $F_i^s(s, t, u)$ $(i = 1, \ldots, 5)$:

$$(3a)\qquad F_1^s \equiv f^s_{++,++} + f^s_{++,--} = 16\pi \sum_{J=0}^{\text{even}} (2J+1)\{\langle + + |T^J(s)| + +\rangle + \langle + + |T^J(s)| - -\rangle\}\, P_J(\cos\theta_s)\,,$$

$$(3b)\qquad F_2^s \equiv f^s_{+-,++} + f^s_{+-,--} = 16\pi \sum_{J=2}^{\text{even}} (2J+1)\{\langle + - |T^J(s)| + +\rangle + \langle + - |T^J(s)| - -\rangle\}\, d^J_{2,0}(z_s)\,,$$

$$(3c)\qquad F_3^s \equiv f^s_{+-,+-} + f^s_{+-,-+} = 16\pi \sum_{J=2} (2J+1)\{\langle + - |T^J(s)| + -\rangle \cdot d^J_{2,2}(z_s) + \langle + - |T^J(s)| - +\rangle\, d^J_{-2,2}(z_s)\}\,,$$

$$(3d)\qquad F_4^s \equiv f^s_{++,++} - f^s_{++,--} = 16\pi \sum_{J=0}^{\text{even}} (2J+1)\{\langle + + |T^J(s)| + +\rangle - \langle + + |T^J(s)| - -\rangle\}\, P_J(\cos\theta_s)\,,$$

$$(3e)\qquad F_5^s \equiv f^s_{+-,+-} - f^s_{+-,-+} = 16\pi \sum_{J=2} (2J+1)\{\langle + - |T^J(s)| + -\rangle \cdot d^J_{2,2}(\cos\theta_s) - \langle + - |T^J(s)| - +\rangle\, d^J_{-2,2}(\cos\theta_s)\}\,,$$

where $f^s_{\lambda_1\lambda_2,\lambda_3\lambda_4}$ stand for the s-channel helicity amplitudes $(\lambda_i = \pm 1)$ and use has already been made of parity, time-reversal invariance and Bose symmetry.

([12]) V. BUDNEV, V. CHERNYAK and I. GINZBURG: *Nucl. Phys.*, **34** B, 470 (1971).

The total cross-section for a given helicity amplitude is

$$\sigma_{\lambda_1\lambda_2}(s) = \frac{1}{s}\,\mathrm{Im}\, f^s_{\lambda_1\lambda_2,\lambda_1\lambda_2}(s, 0)\,. \tag{4}$$

The contribution of a given resonance of mass m_R, width Γ^R and spin J_R is given by

$$\sigma^{(R)}_{\lambda_1\lambda_2}(s) = 16\pi^2\,(2J_R+1)\,\frac{\Gamma^R_{\lambda_1\lambda_2}}{m_R}\,\delta(m_R^2-s)\,. \tag{5}$$

The amplitudes $F^s_{1,2,3,5}$ get contributions from positive-parity intermediate states in the s-channel, while F^s_4 has $P=-1$. As can be directly verified from the partial-wave expansion given in eqs. (3), $t\leftrightarrow u$ interchange gives

$$F^s_{1,2,3,4}(s,t,u) = F^s_{1,2,3,4}(s,u,t)\,, \tag{6a}$$

$$F^s_5(s,t,u) = -F^s_5(s,u,t)\,. \tag{6b}$$

The t-channel helicity amplitudes F^t_i $(i=1,2,\ldots,5)$ are related to F^s_i through the following crossing matrix

$$\begin{bmatrix} F^s_1\\ F^s_2\\ F^s_3\\ F^s_4\\ F^s_5\end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & -1\\ 0 & 1 & 0 & 0 & 0\\ \frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & \frac{1}{2}\\ -\frac{1}{2} & 0 & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2}\\ -\frac{1}{2} & 0 & \frac{1}{2} & -\frac{1}{2} & \frac{1}{2}\end{bmatrix} \begin{bmatrix} F^t_1\\ F^t_2\\ F^t_3\\ F^t_4\\ F^t_5\end{bmatrix}. \tag{7}$$

By using the crossing matrix eq. (7) and the partial-wave expansions eqs. (3), the following kinematical-singularity free amplitudes are found:

$$A_1 \equiv \frac{1}{stu}\,F^s_2\,, \tag{8a}$$

$$A_2 \equiv F^s_1 - F^s_4\,, \tag{8b}$$

$$A_3 \equiv \frac{F^s_1+F^s_4}{2s^2}\,, \tag{8c}$$

$$A_4 \equiv \frac{F^s_3+F^s_5}{2u^2}\,, \tag{8d}$$

$$A_5 \equiv \frac{F^s_3-F^s_5}{2t^2}\,. \tag{8e}$$

By defining $\nu_u = \frac{1}{2}(s-t)$, the crossing properties of the invariant amplitudes $A_i(\nu_u, u)$ are as follows:

$$A_{1,2,4}(\nu_u, u) = A_{1,2,4}(-\nu_u, u)\,, \tag{9a}$$

$$A_3(\nu_u, u) + A_5(\nu_u, u) = A_3(-\nu_u, u) + A_5(-\nu_u, u)\,, \tag{9b}$$

$$A_3(\nu_u, u) - A_5(\nu_u, u) = -A_3(-\nu_u, u) + A_5(-\nu_u, u)\,. \tag{9c}$$

The Regge behaviour of the above amplitudes is given by

$$A_1(\nu_u, u) \underset{\nu_u \text{large}}{\simeq} \left(\frac{\nu_u}{\nu_0}\right)^{\alpha_+(u)-2} \beta_1(u)\xi_+(\alpha)\,, \tag{10a}$$

$$A_2(\nu_u, u) \sim \left(\frac{\nu_u}{\nu_0}\right)^{\alpha_+(u)} \beta_2(u)\xi_+(\alpha)\,, \tag{10b}$$

$$A_4(\nu_u, u) \sim \left(\frac{\nu_u}{\nu_0}\right)^{\alpha_+(u)} \beta_4(u)\xi_+(\alpha)\,, \tag{10c}$$

$$A_3(\nu_u, u) + A_5(\nu_u, u) \sim \left(\frac{\nu_u}{\nu_0}\right)^{\alpha_+(u)-2} \beta_{3+5}(u)\xi_+(\alpha)\,, \tag{10d}$$

$$A_3(\nu_u, u) - A_5(\nu_u, u) \sim \left(\frac{\nu_u}{\nu_0}\right)^{\alpha_-(u)-2} \beta_{3-5}(u)\xi_-(\alpha)\,, \tag{10e}$$

where $\alpha_+(u)$ and $\alpha_-(u)$ are the leading f-A_2 and -A_1 Regge trajectories, respectively, and $\xi_\pm(\alpha)$ are their signature factors. We have omitted writing the pomeron contribution, since it does not enter into the following discussions.

3. – Real photons.

A simple sum rule for real photons can be derived (8-11) by using the asymptotic behaviour of the amplitude $A_3 - A_5$, given in eq. (10e). Writing the dispersion relation

$$\frac{A_3(\nu_u, u) - A_5(\nu_u, u)}{\nu_u} = \frac{2}{\pi}\int_{\nu_0}^{\infty} \frac{d\nu'}{\nu'^2 - \nu_u^2} \operatorname{Im}[A_3(\nu', u) - A_5(\nu', u)] \tag{11}$$

and using $\alpha_-(0) \simeq 0$, we have the superconvergence relation

$$\int_{\nu_0}^{\infty} d\nu \operatorname{Im}[A_3(\nu, 0) - A_5(\nu, 0)] = 0\,. \tag{12}$$

This can be converted in terms of the physical cross-section upon using the identity

$$A_3(\nu_u, u=0) - A_5(\nu_u, u=0) = \tag{13}$$
$$= \frac{1}{\nu^2}[f^s_{++,++}(s, z_s=-1) - f^s_{+-,-+}(s, z_s=-1)] =$$
$$= \frac{1}{\nu^2}[f^s_{++,++}(s, z_s=+1) - f^s_{+-,+-}(s, z_s=+1)] .$$

Thus eq. (12) becomes

$$\int\limits_0^\infty \frac{d\nu}{\nu}[\sigma_{++}(\nu) - \sigma_{+-}(\nu)] = 0 . \tag{14}$$

This is the analog of the Drell-Hearn sum rule for $\gamma\gamma$ scattering and has been discussed extensively in the literature (8-11). We rediscuss it for reasons advanced in the introduction. That is to say we first explicitly check that the « box » diagram, which is certainly a potential source of non-Regge terms, does not invalidate this sum rule. Once this is done, we shall proceed to a saturation of the sum rule and thus be able to compare the integrated low-energy cross-section as given by resonances with the corresponding Regge result.

Now the « box » diagram (fig. 2) is easily computed (13) with intermediate particles being spin-$\frac{1}{2}$ « elementary partons » of charges

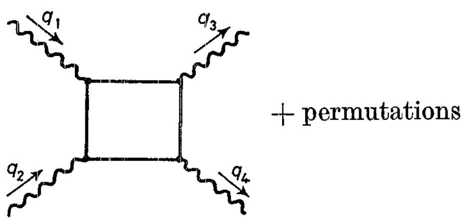

Fig. 2.

e_i (in units of e). Asymptotically, for the amplitudes of interest it gives

$$A_2^{\text{box}}(\nu_u, 0) \underset{\nu_u \text{large}}{\sim} \sum_i \frac{e_i^4}{\pi^2} e^4 , \tag{15a}$$

$$A_3^{\text{box}}(\nu_u, 0) - A_5^{\text{box}}(\nu_u, 0) \underset{\nu_u \text{large}}{\sim} -\frac{1}{2\pi\nu^2}\sum_i e_i^4 e^4 \cdot \tag{15b}$$
$$\cdot [\theta(\nu_u) - \theta(-\nu_u)]\left\{\frac{\pi}{2} + i\left[-3 + \ln\frac{|4\nu|}{m^2}\right]\right\},$$

(13) B. DE TOLLIS: *Nuovo Cimento*, **35**, 1182 (1965).

where m is the mass of the « parton ». Expression (15) does satisfy the superconvergence relation (12) and thus we may proceed to use the sum rule.

We saturate the sum rule eq. (14) with low-lying resonances of $J^P = 0^+$, 0^- and 2^+ to obtain

$$\sum_P \frac{\Gamma_i}{m_i^3} + \sum_S \frac{\Gamma_i}{m_i^3} + 5\sum_T \frac{\Gamma_i(2^+ \to ++)}{m_i^3} \simeq 5\sum_T \frac{\Gamma_i(2^+ \to +-)}{m_i^3}\,. \tag{16}$$

In terms of the coupling constants, the decay widths of scalar, pseudoscalar and tensor mesons are given by

$$\Gamma(\mathrm{P} \to \gamma\gamma) = \frac{e^4}{64\pi} g_\mathrm{P}^2 m_\mathrm{P}^3\,, \tag{17a}$$

$$\Gamma(\mathrm{S} \to \gamma\gamma) = \frac{e^4}{16\pi} g_\mathrm{S}^2 m_\mathrm{S}^3\,, \tag{17b}$$

$$\Gamma(\mathrm{T} \to +-) = \frac{e^4}{80\pi} m_\mathrm{T}^3 |G_{+-}|^2\,, \tag{17c}$$

$$\Gamma(\mathrm{T} \to ++) = \frac{e^4}{80\pi} m_\mathrm{T}^3 \frac{1}{24} |G_{++}|^2\,. \tag{17d}$$

Thus eq. (16) may be rewritten as

$$\sum_i \left\{ g_{\mathrm{P}i}^2 + 4g_{\mathrm{S}i}^2 + \frac{1}{6}|G_{++}^i|^2 \right\} \simeq 4\sum_i |G_{+-}^i|^2\,. \tag{18}$$

The naive quark model and the use of the chirality relation $g_{\varepsilon\gamma\gamma} \simeq g_{\pi\gamma\gamma}$ lead us finally to obtain for the f-meson

$$10 g_{\pi\gamma\gamma}^2 + \frac{6}{25}|G_{++}^f|^2 = \frac{144}{25}|G_{+-}^f|^2\,. \tag{19}$$

From finite-energy sum rules [14] and other independent arguments [11] G_{++}^f is found to be about the same as G_{+-}^f, so that we can estimate, using eqs. (19) and (17c, d),

$$\Gamma(\mathrm{f} \to \gamma\gamma) \simeq 9.2\ \mathrm{keV}\,, \tag{20}$$

to be compared with the FESR value of 4.7 keV (*).

A slightly different argument to justify that G_{++} and G_{+-} are of the same

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([14]) B. SCHREMPP-OTTO, F. SCHREMPP and T. WALSH: *Phys. Lett.*, **36** B, 463 (1971).

(*) The value given in ref. ([14]) for $\Gamma(\mathrm{f} \to \gamma\gamma)$ is 5.7 keV, which seems to be in contradiction with eqs. (7) and (10) of that paper.

order of magnitude results from considering FESR for the amplitudes A_2 and A_3+A_5. From eqs. (10p, d), the residue function β_2 will be much smaller than β_{3+5} because the resonances tend to cancel in A_2, whereas they add in A_3+A_5. This in turn leads to much smaller tensor couplings to photons of the same helicity compared to those with opposite helicity.

The above results allow us to give a reliable evaluation of the integrated resonant cross-section in the low-energy region for ee $\to$ ee + hadrons. As is well known, one has [1-3]

$$\sigma(\mathrm{ee}\to\mathrm{ee}+\text{hadrons})\simeq\left(\frac{\alpha}{\pi}\right)^2\left[\ln\left(\frac{E}{m_e}\right)\right]^2\int_{s_0}^{4E^2}\frac{\mathrm{d}s}{s}\,\sigma_{\gamma\gamma}(s)\,f\left(\frac{s}{4E^2}\right),\tag{21}$$

where $f(y)=-(2+y)^2\ln y-2(1-y)(3+y)$. The pseudoscalar contribution can be explicitly evaluated by considering the physical mesons and widths. For 0^+ and 2^+ mesons, which all lie roughly between $(1\div1.5)$ GeV, an average value for $f(y)$ can be used; thus simplifying the integral in eq. (21). We obtain from eqs. (5) and (19)

$$\int_{0^+,2^+}\frac{\mathrm{d}s}{s}\,\sigma_{\gamma\gamma}(s)\simeq\frac{128\pi^2\Gamma(\pi^0\to\gamma\gamma)}{m_\pi^3}\simeq1.4\ \mu\mathrm{b}\,.\tag{22}$$

Now we can compare our result with the Regge estimate, obtained through factorization [5-15],

$$\sigma_{\gamma\gamma}(s)\simeq0.24\ \mu\mathrm{b}+\frac{0.27}{\sqrt{s}}\ \mu\mathrm{b\ GeV}\,,\tag{23}$$

so that

$$\int_{\sim m_\rho^2}^{\sim3\,\mathrm{GeV}^2}\frac{\mathrm{d}s}{s}\,\sigma_{\gamma\gamma}^{\mathrm{R}}(s)\simeq0.4\ \mu\mathrm{b}\,.\tag{24}$$

The large discrepancy between eqs. (22) and (24) strengthens our belief that the simple resonance $\leftrightarrow$ Regge duality is not valid for this process. Thus, non-Regge terms are required. According to our hypothesis the box diagram is a source of non-Regge terms.

We can estimate this contribution very crudely as follows:

$$\sigma_{\gamma\gamma}^{\mathrm{box}}(s)\sim\frac{4\pi\alpha^2}{s}\sum_i e_i^4e^4\ln\frac{s}{m_Q^2}\,,\tag{25}$$

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[15] J. ROSNER: BNL preprint CRISP 71-26 (1971).

where m_Q stands for the average mass of the quarks. Then, for $\sum_i e_i^4 = \frac{2}{3}$,

$$\int_{s_1}^{s_2} \frac{ds}{s} \sigma_{\gamma\gamma}^{box}(s) \simeq \frac{8\pi\alpha^2}{3} \left\{ \frac{1}{s_1}\left(\ln\frac{s_1}{m_Q^2}+1\right) - \frac{1}{s_2}\left(\ln\frac{s_2}{m_Q^2}+1\right)\right\}. \tag{26}$$

Since eq. (26) depends on m_Q only logarithmically, the result is rather insensitive to the precise value of the quark mass. Choosing $m_Q \sim 300$ MeV, we get

$$\int_{m_\rho^2}^{3\,\text{GeV}^2} \frac{ds}{s} \sigma_{\gamma\gamma}^{box}(s) \approx 0.6\ \mu\text{b}\ , \tag{27}$$

which, when added to eq. (24), improves considerably the comparison with the resonance integral (22). Of course, we should bear in mind that we have used asymptotic formulae for both Regge and box terms while doing these low-energy estimates, and possibly nonleading terms may not be negligible. Similarly, uncertainties of $(20 \div 30)\%$ can be present in eq. (22), due to quark model relations used for the couplings.

We will briefly discuss a sum rule for the amplitude A_2 obtained under the extreme hypothesis that all non-Regge terms are given by the box diagram.

From eqs. (10*b*) and (15*a*) we obtain

$$\sum_i \frac{e_i^4 e^4}{\pi^2} = -\frac{2}{\pi}\int_0^\infty \frac{ds}{s}\left[\operatorname{Im} A_2^R(s,0) + \beta_2(0)\left(\frac{s}{s_0}\right)^{\alpha_+(0)}\right]. \tag{28}$$

If we lack a knowledge of $\beta_2(0)$, the sum rule cannot be tested directly. However, if we make the additional hypothesis that the leading $\alpha_+(0)$ does not couple (appreciably) to $c\bar{c}$ states, then we can try to saturate eq. (28) by the low-lying 0^-, 0^+ and 2^+ $c\bar{c}$ mesons and we get (with $\sum e_c^4 = 16/27$)

$$\frac{4}{27}\frac{\alpha^2}{\pi} \simeq \frac{\Gamma(\eta_c \to \gamma\gamma)}{m_{\eta_c}} - \frac{\Gamma(\chi \to \gamma\gamma)}{m_\chi} - \frac{5\Gamma(2^{+c} \to ++)}{m_{2^+}}\ , \tag{29}$$

which leads to the lower bound (with $m_{\eta_c} = 2.8$ GeV)

$$\Gamma(\eta \to \gamma\gamma) \geqslant \frac{4}{27}\frac{\alpha^2}{\pi} m_{\eta_c} = 7\,\text{keV}. \tag{30}$$

4. – Virtual photons.

Annihilation of virtual photons into hadrons has been studied by many authors ([16]) in different regions of q_1^2, q_2^2 and s which define various kinematic limits corresponding to the possible fixed ratios of these variables. In particular, it has been proposed ([16]) as a test of the algebra of bilocal operators in close analogy with the proposal by GROSS and TREIMAN ([17]) for the process $e^+e^- \to \mu^+\mu^- + X$.

In the following, we will restrict ourselves to the kinematic region in which $Q^2 \to \infty$ with ξ and s fixed, where

$$Q = \frac{1}{2}(q_2 - q_1) \qquad \text{and} \qquad \xi = \frac{q_2^2 - q_1^2}{q_2^2 + q_1^2}.$$

It has been shown ([16-18]) that the light-cone algebra and the algebra of bilocal operators are sufficient in this limit to determine the asymptotic behaviour of the current $\times$ current amplitude which becomes identical to the result obtained for pair creation of massless quarks by two virtual photons (box diagram in the parton model).

More explicitly, one finds that only two independent helicity amplitudes survive in the above limit, namely

$$W_{++,++}(\xi, s) \equiv \tfrac{1}{2}[g_S(\xi, s) + g_P(\xi, s)]\,, \tag{31a}$$

$$W_{++,--}(\xi, s) \equiv \tfrac{1}{2}[g_S(\xi, s) - g_P(\xi, s)]\,. \tag{31b}$$

Thus the full tensor $W^{\mu\nu\lambda\sigma}$ in this limit is reduced to only those to which scalars and pseudoscalars contribute. Furthermore, $g_{S,P}(\xi, s)$ scale ([17]), *i.e.*

$$\lim_{\substack{s\to\infty \\ \xi\ \text{fixed}}} g_S(\xi, s) = g_S(\xi) = \sum_i e_i^4 \xi^4 \int_{-1}^{+1} \frac{dz}{2\pi} \frac{z^2(1-z^2)}{(1-z^2\xi^2)^2}\,, \tag{32a}$$

$$\lim_{\substack{s\to\infty \\ \xi\ \text{fixed}}} g_P(\xi, s) = g_P(\xi) = \sum_i e_i^4 \xi^2 \int_{-1}^{+1} \frac{dz}{2\pi} \frac{(1-z)^2}{(1-z^2\xi^2)^2}\,, \tag{32b}$$

where e_i is the charge of the i-th quark (in units of the electric charge e) and the sum accounts for colour. The r.h.s. in eqs. (32) are the box diagram contributions to the helicity amplitudes $W_{++,\pm\pm}$.

([16]) For a review, see for example H. TERAZAWA: *Rev. Mod. Phys.*, **45**, 615 (1973).
([17]) D. GROSS and S. TREIMAN: *Phys. Rev. D*, **4**, 2105 (1971).
([18]) V. CHERNYAK and V. SERBO: Novosibirsk preprint TP-79 (1973).

It has been observed ([19]) that the s-channel resonance contributions survive in the above limit and indeed exhibit a close similarity with the algebraic result of eqs. (32). In the following, we study this connection in more detail, showing that, in fact, a duality relation exists between the resonance contribution and the quark parton result.

Let us consider first the π^0 contribution $\gamma\gamma$ scattering. If we define the $\pi^0\gamma\gamma$ vertex function as

$$T^{(\pi)}_{\mu\nu}(q_1, q_2) = \varepsilon_{\mu\nu\lambda\sigma} q_1^\varrho q_2^\sigma F_\pi(q_1^2, q_2^2) \tag{33}$$

with $F_\pi(0, 0) = g_{\pi^0\gamma\gamma}$, its contribution to the nonvanishing helicity amplitudes is given by

$$T^{(\pi)}_{++} = -T^{(\pi)}_{--} = i\sqrt{\nu^2 - m_\pi^2 Q^2}\, F_\pi(q_1^2, q_2^2)\,, \tag{34}$$

where $\nu = \frac{1}{2}(q_2^2 - q_1^2)$. The asymptotic behaviour of $F_\pi(q_1^2, q_2^2)$ is determined by the leading light-cone singularity and one has ([20,21])

$$F_\pi(q_1^2, q_2^2) \xrightarrow[q_2^2 \text{ fixed}]{q_1^2\to\infty} \frac{2}{q_1^2} f_\pi \tag{35}$$

and

$$T^{(\pi)}_{++} \xrightarrow[\xi=\pm 1]{Q^2\to\infty} - \pm 2if_\pi\,. \tag{36}$$

Therefore, we find that

$$W^{(\pi)}_{++,\pm\pm} \xrightarrow[\xi^2=1]{Q^2\to\infty} \pm 4\pi f_\pi^2\, \delta(s - m_\pi^2)\,. \tag{37}$$

A similar calculation can be repeated for the scalar SU_3 singlet ε-meson and one finds ([22]) through the light-cone expansion

$$T^{(\varepsilon)}_{++} \xrightarrow[\xi^2=1]{Q^2\to\infty} i\frac{8}{3} f_\varepsilon \tag{38}$$

and

$$W^{(\varepsilon)}_{++,\pm\pm} \xrightarrow[\xi^2=1]{Q^2\to\infty} \frac{64\pi}{9} f_\varepsilon^2\, \delta(s - m_\varepsilon^2)\,. \tag{39}$$

([19]) G. Köpp, T. Walsh and P. Zerwas: *Nucl. Phys.*, **70** B, 461 (1974).
([20]) R. Brandt and G. Preparata: *Phys. Rev. Lett.*, **25**, 1530 (1970).
([21]) E. Etim and M. Greco: CERN preprint TH-2174 (1976).
([22]) M. Greco and H. Inagaki: *Phys. Lett.*, **65** B, 267 (1976).

Summing over the entire SU_3 nonets of pseudoscalar and scalar mesons, we get finally through eqs. (37) and (39)

$$\int_{P,S} \mathrm{d}s\, W_{++,--}(s,\xi^2=1) = -16\pi\left(f_\pi^2 - \frac{2}{3}f_\varepsilon^2\right) = -\frac{16\pi}{3}f_\pi^2\,, \tag{40}$$

where in the last line we have used the chiral relation $f_\varepsilon \simeq f_\pi$.

On the other hand, from eqs. (31*b*) and (32), we find that in the scaling limit

$$W_{++,--}(\xi^2=1) = -\frac{1}{2\pi}\sum_i e_i^4\,. \tag{41}$$

The quality relation between the resonance contributions (40) and the algebra of bilocal current result (41) follows directly upon averaging the latter over a suitable range of s. With $\Delta s = 2m_\rho^2$ the equation

$$\int \mathrm{d}s\, W^{\mathrm{R}(P,S)}_{++,--}(s,\xi^2=1) = \int \mathrm{d}s\, W^{\mathrm{box}}_{++,--}(\xi^2=1) \tag{42}$$

is satisfied with

$$\sum_i e_i^4 = \frac{1}{3}\frac{8\pi^2}{f_\rho^2} = \frac{1}{3}R\,. \tag{43}$$

In deriving the last line, the KSFR relation $2f_\rho^2 f_\pi^2 = m_\rho^2$ has been used and the last equality in eq. (43) holds in the EVMD model ([23]). In the fractionally charged three-triplet model, one has for comparison $\sum_i e_i^4 = \frac{2}{3} = \frac{1}{3}R$. Thus the proposed relation is completely self-consistent.

Due to the similarities between 1^+ and 2^+ resonance contributions (ref. ([19,24])) to $W_{++,--}$ and those of the 0^- and 0^+ mesons, it is very likely that the duality result eq. (42) holds generally for all resonances contributing to virtual $\gamma\gamma$ scattering in this limit. In turn, the above result reinforces our previous conclusion that the hadronic Regge $\leftrightarrow$ resonance duality is not valid in real $\gamma\gamma$ scattering.

The above results can be extended to include charm through the contributions of the 0^- and 0^+ $c\bar{c}$ states, η_c and χ. From ref. ([21,22]), one finds that

$$T^{(\eta_c)}_{\pm\pm} \xrightarrow[\xi^2=1]{Q^2\to\infty} \pm\frac{8\sqrt{2}}{3}\,i f_{\eta_c}\,, \tag{44a}$$

$$T^{(\chi)}_{\pm\pm} \xrightarrow[\xi^2=1]{Q^2\to\infty} i\frac{16}{9}f_\chi \tag{44b}$$

([23]) A. Bramon, E. Etim and M. Greco: *Phys. Lett.*, **41** B, 609 (1972); M. Greco: *Nucl. Phys.*, **63** B, 398 (1973).

([24]) M. Greco: talk given at the *XII Rencontre de Moriond, Flaine, March 1977*; Frascati preprint LNF-77/11 (1977).

and

$$W^{(\chi,\eta_c)}_{++--} \xrightarrow[\xi^2=1]{Q^2\to\infty} \frac{256}{81}\pi f_\chi^2 \delta(s-m_\chi^2) - \frac{128}{9}\pi f_{\eta_c}^2 \delta(s-m_{\eta_c}^2)\,. \tag{45}$$

If we use $\sum_i e_i^4 = 16/27$, $f_\chi^2 \simeq 2f_{\eta_c}^2$ [22] and $f_{\eta_c} = m_\psi/\sqrt{2}\, f_\psi$ [21], the duality sum rule eq. (42) holds provided $\Delta s \simeq 10\ \text{GeV}^2$, which is quite consistent with the masses of η_c and χ mesons.

5. – Conclusion.

In this paper we have presented arguments based on various sum rules, asserting that, both for real as well as for virtual photons, the resonances occuring in photon-photon scattering are not dual to the Regge terms. Since non-Regge terms may be present in the absorptive parts as well, the low-energy cross-section estimates based on the duality with the Regge-asymptotic behaviour—the latter obtained through factorization—are probably quite erroneous. Our results suggest that the integrated $\gamma\gamma$ cross-section in the low-energy region is larger by about a factor of three.

Appendix

In this appendix we first compute the various divergences of the 4-current absorptive parts for charged SU_2 currents. As stated in the introduction, these are in general not zero but are related to smaller (2- and 3-) current amplitudes. Next we show by an explicit calculation that a model which includes only the « box diagrams » (1-fermion loop) is a consistent one for the absorptive parts, in that it satisfies the above divergence conditions. This is of interest to us in the text of the paper in which we advocate that it provides *the* non-Regge part of the 4-current absorptive amplitude.

The absorptive part of the forward ($q_1 = q_3$, $q_2 = q_4$) amplitude for charged SU_2 currents can be decomposed as

$$W^{abcd}_{\mu\nu\lambda\sigma} = \sum_n (2\pi)^4\delta^4(q_1+q_2-p_n)\int \mathrm{d}^4x\,\mathrm{d}^4y\, \exp[iq_1(x-y)]\cdot \\ \cdot\langle 0|\overline{T}[J^c_\lambda(y)J^d_\sigma(0)]n\rangle\langle n|T[J^a_\mu(x)J^b_\nu(0)]0\rangle\,. \tag{A.1}$$

Contracting eq. (A.1) with q_1^μ gives, after an integration by parts,

$$q_1^\mu W^{abcd}_{\mu\nu\lambda\sigma} = i\sum_n (2\pi)^4\delta^4(q_1+q_2-p_n)\int \mathrm{d}^4y\,\exp[-iq_1\cdot y]\cdot \\ \cdot\langle 0|\overline{T}(J^c_\lambda(y)J^d_\sigma(0)|n\rangle \mathrm{d}^4x\,\frac{\partial}{\partial x^\mu}\langle n|T(J^a_\mu(x)J^b_\nu(0))|0\rangle\,. \tag{A.2}$$

Using the SU_2 equal-time algebra

$$[J_0^a(x), J_\mu^b(y)]\delta(x^0-y^0) = i\varepsilon^{abc}\delta^4(x-y)J_\mu^c(x)\,,$$

we obtain

$$\frac{\partial}{\partial x^\mu}\langle n|T(J_\mu^a(x)J_\nu^b(0))|0\rangle = i\varepsilon^{abc}\langle n|J_\nu^c(x)|0\rangle\delta^4(x)\,.$$

Thus eq. (A.2) simplifies to

$$\text{(A.3)}\qquad q_1^\mu W_{\mu\nu\lambda\sigma}^{abcd} = -\sum_n (2\pi)^4\delta^4(q_1+q_2-p_n)\int d^4y\langle 0|T(J_\lambda^c(y)J_\sigma^d(0)|n\rangle\cdot$$
$$\cdot\exp[-iq_i y]\,\varepsilon^{abe}\langle n|J_\nu^e(0)|0\rangle = -\varepsilon^{abe}\int d^4x\, d^4y \exp[-iq_i x + i(q_1+q_2)y]\cdot$$
$$\cdot\langle 0|\overline{T}(J_\lambda^c(x)J^d(0)J_\nu^e(y))|0\rangle = -\varepsilon^{abe}\tau_{\nu\lambda\sigma}^{ecd}(q_1+q_2, q_1, q_2)\,,$$

where $\tau_{\mu\nu\lambda}^{abc}(q_1, q_2, q_1+q_2)$ is the absorptive part of the 3-current amplitude defined as

$$\text{(A.4)}\qquad \tau_{\mu\nu\lambda}^{abc}(q_1, q_2, q_1+q_2) = \int d^4x\, d^4y \exp[+iq_i x - i(q_1+q_2)y]\cdot$$
$$\cdot\langle 0|J_\lambda(y)\,T(J_\mu^a(x)J_\nu^b(0))|0\rangle\,.$$

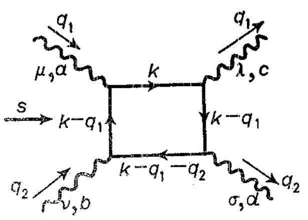
Contracting with another momentum, other Ward identities may be obtained. For example, if we contract with q_2, we obtain, upon an integration by parts and the use of the equal-time algebra,

$$\text{(A.5)}\qquad q_1^\mu q_2^\sigma W_{\mu\nu\lambda\sigma}^{abcd} = \varepsilon^{abe}\varepsilon^{cdf}\int d^4x \exp[i(q_1+q_2)x]\langle 0|J_\nu^e(0)J_\lambda^f(x)|0\rangle =$$
$$= \varepsilon^{abe}\varepsilon^{cdf}\,\mathrm{Im}\,\pi_{\nu\lambda}^{ef}(q_1+q_2)\,,$$

where $\mathrm{Im}\,\pi_{\nu\lambda}^{ef}(q_1+q_2)$ is the imaginary part of the 2-current amplitude. Clearly all other Ward identities can similarly be obtained.

Now we calculate the absorptive parts in the 1-loop approximation to show that these Ward identities are indeed satisfied for such a model, thus showing their internal consistency.

As a prototype of the general case, consider the absorptive part of the forward scattering amplitude of 4 charged SU_2 currents for the special kinematical configuration $q_i^2 = 0$ (this is only to reduce the algebra, the result is valid for arbitrary (spacelike) masses q_i^2 of the currents).



$+$ permutations.

Fig. 3

The absorptive part in the s-channel, of the diagram shown in fig. 3, is given by

$$(A.6)\qquad W^{abcd}_{\mu\nu\lambda\sigma} = \pi^2 \operatorname{Tr}\left(\left[\frac{\tau_b}{2},\frac{\tau_a}{2}\right]\left[\frac{\tau_c}{2},\frac{\tau_d}{2}\right]\right)\int\frac{d^4k}{(2\pi)^4}\cdot$$
$$\cdot \operatorname{Tr}\{\gamma_\mu \not{k}\gamma_\lambda(\not{k}-\not{q}_1)\gamma_\sigma(\not{q}_1+\not{q}_2-\not{k})\gamma_\nu(\not{k}-\not{q}_1)\}\,\delta(k^2)\,\delta[(Q-k)^2]\left[\frac{1}{(k-q_1)^2}\right]^2,$$

where $Q = q_1 + q_2$ and $s = Q^2$. As said before, for algebraic simplicity, we have let $q_i^2 = 0$ and, since there is no infra-red divergence, we have also set the fermion mass to be zero as well.

Now let us take two « divergences », *i.e.* compute

$$(A.7)\qquad q_1^\mu q_2^\sigma W^{abcd}_{\mu\nu\lambda\sigma} = \pi^2 \operatorname{Tr}\left(\left[\frac{\tau_b}{2},\frac{\tau_a}{2}\right]\left[\frac{\tau_c}{2},\frac{\tau_d}{2}\right]\right)\int\frac{d^4k}{(2\pi)^4}\,\frac{\delta(k^2)\,\delta(s-2kQ)}{4(kq_1)^2}\,J_{\nu\lambda},$$

where

$$(A.8)\qquad J_{\nu\lambda} = \operatorname{Tr}\{\not{q}_1\not{k}\gamma_\lambda(\not{k}-\not{q}_1)\not{q}_2(\not{q}_1+\not{q}_2-\not{k})\gamma_\nu(\not{k}-\not{q}_1)\}\,.$$

If we use $q_i^2 = 0$ and the two Dirac delta-functions present in eq. (A.7), $J_{\nu\lambda}$ of eq. (A.8) can be reduced to

$$(A.9)\qquad J_{\nu\lambda} = -16(kq_1)^2\left\{k_\lambda Q_\nu + k_\nu Q_\lambda - 2k_\nu k_\lambda - \frac{2}{s}\eta_{\lambda\nu}\right\}.$$

Substituting eq. (A.9) into (A.7) and evaluating some integrals, we are led to

$$(A.10)\qquad q_1^\mu q_2^\sigma W^{abcd}_{\mu\nu\lambda\sigma} = \frac{1}{24\pi}\operatorname{Tr}\left(\left[\frac{\tau_b}{2},\frac{\tau_a}{2}\right]\left[\frac{\tau_c}{2},\frac{\tau_d}{2}\right]\right)[Q^2\eta_{\nu\lambda} - Q_\nu Q_\lambda]\,.$$

Now consider the absorptive part of the 2-current amplitude in the 1-loop approximation:

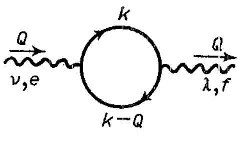


Fig. 4

For the diagram in fig. 4, we have

$$(A.11)\qquad \operatorname{Im}\pi^{ef}_{\nu\lambda} = \pi^2 \operatorname{Tr}\left(\frac{\tau_e}{2}\,\frac{\tau_f}{2}\right)\int\frac{d^4k}{(2\pi)^4}\,\delta(k^2)\,\delta(s-2kQ)\operatorname{Tr}\{\gamma_\nu\not{k}\gamma_\lambda(\not{k}-(\gamma\cdot Q))\} =$$
$$= 4\pi^2 \operatorname{Tr}\left(\frac{\tau_e}{2}\,\frac{\tau_f}{2}\right)\int\frac{d^4k}{(2\pi)^4}\,\delta(k^2)\,\delta(s-2kQ)\left\{(k_\lambda-Q_\lambda)k_\nu + \eta_{\nu\lambda}\frac{s}{2} + (k_\nu-Q_\nu)k_\lambda\right\} =$$
$$= \frac{1}{24\pi}\operatorname{Tr}\left(\frac{\tau_e}{2}\,\frac{\tau_f}{2}\right)[Q^2\eta_{\nu\lambda} - Q_\nu Q_\lambda]\,.$$

Using eqs. (A.10) and (A.11), we get

$$q_1^\mu q_2^\sigma W_{\mu\nu\lambda\sigma}^{abcd} = \varepsilon^{abe} \varepsilon^{cdf} \operatorname{Im} \pi^{ef}(Q) \,, \tag{A.12}$$

which is the required result (see eq. (A.5)).

● RIASSUNTO

Si discute la possibilità che le risonanze siano duali ai termini di Regge nella diffusione fotone-fotone, per fotoni sia reali che virtuali. Varie regole di somma suggeriscono che tale dualità non sia valida.

Правила сумм и члены не реджеевского типа в фотон-фотонном рассеянии.

Резюме (*). — Мы обсуждаем, являются ли резонансы дуальными с членами Редже в процессах реального и виртуального фотон-фотонного рассеяния. Рассмотрение различных правил сумм приводит нас к предположению, что дело обстоит не так.

(*) *Переведено редакцией.*