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F. Palumbo and V. I. Zakharov[x]: ELECTRONIC WIDTHS AND S-D COUPLING IN CHARMONIUM.

ABSTRACT: - It is shown that the electronic width of a pure S-state is not shared by the two states resulting from S-D coupling, and that relativistic corrections to the electronic widths are divergent with some tensor potentials otherwise accettable. The problem of S-D coupling is considered in the framework of dispersion sum rules.

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Within the perturbative scheme usually adopted for the tensor potential, one definite S-wave is coupled to one definite D-wave, and the electronic width of the pure S-wave is shared by the states resulting from S-D mixing. If S-D mixing is large, both states should have large electronic width. Now the experimental value of the electronic width of ψ' is rather small with respect to the theoretical expectation within models with no S-D mixing[1], requiring a large S-D mixing, but there is no other partner resonance with a comparable width.

The above considerations are based on nonrelativistic models. It is the purpose of this paper to show that such models in their present status are not adequate for a proper treatment of the tensor potential.

The first observation concerns the admissible behaviour at the origin of the tensor potential. In a field theoretical derivation such behaviour if like r^{-3}. Now for a non relativistic Hamiltonian with potentials more singular than r^{-2} there are two possibilities: Either it is unbounded from below, or it generates wave functions vanishing at the origin faster than any power[2]. Sometimes such singular potentials are used in the literature under the assumption that a proper regularization would not appreciably change the energy levels. It should be noted, however, that the electronic widths would then be defined by the very regularizing procedure, and it impossible to talk about electronic widths in the absence of a definite regularization procedure. This rises a first problem about the connection between field theory and non relativistic approximation.

We will show below that another problem originates from the fact that with tensor potentials behaving i) like const and ii) like r^{-1} relativistic corrections are divergent.
(For the sake of brevity we do not discuss the marginal case r^{-2}).

We will assume all the potentials to have power behaviour at the origin (as expected from field theory apart from logarithimic modifications). Moreover we will assume the central potential to behave like r^{-1}, (although a different behaviour could be discussed in the same way), because this is the relevant case. It is then possible to show that the two independent solutions of the

Schrodinger equation have the following behaviour

$$\text{case i)} \qquad \begin{aligned} R_S^{(1)} &\sim 1 + a_S^{(1)}\, r + b_S^{(1)}\, r^2 + c_S^{(1)}\, r^2 \ln r \\ R_D^{(1)} &\sim b_D^{(1)}\, r^2 + c_D^{(1)}\, r^2 \ln r \\ R_S^{(2)} &\sim r^4 \\ R_D^{(2)} &\sim r^2 \end{aligned} \qquad (1)$$

$$\text{case ii)} \qquad \begin{aligned} R_S^{(1)} &\sim 1 + a_S^{(1)}\, r + b_S^{(1)} r^2 + c_S^{(1)}\, r^2 \ln r \\ R_D^{(1)} &\sim \quad a_D^{(1)}\, r + b_D^{(1)} r^2 + c_D^{(1)} r^2 \ln r \\ R_S^{(2)} &\sim r^3 \\ R_D^{(2)} &\sim r^2, \end{aligned} \qquad (2)$$

where the constants are in general different from zero and can be expressed explicitely in terms of the parameters of the potential[3]. The physical states are those superpositions of the two independent solutions which vanish at infinity, showing that the magnitude of the mixing depends on the behaviour of the potential at infinity. It should be noted that the behaviour of eqs. (1) and (2) differs from the behaviour obtained in perturbation theory if only one S-wave and one D-wave are considered. On the other hand, if one includes in the perturbative expansion all the eigenstates of the Hamiltonian in the absence of tensor potential one can reproduce in explicit exemples the above behaviour[4]. The continuum of states is essential in order to reproduce the correct behaviour at r=0.

The additional problem previously mentioned is that matrix elements of the annhilation current with the functions $R^{(1)}$ are divergent both in case i) and ii). This follows immediately from the expression of the current where the first relativistic corrections are retained

$$j_i = \bar{c}\, \gamma_i\, c \sim \chi \left\{ \sigma_i - \frac{1}{3} \frac{q^2}{m^2}\, \sigma_i + \sum_{k=1}^{3} \sigma_k \frac{1}{2m^2} \left(p_i p_k - \frac{1}{3}\, \delta_{ik}\, p^2 \right) \right\} \chi \qquad (3)$$

Here χ are Pauli spinors and σ_i Pauli matrices. The first term is the nonrelativistic approximation which annhilates only S-waves,the second term is the relativistic correction to it, and the third term (which is also relativistic) annhilates D-waves. These relativistic corrections are divergent, involving expressions like $\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 R)\Big|_{r=0}$. There is nothing wrong, of course, with the relativistic calculation of the electronic width, but something goes wrong with the non relativistic calculation.

We conclude that it is not possible at present to calculate the electronic widths within the nonrelativistic approximation taking into account the relativistic corrections. This is a serious limitation to the extent that the S-D mixing in strong. It should be noted that there are no counterarguments left to a large S-D mixing. It is no longer true, in fact, that the width of a pure S-state should be shared by two S-D coupled states, as required by two-levels perturbation theory, because of the necessary presence of all the continuum of states. It is then possible to have one resonance with large S-D mixing and no partner resonance with large electronic width.

The last conclusion raises one additional problem: How can we determine the S-D mixing? In the framework of two-level perturbation theory it is directly determined by the electronic widths of the states resulting from the mixing. Now we see that there is not simple relation between these widths and the S-D mixing.

We can use dispersion sum rules in order to get additional information.

Let us consider sum rules for vacuum polarization induced by the interference of the e. m. current j_μ and the D-current j^D_μ of charmed quarks

$$j_\mu = e\, Q_c\, \bar{c}\, \gamma_\mu\, c$$
$$j^D_\mu = \frac{e\, Q_c}{2m_c^2}\, \bar{c}\, \gamma_\nu\, c(q_\mu\, q_\nu - \frac{1}{3}\, q^2 \eta_{\mu\nu}). \qquad (4)$$

Here Q_c and m_c are the charge and mass of the charmed quark, $e^2 = 4\pi\alpha$, $\eta_{\mu\nu} = g_{\mu\nu} - \frac{P_\mu\, P_\nu}{p^2}$, P_μ and q_ν being the sum and half the difference of the quarks momenta. The current j^D_μ has been defined in such a way that its non-

relativistic approximation coincides with the D-current of eq. (3).

The sum rules are (for details of derivation see ref. (5))

$$\frac{\alpha^2 A_n}{9\pi (4m_c)^n} = \sum_{\text{resonances}} \frac{\sqrt{\Gamma_e(R)\ \Gamma_D(R)}}{m_R^{2n+1}} + (\text{continuum}) \qquad (5)$$

where $\Gamma_e(R)$ is the electronic width of a resonance with mass m_R, $\Gamma_D(R)$ is the width induced by the j^D_μ-current and (continuum) stands for the contribution of pair production of charmed particles. The coefficients A_n can be found from an explicit calculation the quark loop of the figure

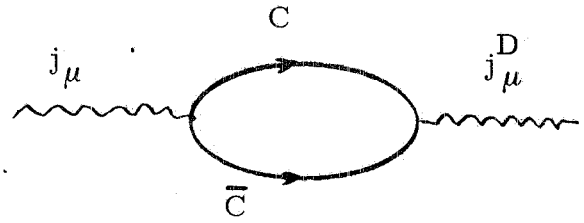

Fig. 1 - Vacuum polarization induced by the e.m. current j_μ and the D current j^D_μ.

$$An = 2\,Q_c^2 \int_0^1 dv\, v^6 (1-v^2)^{n-2}, \qquad n \geq 2 \qquad (6)$$

If one assumes that above threshold of pair production the physical cross section coincides with the quark cross section, the sum rules (5) can be interpreted as a kind of duality relations

$$\sum_{\text{resonances}} \frac{\sqrt{\Gamma_e(R)\ \Gamma_D(R)}}{m_R^{2n+1}} = \frac{\alpha^2}{9\pi (4m_c^2)^n}\, 2Q_c^2 \int_0^{v_o} dv\, v^6 (1-v^2)^{n-2}, \qquad (7)$$

where $v_o^2 = 1 - 4m_c^2/s_{\text{threshold}}$, $s_{\text{threshold}} \sim (4\ \text{GeV})^2$ being the physical threshold, and m_c being the mass of charmed quarks at short distances, which has been determined from sum rules for electronic widths[5] to be $m_c = 1.25$ GeV.

Eq. (7) involves Γ_D in a fully relativistic way, and can be related to the non relativistic S-D coupling if a consistent nonrelativistic scheme can be devised.

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