Invited paper at the "Desy Study Week on Future High Energy e^+e^- Rings"

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-77/13(R)
19 Aprile 1977

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G. Salvini(*) and S. Tazzari: ON THE OPTIMIZATION OF $e^+ e^-$ RINGS IN THE 2x40 ÷ 2x100 GeV RANGE.SOME FUTURE PERSPECTIVES. -

Abstract: - Some results are obtained on the cost and main parameters of a high-energy $e^+ e^-$ storage ring, through an optimization procedure carried out along lines similar to those of previous studies, with the main difference that mean radii R and costs are estimated under the assumption that the machine does not always run at its maximum energy E_{max}. We arrive at the conclusion that, if necessary, a mean radius R(E) of 4.5 $(E_{GeV}/100)^2$ km could be possible. Some estimates are given of the maximum energies attainable in case some, as of now unsolved, technical problems can be solved in the future.

The possibility of using an intermediate ring (~ 2x50 GeV c.m.) for a 2x100 GeV c.m. final energy machine, rather than a high energy injector, is suggested.

1. - INTRODUCTION

We have analysed the optimization of $e^+ e^-$ storage rings at energies from 2x40 to 2x100 GeV in the c.m. The criteria are similar to those indicated by B. Richter(1) and E. Keil(2) with some difference, specified in the following, the most important one being the use of the fact that the machine is not likely to work continuosly at maximum energy. The main costs taken into account are:

Tunnel, magnet, donut etc.	(cost/Km)
R.F. power installation (Klystrons etc)	(cost/MW)
R.F. Cavities	(cost/Km)
Electricity (in ten Years)	(cost/MW year)

We shall not consider the problems and costs of new sites. We shall however find that a limited increase in cost may allow quite a large reduction in R, so that the optimization is likely to be dominated by the actual conditions of the site, and/or by other technical considerations such as the possibility of designing a suitable lattice, expandibility etc.

The calculation convinced us that optimization studies are more ambiguous than one could think at first. In this connection it is worthwhile to distinguish between what one could call "absolute" and "relative" optimization. In the first case one proceeds as if he had been offered an ideal site, as

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(o) - Invited paper at the Desy Study Week on Future High Energy e^+e^- Rings - February 1977.

large as desired. This is the procedure followed in Ref. [1] and for LEP; it is very reasonable under present circumstances and leads to very large radii (meaning also that one can hope to push the machine above 2x100 GeV at a later stage).

A "relative" optimization procedure must be adopted if one wishes to get the best out of a given, limited site. In this case the overall cost curve as a function of R must be carefully studied, to know what has to be payed when, for a given energy, one moves away from the absolute optimum.

We all like, of cours, the perspective of building an $e^+ e^-$ ring with the largest possible radius (6-8 Km), guaranteeing the best exploitation of $e^+ e^-$ physics, with a machine tunnel which could possibly be large enough to accomodate the future Proton fixed target world machine (V A B). But it is nevertheless important to analyse which energies one can reach in a site which has the CERN dimensions (either the present CEEN Meyrin, or some possible extension around it).

The case for an $e^+ e^-$ ring inside the present SPS tunnel[3] and of its possible energy extension[4] does not fit into our present discussion of optimization, its parameters being already conditioned by the existing tunnel.

In the following paragraph we develop the cost equation according to our assumptions.

2. - MAIN EQUATIONS RELATING THE VARIOUS MACHINE PARAMETERS.

2. 1 - We take the expression of luminosity, L, from ref. [1]:

(1) $L = a \dfrac{\varrho P_B}{E^3 \beta_y^*}$ with: a = constant, ϱ = magnet radius, P_B = R F power to both beams, E = energy of each beam, β_y^* = β_y value at crossing.

The required R. F voltage per turn is

(2) $V = b_q \dfrac{E^4}{\varrho}$ where q = overvoltage factor, b = constant.

The power dissipated in the cavities is

(3) $P_D = \dfrac{V^2}{Zl}$ where: Z = Shunt impedence per unit cavity length, l = total length of the cavity structure;

and the voltage gradient is

(4) $G = \dfrac{V}{l}$.

We also set

$R = g\varrho$ where: R is the average radius in the arcs and g is assumed to be a constant factor (≈ 1), so that we can write

(5, a) $$P_B = \frac{L}{a} \beta_y^* g \frac{E^3}{R} ,$$

(5, b) $$G = \frac{V}{l} = bq\, g \frac{E^4}{Rl} ,$$

(5, c) $$P_D = \frac{b' q^2 g^2 E^8}{ZR^2 l} = \frac{b' q\, g\, E^4 G}{bZR} .$$

where b' is a constant.

If we measure our variables in the following units:

L	in units of 10^{32} cm^{-2}/s,
R, p, l	in Km,
P_B, P_D	in MW,
β_y^*	in cm,
E	in GeV,
Z	in MΩ/m;

the values of the constant are

$$\frac{1}{a} = 1.36 \times 10^{-5} \qquad b = 8.85 \times 10^{-8}$$

We can therefore write a cost equation similar to that of Ref. (1):

$$(6) \qquad C_T = C_c + C_p = \left[K_1 \times 2\pi R + K_2 (P_B + P_D) + (K_3 + K_1) l \right] + \left[K_4 \frac{T(P_B + P_D)}{\eta_{RF}} \right] + F,$$

where C_c is the capital cost and C_p the capitalized 10-year operating cost K_1 through K_4 are the unit costs of the various items, and are taken from Ref. (1) (see Table I). T is the total

TABLE I
(Unit costs taken from Ref. (1)).

Main ring (tunnel, magnets installation, controls)	K_1 = 12.8 M\$/km
RF power capital cost	K_2 = 0.58 M\$/MW
RF cavity	K_3 = 81 M\$/km
Power bill	K_4 = 50 \$/Mw·h
Assumed running time	T = 5000 h/y
RF system efficiency	η_{RF} = 0.75

10-year running time - F is a fixed cost and η_{RF} is the RF system efficiency; term $K_1 l$ has been added, assuming that to first approximation the unit cost of the ring portion dedicated to the RF is the same as that for the arcs, in order to allow for possible race-track like configurations of the ring. Equation (6) is of course subject to the same limitations as the analogous equation of Ref.(1), and in particular to the assumption that unit costs are constant over the whole range of parameter values spanned by the optimization. Using (5 a, b, c) we can reduce equation (6) to two variables only (R, G) and we obtain ($b_D = 10^3 b = 8.85 \times 10^{-5}$):

$$(7) \qquad C_T = 2\pi K_1 R + \left(K_2 + \frac{K_4 T}{\eta_{RF}} \right) \left(\frac{L \beta_y^* g}{aE} + b_D \frac{qg}{Z} G \right) \frac{E^4}{R} + K_3 bqg \frac{E^4}{RG} + F.$$

A minimum is obtained when

(8) $$G = G^{(o)} = (10^{-3}\, Z \; \frac{K_3 + K_1}{K_2 + \frac{K_4 T}{\eta_{RF}}}), \quad \text{note that G is independent of R and E,}$$

and

(9) $$R = R^{(o)} = E^2\,(U_1 + \frac{U_2}{E})^{1/2},$$

with

$$U_1 = \frac{1}{\pi K_1}\left[\frac{b^1 q^2 g^2}{Z}\,(K_1 + K_3)(K_2 + K_4\,\frac{T}{\eta_{RF}})\right]^{1/2},$$

$$U_2 = \frac{g\,B_y^* L}{a}\;\frac{K_2 + K_4 \frac{T}{\eta_{RF}}}{2\,\pi\,K_1}\,.$$

Therefore, as discussed in Ref. (1)

$$R^{(o)} \;\alpha\; E^2 \qquad \text{for } E \gg \frac{U_2}{U_1} \quad (\frac{U_2}{U_1} \approx 20).$$

Also note that, from (5,b) and (8) it follows that

$$1^{(o)} \alpha \; \frac{E^4}{R^{(o)}} \;\tilde{\alpha}\; E^2, \qquad \text{so that} \qquad (\frac{R}{1})^o \approx \text{constant.}$$

2.2 - In this derivation it has been assumed, following Ref. (1), that the machine always works (during its entire 10-year operating time) at maximum energy and luminosity. This is probably an unrealistic view; we therefore tried to explore the effects of a more true-to-life approach, such as to assume that, during its capitalized 10-year lifetime, the machine produces a costant integrated luminosity over an energy range extending from E_{MAX} down to $\varepsilon_o\, E_{MAX}$ ($\varepsilon_o \leq 1$).

Let ε be defined as

$$\varepsilon = \frac{E}{E_{MAX}} \qquad \text{whereby} \qquad \varepsilon_o = \frac{E_{MIN}}{E_{MAX}}$$

E_{MAX} and E_{MIN} being the upper and lower bounds of the explored energy region.
The overall (10-year) running time being T, let us define an "equivalent luminosity", Leq, such that the overall integrated luminosity over the $\varepsilon_o \div 1$ energy range is Leq x T.
Luminosity is a function of energy that we call L (ε).
According to the proposed model one must have

(10) $$\int_o^T dt = \frac{\text{Leq. } T}{1 - \varepsilon_o} \int_{\varepsilon_o}^1 \frac{d\varepsilon}{L(\varepsilon)},$$

and therefore

(11) $$\text{Leq.} = \frac{1 - \varepsilon_o}{\int_{\varepsilon_o}^1 \frac{d\varepsilon}{L(\varepsilon)}}$$

and finally

(12) $$\frac{dt}{d\varepsilon} = \frac{L_{eq.}\ T}{(1-\varepsilon_o)L(\varepsilon)} ,$$ t being the time spent at a given ε.

The capitalized operating cost C_p, must now be evaluated, by weighting the cost of power with equation (10). Let us write the dependence of P_B and P_D on energy (for a given maximum energy ring) in the form (from eq. (5)):

$$P_B^{(E)}(\varepsilon) = P_B\, E^3\, \frac{L(\varepsilon)}{L} , \qquad P_D^{(E)}(\varepsilon) = P_D\, E^8$$

where P_B and P_D are the values at maximum energy and luminosity and L is the maximum luminosity.

The operating cost C_p becomes

$$C_p = \frac{K_4}{\eta_{RF}} \int_{\varepsilon_o}^{4} (P_B^{(E)}(\varepsilon) + P_D^{(E)}(\varepsilon)) \frac{dt}{d\varepsilon} d\varepsilon = \frac{K_4 T}{\eta_{RF}} \frac{L_{eq.}}{(1-\varepsilon_o)} \int_{\varepsilon_o}^{1} \frac{P_B^{(E)}(\varepsilon)+P_D^{(E)}(E)}{L(\varepsilon)} d\varepsilon.$$

This can be written in form

$$C_P = \frac{T}{\eta_{RF}} (K_4^{(B)} P_B + K_4^{(o)} P_D),$$

where :

$$K_4^{(B)} = K_4 \left(\int_{\varepsilon_o}^{1} \frac{\varepsilon^3}{L} d\varepsilon\right) / \left(\int_{\varepsilon_o}^{1} \frac{d\varepsilon}{L(\varepsilon)}\right), \qquad K_4^{(D)} = K_4 \left(\int_{\varepsilon_o}^{1} \frac{\varepsilon^8}{L(\varepsilon)} d\varepsilon\right) / \left(\int_{\varepsilon_o}^{1} \frac{d\varepsilon}{L(\varepsilon)}\right).$$

The overall cost equation therefore becomes

(13) $$C_T = 2\pi K_1 R + \left[(K_2 + \frac{K_4^{(B)} T}{\eta_{RF}} \frac{L_M \beta_y^* g}{a E} + (K_2 + \frac{K_4^{(D)} T}{\eta_{RF}} b_D \frac{qg}{Z} G + K_3 \frac{bqg}{G} \right] \frac{E^4}{R} + F,$$

and (8) and (9) become:

(14) $$G^{(o)} = \left(10^{-3} Z \frac{K_3 + K_1}{K_2 + \frac{K_4^{(D)} T}{\eta_{RF}}}\right)^{1/2},$$

(15) $$R^{(o)} = E^2 \left(U_1' + \frac{U_2'}{E}\right)^{1/2},$$

U_1' is obtained from U_1 by replacing K_4 by $K_4^{(D)}$,

U_2' is obtained from U_2 by replacing K_4 by $K_4^{(B)}$.

3. - SOME BASIC CURVES

The average radius is larger than ϱ, the magnetic radius, and in the following we have assumed $R/\varrho = 1.2$, except when differently specified. Fig. 1 gives an example of the expected approximate relations between Energy and cost. Fig. 2 shows some of the most significant curves as obtained from cost function (13) together with curves obtainable from Ref. (1,2). The specific parameters of each curve are specified in the figure captions. In abscissa we plot radius R (R=1.2ϱ) in Km and in ordinates the total cost in million dollars.

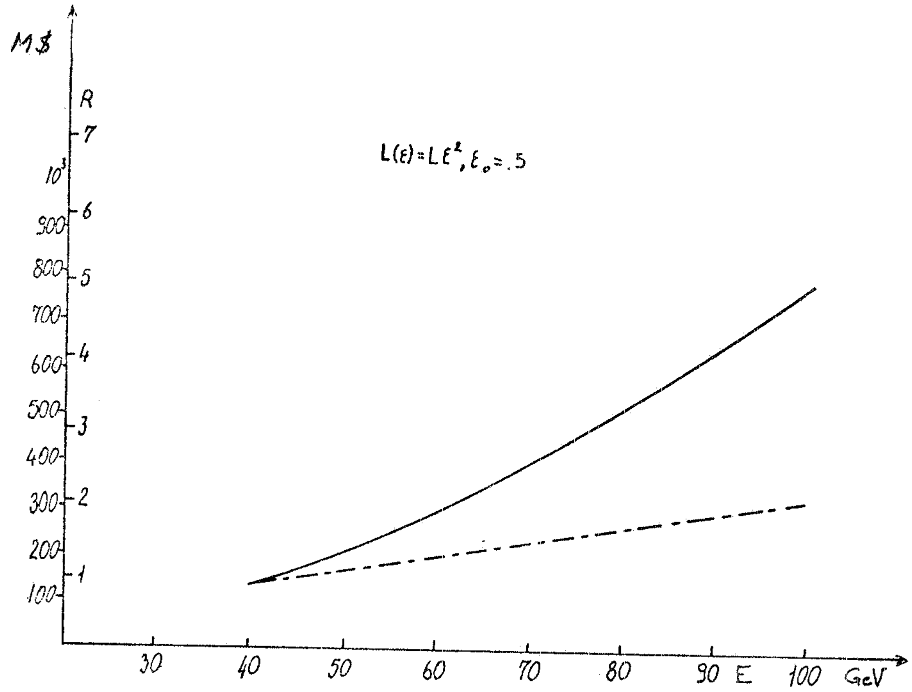

FIG. 1 - Cost of optimized machine vs energy (full curve). Linear behaviour (dash-dot curve) is shown for comparison.

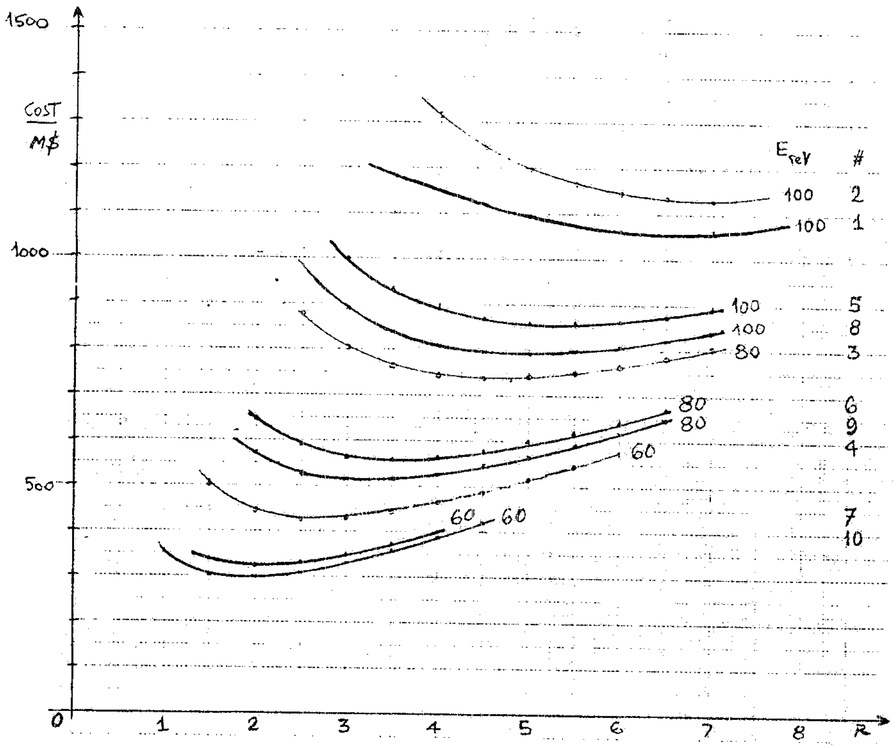

FIG. 2 - Cost vs radius for various energies. Curve 1 : case C of Ref. (1) ; Curves 2, 3, 4 : $\varepsilon_0 = 1$, $q = 1.13$, $g = 1.2$ - 100 GeV, 80 GeV, 60 GeV ; Curves 5, 6, 7 : $L(\varepsilon) = L_M$ = constant, $\varepsilon_0 = 0.5$, other parameters are the same as for curves 2, 3, 4 respectively ; Curves 8, 9, 10 : $L(\varepsilon) = L_M \varepsilon^2$, $\varepsilon_0 = 0.5$, other parameters are the same as for curves 2, 3, 4 respectively. If for case 8 a value of $G = 0.67$ is imposed, the result becomes practicaly coincident with curve 5. (N.B. - The cost figure do not include term F of equation (13)).

As expected, curves 2, 3, 4 of Fig. 2, corresponding respectively to 100, 80, 60 GeV per beam, give the highest costs C_T and radii R. They correspond to case $\varepsilon_o = 1$, i. e. to the hypothesis that the machine is continuously operated at maximum energy E_{MAX}.

The other curves correspond to the hypothesis, which as earlier said we consider more realistic, that the running time versus energy behaviour is that described in the previous paragraph. One notices a remarkable drop in the cost-radius curve. The change in G is a consequence of formula (14). We believe that values G=1. 16 ÷ 1. 3 M volt/m are still acceptable.

It appears rather clearly from this set of curves (as also remarked to one of us by Dr. Keil, with respect to capital costs) that the minima are rather flat.

The whole set of curves of Fig. 2 can be reduced to a more compact from by nothing that P_D, L, R, appearing in the leading terms of the cost equation, scale with good approximation like E^2. This suggests to plot curves 2-10 of Fig. 2 in normalized coordinates, R_{norm} and C_{norm} defined as:

$$C_{T\,norm} = C_T \left(\frac{100}{E_{GeV}}\right)^2 \quad , \quad R_{norm} = R \left(\frac{100}{E_{GeV}}\right)^2 .$$

The result are shown in Fig. 3.

A further step towards simplification is given in Fig. 4 where, within a ± 3% error, we take a simple curve (the 100 GeV curve) to represent all anergies in the 60 to 100 GeV range in normalized units. The abscissa and ordinate are directly those for the 100 GeV per beam case. From Figg. 2, 3, 4 we can derive the following conclusions:

- The changes in total cost are not significant over a wide range of R's, and for a given energy, a large variety of conditions may maintain the total cost to within a 10-15% fluctuation (Remember that the total cost also contains the fixed cost F (general facilities etc.) which has been estimated in 200 MS in Ref. (1) and which we did not add to Figg. 1-3)
- The choice of the radius is rather free, from the point of view of cost, both to the left (in case we we have site limitations) and to the right (in case we have plenty of space, and we wish to maintain a degree of expandibility (§ 4) to C.
- The assumptions made on the operating costs (values of ε_o , L) are rather important in determining total cost and radius.

These indications tend to confirm that cost optimization is not a stringent requirement and that the real site problems may be by far more decisive than any such exercise in optimization. It could perhaps be better to try a precise application on a few real sites and look for the optimum in those particular cases.

Among the problems one could run into when shrinking the radius and therefore increasing the R. F. power, it seems worthwhile to recall the following:

- The optimization contains the hypothesis that $\nu^3 \varrho$ can be kept constant (for constant luminosity) when varying ϱ (ν is the β-tron wavenumber, see Ref. (1). While at a first look this does not seem unfeasible, the matter has to be further investigated.
- The amount of R. F. power to be installed depends on the value of E_{MAX}. In Fig. 3 (in Megawatts) and in Fig. 4 (in "normalized" units $(P_B+P_D)_{norm} = (P_B+P_D).(\frac{E_{GeV}}{100})^2$) we have indicated the Megawatts to be installed for a given energy and radius. This point may pose a limit on the reduction in R, since to install a several hundred MW station on a site may be an absolute major enterprise.
- The increase of the percentage energy loss per turn. For instance $U_o/E \cong 2.7 \times 10^{-2}$ for an 80 GeV, 2 Km radius ring to be compared to 1.5×10^{-2} for an optimized 100 GeV ring (~7 Km radius)
- The increasing amount of synchrotron radiation power per meter, (going ≑ $1/R^2$ at given energy).
- The somewhat reduced energy resolution σ ·

$$(\sigma \doteqdot R^{-1/2})$$

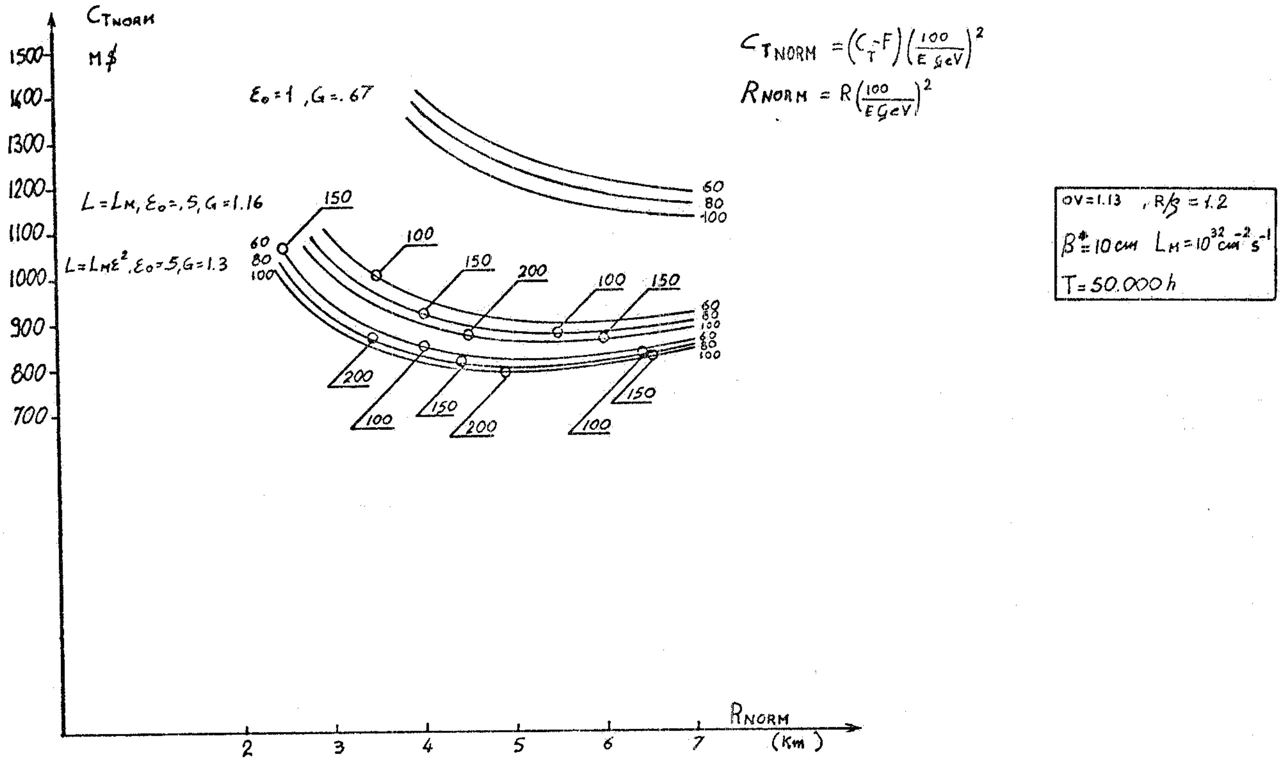

FIG. 3 - Normalized cost vs radius curves.

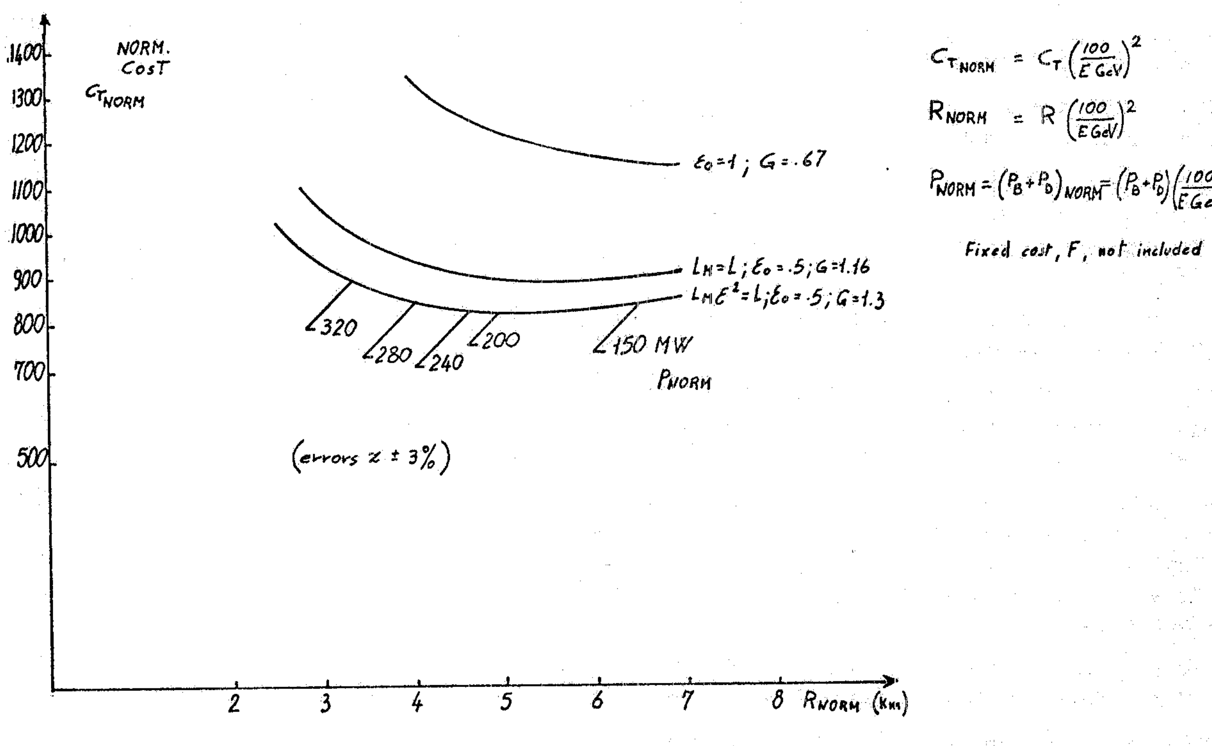

FIG. 4 - Normalized cost vs radius. Single curve taken to represent all energies in the 60 GeV to 100 GeV range.

Just to give a point of reference for our discussions, in case the radius must shrink, we could think of a grand Radius RM (for $e^+ e^-$ ring in a given site and with energy E) such as

$$RM = R + \sqrt{2}\,\frac{l}{8} = 4\left(\frac{E}{100}\right)^2 + \frac{\sqrt{2}}{8}\, l_o \left(\frac{E}{100}\right)^2 \frac{0.7}{G} =$$

$$= \left[4 + \frac{\sqrt{2}}{8}\, l_o \frac{0.7}{G}\right] \left(\frac{E}{100}\right)^2 \text{ Km},$$

l_o being the length of the cavities for G=0.7 at 100 GeV.

For the case G=1, one has:

$$RM = (4.5)\left(\frac{E_{GeV}}{100}\right)^2.$$

For E_{GeV} = 70, we get RM $\simeq$ 2.3 Km. This, for instance, is probably still a reasonable size for the CERN Meyrin area and surroundings.

4. - FUTURE PERSPECTIVES AT GIVEN RADIUS

The question is: once we have chosen a value of R, optimized for an energy E_{max} with present technologies what hopes can one have to push the machine to higher energies in the future? We can only consider a series of possibilities, which are still far from being guaranteed or tested.

- To save power and increase the gradient G (MVolt/m) by amplitude modulation of the cavities. (W. Schnell[5]).
  In this case one can hope both to reduce P_D and to increase the possible maximum value of G. Should one go from G = 0.7 to G = 2.0, energy could be increased of the order of $(2.0/0.7)^{1/4}$ = 1.30. P_D would increase by a factor $(1.30)^{8/m} \simeq 8/m$, m being the reduction one can hope to obtain by amplitude modulation. As W. Schnell remarked, high values of m could be obtained through a (still to be developed) fast tuner. With these methods, a 2x70 GeV ring could perhaps be pushed to 2x90 GeV.
- Another possibility to be investigated is the method suggested in Ref. (4).
  In this case the ring should have a varying magnetic field, the idea being to stay at maximum energy for a short time (6% duty cycle) with obviuos reduction in luminosity.
  An increase in energy of the order of 50% does not seem impossible. One difficult point would be the donut because of eddy currents.
  This method could possibly lead-to a 2x150 GeV machine, with strongly reduced luminosity above 100 GeV.
- Superconducting cavities. We shall learn at this meeting what we can expect to becomes available within the eighties.
  In the optimistic case, a value G=4 could be within reach.
  This would correspond to energies $(4/0.7)^{1/4} \simeq 1.55$ times the maximum conventional energies, which means about 310 GeV c.m. for LEP. Of course, with s.c. cavities we have a new free parameter namely luminosity.
  In fact it pays to reduce the beam current (i) since $P_B \gg P_D$.

A real limit to all energy increase could of course eventually be set by the ring lattice elements.

5. - A POSSIBILITY TO BE STUDIED: AN INTERMEDIATE ENERGY RING.

It is an experimental fact that the typical lowest useful operating energy of existing storage rings is $1/2 \div 1/3$ of their maximum energy, E_{MAX} because of the steep dependence of luminosity on energy.

For machines at present in the design or construction stage, methods have been proposed to control beam size, so as to keep luminosity versus energy constant down to about one half of

E_{MAX}. These methods have not been fully tested and, for instance in the case of LPT, require the machine aperture to be enlarged quite considerably.

It has also to be remarked that the low bending fields of very high energy future electron rings require a relatively high energy injector: for instance the proposed injector for LEP is a 20 GeV synchrotron that has to cope with an injection field in the main ring of only 100 Gauss.

Given this situation it is important to recall that the cost of an $e^+ e^-$ ring is proportional to E^2. A 2x50 GeV ring would therefore cost only ~ 1/4 of 2x100 GeV ring(*) (actually something more when fixed costs are added in).

It seems therefore worthwile to suggest that the possibility of a multistage machine (e.g.: 5 GeV booster, 50 GeV ring, 100 - 150 GeV ring, see Fig. 5) be studied in more detail. In fact it is not altogether obvious "a priori" that the cost difference of such a facility with respect to a more conventional one (possibly with a much larger aperture) would be such as to outweight its possible advantages.

One may add that a 50 GeV machine could already be in the area of weak interactions and therefore provide invaluable information for future extensions.

ACKNOWLEDGMENTS. -

We gratefully acknowledge useful discussion with M. Bassetti and M. Preger.

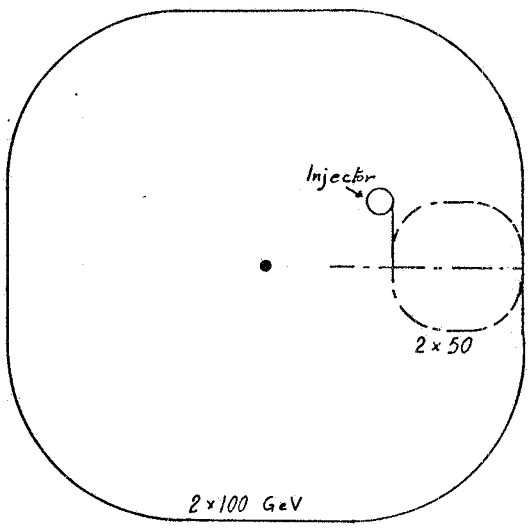

FIG. 5 - Relative size of a 2 x 100 GeV and a 2 x 50 GeV machine.

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(*) Just as a curiosity and an ispiration to possible future systems we notice that if the E^2 law were followed exactly the total cost C of a system of rings with energies E, E/2, E/4, ... each covering its appropriate luminosity range would be C=(cost at energy E)x(1+1/4+...)=$\frac{4}{3}$ that is 1/3 more than the cost of the ring with E_{MAX}=E.

APPENDIX - AFTER THE STUDY WEEK.

The same calculation has been re-run using the numbers presented for LEP by Eberhard Keil. From LEP costs we derive:

$2\pi K_1$ = 76.03 M$/KM

K_2 = 0.92 M$/MW

K_3 = 42.8 M$/MW (cavities - tunnel)

K_4 = K_4/ε = 56.5 $/MW-h (including klystron replacement cost).

We have assumed a running time of 5000 hours per year, an average to bending radius factor of 1.3 and an overvoltage of 1.18.

The total cost C_T under those condition is plotted in Fig. 6 versus the bending radius R for different maximum energies.

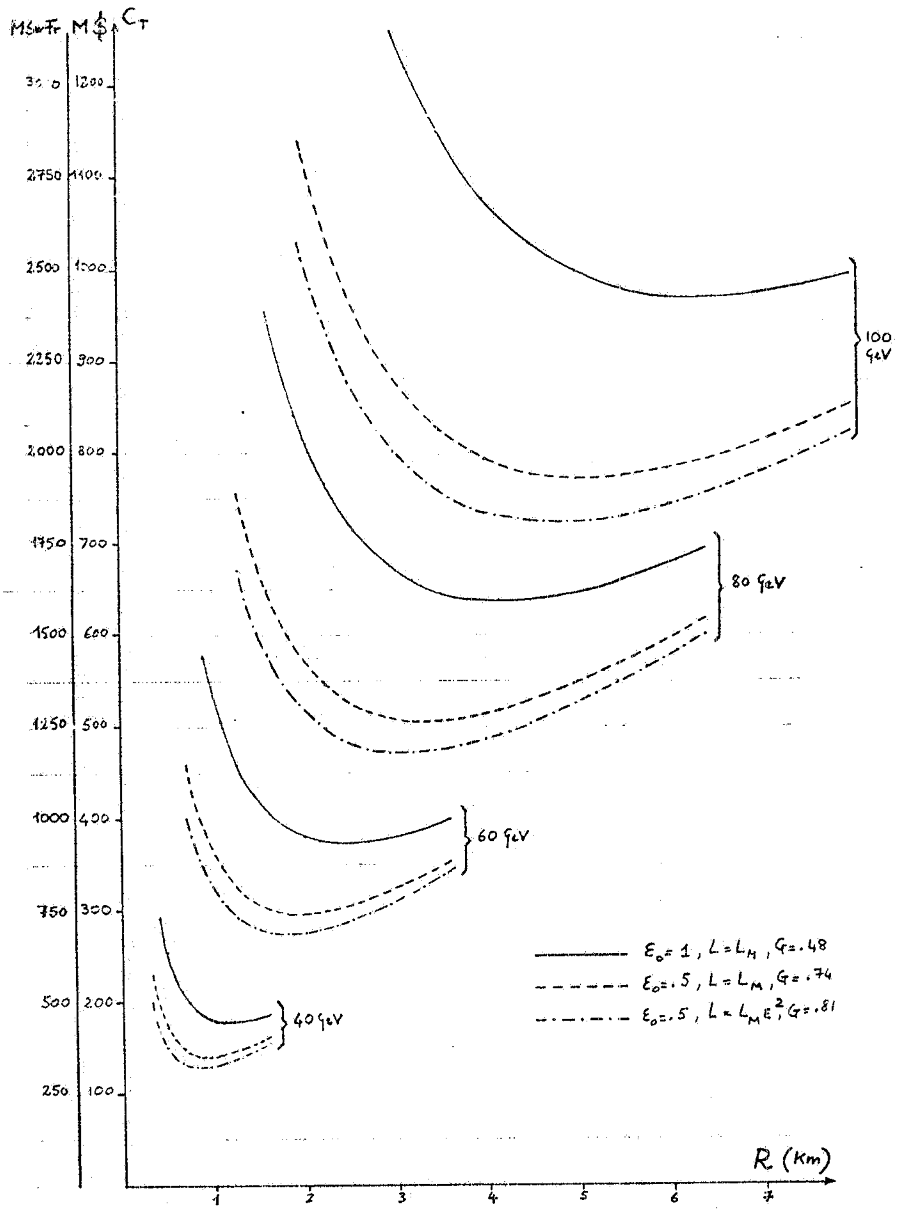

FIG. 6 - Cost vs radius for various energies using LEP unitary costs.