Talk given at the
XIIth Rencontre de
Moriond, Flaine

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M. Greco: RADIATIVE DECAYS OF OLD AND NEW MESONS.

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M. Greco: RADIATIVE DECAYS OF OLD AND NEW MESONS. -

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Abstract: A unified scheme is presented for radiative decays of pseudoscalar, scalar, axial and tensor mesons, where the basic currents are dominated by vector mesons and exhibit a quark structure asymptotically. Very good predictions are obtained for SU(3) meson decays, including those for which the quark model failed. Applied to the new mesons this scheme predicts radiative widths rather smaller than non relativistic calculations, in closer agreement with experiments.

1. - INTRODUCTION

I wish to discuss in some detail a recently proposed scheme[(1-3)] for the radiative decays of the old and new mesons. In this talk I will review some published and yet unpublished work on the argument, trying to make clear the basic ideas of the model and presenting a schematic derivation of the results. Much of the technical details can be found in the original papers[(1-3)].

The problem of radiative meson decays has been mostly studied[(4)] over the past few years in the framework of SU(3) or the naive quark model (QM), and vector meson dominance (VDM). Recent experimental data[(5-8)] however have been found difficult to be reconciled with theory, in particular the decays $\varrho \to \pi\gamma$, $K^{*o} \to K^o \gamma$ and $\varphi \to \eta\gamma$. Furthermore simple VMD is well known to relate only approximately the various decays, and sizeable currections are needed which are usually attributed to higher mass contributions.

From a different point of view it is well known that low energy theorems, together with the introduction of triangle and quadrangle anomalies[(9)], give a very good description of some decays as $\pi^o \to \gamma\gamma$, $\eta\,\eta' \to \gamma\gamma$, $\eta \to \pi^+\pi^-\gamma$, etc. In addition the quark current algebra fixes well defined constraints to the asymptotic behaviour of the vector functions $F_i(q_1^2, q_2^2)$ as $q_{1,2}^2 \to \infty$, in contrast to the usual field-current identity formulation of VMD, or any finite generalization of it.

On the basis of the phenomenological equivalence between these two approaches, and motivated by the remarkable agreement between the quark parton model and the method of infinite vector meson saturation in the description of deep inelastic processes we then consider a relativistic model in which the basic currents of SU(3) [and SU(4)] are dominated by vector mesons and exhibit a quark structure asymptotically. The model is not completely defined by this requirement, however, because the purely hadronic part of the amplitudes involves a large degree of unknown. Guided by analitycity we abstract from the dual resonance model a prescription for implementing the saturation with vector meson poles and deduce therefrom the strong interaction couplings. Low energy theorems and sum rules are used in addition to constrain the $\gamma(q_1^2)\,\gamma(q_2^2)M$ vertices, where M is a meson with quantum numbers $0^{-+}, 0^{++}, 1^{++}$ and 2^{++}.

The model gives very good predictions for SU(3) meson decay, including those for which the quark model by itself fails. The results for scalar and tensor meson decay also agree with previous estimates based on FESR. Applied to the radiative decays of the new mesons this scheme avoids the difficultics of non-relativistic calculations and predicts rather smaller widths, in closer agreement with experiments.

The main issue of this relativistic approach is that of providing a unifying scheme for all SU(3) and SU(4) meson decays, with a particularly simple mechanism to account for the deviations from the exact symmetry. In this respect, and also at the light of the phenomenological success of the model, it is really remarkable the level of accuracy of the dual vertex in extrapolating from the asymptotics of quark current algebra to the small q^2 region governed by the low energy theorems.

2. - KINEMATICS AND NOTATIONS

The vertex function $T_M^{\mu\nu}(q_1,q_2)$ for a (virtual) photon pair going into a meson M of positive charge conjugation is given by

$$T_M^{\mu\nu}(q_1,q_2) = i\int d^4x\, e^{iQx} \langle M(P)/T\left[J^\mu(\tfrac{x}{2})\, J^\nu(-\tfrac{x}{2})\right]/0\rangle, \tag{2.1}$$

where $Q = \frac{1}{2}(q_2-q_1)$ and $P = (q_1+q_2)$. Other kinematical variables used are $\nu = PQ$ and $\xi = \nu/Q^2$. The various helicity amplitudes are then defined as

$$T_M^{\lambda_1\lambda_2}(q_1,q_2) = T_M^{\mu\nu}(q_1,q_2)\, \varepsilon_\mu^{\lambda_1}(q_1)\, \varepsilon_\nu^{\lambda_2}(q_2),\quad \lambda_{1,2} = \pm 0 \tag{2.2}$$

and the known invariance principles can be used to reduce the number of the independent ones. For later purpose we also define the absorptive part of the forward ($q_1=q_3$, $q_2=q_4$) current-current scattering amplitude as

$$W^{\mu\nu\lambda\sigma}(q_1,q_2) = \frac{1}{2}\sum_n (2\pi)^4\, \delta^4(q_1+q_2-P_n)\, T_n^{\mu\nu}(q_1,q_2)\, T_n^{\lambda\sigma *}(q_1,q_2) =$$

$$= \frac{1}{2}\int d^4x\, d^4y\, d^4z\, e^{-\frac{i}{2}\left[(q_2-q_1)(x-y)+(q_2+q_1)z\right]} \tag{2.3}$$

$$\cdot\langle 0/\,\overline{T}\left[J^\lambda(\tfrac{x}{2})\, J^\sigma(-\tfrac{x}{2})\right] T\left[J^\mu(\tfrac{y}{2}+z)\, J^\nu(-\tfrac{y}{2}+z)\right]/0\rangle .$$

As already stated in the introduction the large Q^2 behaviour implied by the quark current algebra will be used to constrain our dual-type vertices. Then the light cone expansion of the time order product of two electromagnetic currents is[10]

$$T\left[J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right)\right] = -\left\{s_{\mu\varrho\nu\sigma}\left[J^\sigma_{Q^2}\left(\frac{x}{2},-\frac{x}{2}\right) - J^\sigma_{Q^2}\left(-\frac{x}{2},\frac{x}{2}\right)\right]\right. \tag{2. 4}$$

$$\left. - i\,\varepsilon_{\mu\varrho\nu\sigma}\left[J^{5\sigma}_{Q^2}\left(\frac{x}{2},-\frac{x}{2}\right) + J^{5\sigma}_{Q^2}\left(-\frac{x}{2},\frac{x}{2}\right)\right]\right\}\frac{\delta}{\delta x_\varrho}\, D_F(x),$$

where $s_{\mu\varrho\nu\sigma} = g_{\mu\varrho}\, g_{\nu\sigma} + g_{\mu\sigma}\, g_{\varrho\nu} - g_{\mu\nu}\, g_{\varrho\sigma}$ and $\frac{\delta}{\delta x_\varrho} D_F(x) = \frac{i}{2\pi^2 i} \times \frac{x_\varrho}{(x^2 - i\varepsilon)^2}$.

$J^{(5)\sigma}_{Q^2}\left(\frac{x}{2},-\frac{x}{2}\right)$ are bilocal vector and axial-vector currents given in the free quark model by

$$J^{(5)\sigma}_{Q^2}\left(\frac{x}{2},-\frac{x}{2}\right) = \bar{q}\left(\frac{x}{2}\right)\gamma^\sigma(\gamma_5)\, Q^2\, q\left(-\frac{x}{2}\right),$$

and the usual currents are the local limits of the bilocal operators. The insertion of (2. 4) in (2. 1) will then lead to the asymptotic behaviour of the resonance form factors, as discussed below for the various cases of interest.

3. - PSEUDOSCALAR MESONS[1]

Let us consider the case $P = \pi^0$ to begin with. The form factor $F_\pi(q_1^2, q_2^2)$ is defined as

$$T_\pi^{\mu\nu} = \varepsilon^{\mu\nu\lambda\tau}\, q_{1\lambda}\, q_{2\tau}\, F_\pi(q_1^2, q_2^2), \tag{3. 1}$$

with $g_{\pi^0\gamma\gamma} = F_\pi(0,0)$. In terms of it the only independent helicity amplitude $T_\pi^{++} = -T_\pi^{--}$ takes the form

$$T_\pi^{++} = i\sqrt{\nu^2 - m_\pi^2 Q^2}\, F_\pi(q_1^2, q_2^2). \tag{3. 2}$$

As well known, in the limit of massless pions $F_\pi(0,0)$ is related to PCAC anomaly S_π[9]

$$F_\pi(0,0\,;\,p^2=0) = -\frac{S_\pi}{2\pi^2 f_\pi}\,; \tag{3. 3}$$

and in the fractionally charged colour model $|S_\pi| = 1/2$, whence extrapolating to $p^2 = m_\pi^2$ one obtains a good prediction for $\Gamma(\pi^0 \to \gamma\gamma)$. The use the Crewther relation[11], which in the quark model reads as $S_\pi = R/4$, with R the usual ratio $\sigma(e^+e^- \longrightarrow \text{hadrons})/\sigma(e^+e^- \longrightarrow \mu^+\mu^-)$, leads eq. (3. 3) to

$$F_\pi(0,0\,;\,p^2=0) = -\frac{R}{8\pi^2 f_\pi} \tag{3. 4}$$

According to the Bjorken-Johnson-Low theorem[12], the asymptotic limit of $F_\pi(q_1^2, q_2^2)$ for $q_{10}, q_{20} \longrightarrow \infty$, with $|\vec{q}_1|, |\vec{q}_2|$ fixed is determined by the commutator of the currents,

$$q_{10}\, q_{2i}\, \varepsilon_{0ijk}\, F_\pi(q_1^2, q_2^2) \xrightarrow[\text{BJL}]{} -\frac{1}{q_{10}} \int d^3x\, e^{-i\vec{q}_1 \cdot \vec{x}} \cdot \langle 0/ \left[J_j(0,\vec{x}), J_K(0) \right] /\pi\rangle, \tag{3.5}$$

which in the fractionally charged coloured quark model leads to

$$F_\pi(q_1^2, q_2^2) \longrightarrow \frac{2 f_\pi}{q_1^2}. \tag{3.6}$$

Simple VMD assumes only ϱ and ω saturation of $F_\pi(q_1^2, q_2^2)$ leading to $F_\pi(0,0) = 2\, g_{\varrho\omega\pi} / f_\varrho f_\omega$, which is in rough agreement with experiments. However in the usual field-current identify formulation of VMD the commutators in (3.5) vanishes identically, leading to a $(1/q_1^4)$ asymptotic behaviour of $F_\pi(q_1^2, q_2^2)$, in disagreement with (3.6).

Motivated by our previous[13] successful descriptions of deep inelastic processes in the extended VMD we impose on $F_\pi(q_1^2, q_2^2)$ to have the asymptotics of quark model (eq. 3.6) when saturated with infinite series of vector meson poles. Furthermore we require it to have the analicity in q_1^2 and q_2^2 of a strong interacting vertex. That, of course, does not define the vertex function, because the large degree of ambiguity involved in the hadronic cuplings. However, in order to keep it as little ad hoc as possible, we adress ourselvs to the dual resonance model (DRM) as a realistic model. Following then procedure of Ademollo and Del Giudice[14] we abstract from DRM, the following form

$$\begin{aligned} F_\pi(q_1^2, q_2^2) &= k \int_0^1 dx\, dy\, x^{-\alpha(q_1^2)}\, y^{-\alpha(q_2^2)} (1-x)^{\gamma-1} (1-y)^{\gamma-1} (1-xy)^{\beta-2\gamma} = \\ &= k\, B\left[1-\alpha(q_1^2), \gamma\right] B\left[1-\alpha(q_2^2), \gamma\right] x \\ &\quad x\ {}_3F_2\left[1-\alpha(q_1^2),\ 1-\alpha(q_2^2),\ 2\gamma-\beta;\ 1+\gamma-\alpha(q_1^2), 1+\gamma-\alpha(q_2^2);\ 1\right], \end{aligned} \tag{3.7}$$

where $\alpha(q^2)$ are the vector meson Regge trajectories, β and γ are two fixed parameters, and k is normalization constant. The large Q^2 behaviour of (3.7) is given by

$$F_\pi(q_1^2, q_2^2) \longrightarrow k \frac{\Gamma(\gamma)\, \Gamma(\beta)\, \Gamma(\gamma-\beta)}{\Gamma(2\gamma-\beta)} \left(\frac{1}{-\alpha' q_1^2}\right)^\beta, \tag{3.8}$$

with $\gamma > \beta$, the limiting procedure being $q_1^2 \longrightarrow \infty$ with q_2^2 fixed, and then $q_2^2 \longrightarrow \infty$[1,2].

The most general form of $F(q_1^2, q_2^2)$ compatible with our requirements is obtained by multiplying the integrand in the r. h. s. of eq. (3. 7) by an arbitrary function f (x,y). However, because the large Q^2-behaviour of the integral in (3. 7) comes from the region x = y = 1, the leading term in the expansion of f(x,y) around x = y = 1 only redefines k, leaving therefore the value of β unchanged. All the other non-leading terms, with increasing power in $(1/Q^2)$, would introduce satellites with additional free parameter and therefore such generalization would only spoil the predicting power of the model. The striking phenomenological success of the form (3. 7), as discussed below, is the most remarkable and puzzling aspect of the model.

The asymptotic behaviour (3. 6) and the low energy theorem (3. 4), with $R = 8\pi^2/f_\varrho^2$ [13], are then used to fix the parameters involved, namely $\beta = 1$, $\gamma = 3/2$ and obtain the further relation

$$g_{\varrho\omega\pi} = - \frac{4 f_\varrho f_\pi f_\omega}{\pi \, m_\varrho^2} , \tag{3. 9}$$

which leads to a very good prediction for $\Gamma(\omega \longrightarrow 3\pi)$. Furthermore one gets $g_{\omega\pi\gamma} = \frac{\pi}{4} \frac{g_{\varrho\omega\pi}}{f_\varrho}$, obtaining the right correction to VMD[4].

All SU(3) meson decays can be calculated in a similar way and that is done in detail in ref. (1). All our predictions are displayed in the Table 1. A quadratic $\eta - \eta'$ mixing angle $\theta \simeq -10^\circ$ has been used. The agreement with experiments is excellent with the only exception of $\varrho \longrightarrow \pi\gamma$. Furthermore our results coincide with those of the naive quark model[4], giving in addition a very simple suppression mechanism operative in the decay $\varphi \longrightarrow \eta\gamma$ and $k^* \longrightarrow k\gamma$. This comes about from the φ family dominance of the corresponding vertex function. The coupling constant is given in fact by

$$g_{\varphi\eta\gamma} = \frac{\sqrt{2}}{3} \; g_{\omega\pi\gamma} \left[\frac{2}{\pi} B \left(\frac{3}{2} \, , \, 1 - \alpha_\varphi(0) \right) \right], \tag{3. 10}$$

and the factor in the brackets is responsable for the modification of the quark model result and is about a factor two smaller. If $\alpha_\varphi(q^2) \equiv \alpha_\varrho(q^2)$ this factor is, of course, unity. This reduction mechanism is the same operating in the new particles decays leading to radiative widths considerably smaller than non-relativistic calculations.

TABLE 1

Decay widths		Theory	Experimental data[30]	Quark model[4]
$\Gamma(\omega \to 3\pi)$	(MeV)	8. 8	9.00 ± 0.06	input
$\Gamma(\omega \to \pi^0 \gamma)$	(MeV)	0. 9	0.87 ± 0.05	input
$\Gamma(\omega \to \eta \gamma)$	(MeV)	7. 3	$3.0 {}^{+2.5}_{-1.8}$ [8]	7.3 ± 0.5
$\Gamma(\rho \to \pi \gamma)$	(keV)	95	35 ± 10 [5]	93 ± 6
$\Gamma(\rho \to \eta \gamma)$	(keV)	55	50 ± 13 [8]	55 ± 4
$\Gamma(\varphi \to 3\pi)$	(MeV)	input (θ_V)	0.66 ± 0.06	input (θ_V)
$\Gamma(\varphi \to \pi^0 \gamma)$	(keV)	4. 2	5.9 ± 2.1	4.2 ± 0.3
$\Gamma(\varphi \to \eta \gamma)$	(keV)	45	63 ± 15 [7] 55 ± 12 [8]	175 ± 13
$\Gamma(\pi^0 \to \gamma\gamma)$	(eV)	9. 0	7.8 ± 0.9	input
$\Gamma(\eta \to \gamma\gamma)$	(keV)	0. 43	0.324 ± 0.046	0.375 ± 0.042
$\Gamma(\eta' \to \gamma\gamma)$	(keV)	7. 3	BR = 1.9 ± 0.3	6.35 ± 0.73
$\Gamma(\eta \to \pi^+\pi^-\gamma)$	(eV)	41	42 ± 7	41 ± 16
$\Gamma(\eta' \to \pi^+\pi^-\gamma)$	(keV)	120	BR = 27.4 ± 2.2	118 ± 9
$\Gamma(\eta' \to \omega \gamma)$	(keV)	11	–	11 ± 1
$\Gamma(K^{0*} \to K^0 \gamma)$	(keV)	56	75 ± 35 [6]	217 ± 16
$\Gamma(K^{+*} \to K^+ \gamma)$	(keV)	14	< 80	52 ± 4

More explicity, with $\eta_c \equiv X\,(2.82)$ the vertex function $F_{\eta_c}(q_1^2, q_2^2)$ is simply obtained from (3. 7) by dominating the currents with vector mesons of the ψ family, namely defining $\alpha(q^2)$ as the ψ trajectory with $1/\alpha_c' = m_{\psi'}^2 - m_\psi^2 \simeq 4\,\mathrm{Gev}^2$, $\alpha_c(0) \simeq -3/2$. The parameters β and γ are the same as before due to SU(4) asymptotic symmetry, and the normalization constant k is fixed by the asymptotic behaviour

$$F_{\eta_c}(q_1^2, q_2^2) \longrightarrow \frac{8\sqrt{2}}{3}\;\frac{f_{\eta_c}}{q_1^2}\,, \tag{3.11}$$

in analogy with (3. 6). The η_c decay constant f_{η_c} is defined as

$$\langle 0 / \frac{J_{5\lambda}^{(0)} - \sqrt{3}\; J_{5\lambda}^{(15)}}{2} / \eta_c\,(p)\rangle = i\, f_{\eta_c}\, p_\lambda\,, \tag{3.12}$$

and, using asymptotic chiral SU(4), can be estimated[1] to satisly the relation

$$2\; f_{\eta_c}^2\, f_\psi^2 \simeq m_\psi^2\,, \tag{3.13}$$

analogous to the KSFR relation.

The radiative decays can be then absolutely predicted

$$\Gamma(\psi\to\eta_c\gamma)\simeq 0.6\ \text{keV};\quad \Gamma(\psi'\to\eta_c\gamma)\simeq 4\ \text{keV};\quad \Gamma(\eta_c\to\gamma\gamma)\simeq 0.5\ \text{keV}. \qquad (3.14)$$

The experimental upper limit is $\Gamma(\psi\to\gamma\eta_c)<3$ keV, and the theoretical expectations based on non-relativistic calculations [16] are of the order of 20 keV.

The decays $\psi,\psi'\to\eta(\eta')\gamma$ can be considered introducing a small mixing among the η,η' and η_c mesons. Writing $\eta_c=c\bar{c}+\alpha\eta+\beta\eta'$, from the experimental values $\Gamma(\psi\to\gamma\eta)\simeq 70$ eV[17], $\Gamma(\psi\to\gamma\eta')\simeq 170$ eV[18] one obtains $\alpha^2\simeq 0.8\times10^{-3}$ and $\beta^2=2.4\times10^{-3}$ which in turn leads to $\Gamma(\psi'\to\gamma\eta)\simeq 42$ eV and $\Gamma(\psi'\to\gamma\eta')\simeq 100$ eV, which is consistent with the experimental upper bound $\Gamma(\psi'\to\eta,\eta'\gamma)<250$ eV[17,18]. The mixing parameters α and β agree within a factor of two with the asymptotic freedom estimates by Fritzsch and Jackson[19].

For later purposes we close this section by nothing that in the scaling limit $Q^2\to\infty$, $\xi\to\pm1$ we have

$$T_\pi^{++}\longrightarrow \pm 2 i f_\pi, \qquad T_{\eta c}^{++}\longrightarrow \pm\frac{8\sqrt{2}}{3}\, i\, f_{\eta c} \qquad (3.15)$$

and therefore

$$W_\pi^{++--}\longrightarrow -4\pi f_\pi^2\,\delta(s-m_\pi^2),\qquad W_{\eta c}^{++--}\longrightarrow -\frac{128}{9}\pi f_{\eta c}^2\,\delta(s-m_c^2), \qquad (3.16)$$

where W_P^{++--} is the pseudoscalar contribution to the absorptive part of the off-shell photon-photon forward scattering amplitude (eq. 2.3).

4. SCALAR MESONS[2]

Let us consider the $\sigma\gamma\gamma$ vertex, where σ indicates a scalar SU(3) singlet. We define

$$T_\sigma^{\mu\nu}(q_1,q_2)=i\,A^{\mu\nu}F_\sigma(q_1^2,q_2^2)+i\,A'^{\mu\nu}F'_\sigma(q_1^2,q_2^2), \qquad (4.1)$$

with

$$A_{\mu\nu}=Q^2P_\mu P_\nu+P^2Q_\mu Q_\nu-(P.Q)(P_\mu Q_\nu+P_\nu Q_\mu)+\left[(P.Q)^2-Q^2P^2\right]g_{\mu\nu},$$
$$A'_{\mu\nu}=-\frac{1}{4}P_\mu P_\nu+Q_\mu Q_\nu+\frac{1}{2}(P_\mu Q_\nu-P_\nu Q_\mu)-(Q^2-\frac{1}{4}P^2)\,g_{\mu\nu}. \qquad (4.2)$$

The two form factors $F_\sigma(q_1^2,q_2^2)$ and $F'_\sigma(q_1^2-q_2^2)$ are related to the two independent helicity amplitudes as

$$T_\sigma^{++}=i\,(\nu^2-m_\sigma^2Q^2)F_\sigma(q_1^2,q_2^2)-i\,(Q^2-\frac{1}{4}m_\sigma^2)\,F'_\sigma(q_1^2,q_2^2)$$
$$T_\sigma^{oo}=i\sqrt{(Q^2+1/4\,m_\sigma^2)^2-\nu^2}\;F'_\sigma(q_1^2,q_2^2), \qquad (4.3)$$

and the coupling constant $\sigma\gamma\gamma$ for the $\sigma\longrightarrow\gamma\gamma$ decay is given by

$$g_{\sigma\gamma\gamma}=1/4\left[\,m_\sigma^2\,F_\sigma(0,0)+2F'_\sigma(0,0)\right] \qquad (4.4)$$

The application of the canonical trace anomaly of the energy momentum tensor leads to the low energy theorem[11, 20]

$$F'(0,0) = \frac{R}{6\pi^2 f_\sigma}, \qquad (4.5)$$

with the usual definition of R. Eq. (4.5) will be used to normalize our dual-type vertex, as in the π^o case.

In order to find the large Q^2 behaviour, let us insert the light cone expansion (2.4) in eq. (2.1), with M=σ. The leading contribution comes from the $s_{\mu\varrho\nu\sigma}$ part. The expansion of the bilocals near x=0 gives:

$$\langle\sigma(P)/\left[J^\sigma_{Q^2}\left(\frac{x}{2},-\frac{x}{2}\right) - J^\sigma_{Q^2}\left(-\frac{x}{2},\frac{x}{2}\right)\right]/0\rangle = 2ix_\varrho\langle\sigma(P)/\theta^{\sigma\varrho}_{Q^2}(0)/0\rangle \qquad (4.6)$$

where in the quark model:

$$\theta^{\sigma\varrho}_{Q^2}(x) = \sum_q \frac{i}{2}\,\bar{q}(x)\,\overleftrightarrow{\partial}^\varrho\, Q^2\, q(x). \qquad (4.7)$$

Extracting from $\theta^{\sigma\varrho}_{Q^2}(x)$ the singlet piece $\theta^{\varrho\sigma}_{Q^2} \sim \frac{2}{9}\theta^{\varrho\sigma}$ and recalling the definition of f_σ, namely $\langle\sigma(P)/\theta^\mu_\mu(0)/0\rangle = i\, m^2_\sigma f_\sigma$, one finally finds in the large Q^2 limit

$$F_\sigma(q_1^2,q_2^2) \longrightarrow \frac{1}{m^2_\sigma} F'_\sigma(q_1^2,q_2^2) \longrightarrow \frac{8}{3} f_\sigma \frac{1}{Q^2}. \qquad (4.8)$$

Under the assumption of smooth extrapolation in q^2 this gives $F_\sigma(0,0)=F'_\sigma(0,0)/m^2_\sigma$, and therefore eqs. (4.4-5) lead to

$$g_{\sigma\gamma\gamma} = \frac{R}{8\pi^2 f_\sigma} \qquad (4.9)$$

in exact analogy to the π^o case ($g_{\pi^o\gamma\gamma} = -R/8\pi^2 f_\pi$).

Considering now a vector meson dominated vertex of the form of eq. (3.7) the low energy theorem (4.5) and the large Q^2 behaviour (4.8) fix $\beta = 2$ and $\alpha = 5/2$ and one finally finds ($m_\sigma \simeq 700$ MeV)

$$\Gamma(\sigma \longrightarrow \gamma\gamma) \simeq 6 \text{ keV},$$

with $f_\sigma \simeq f_\pi$, in good agreement with finite energy sum rules estimates[21].

Let us consider now the decay involving the $\chi(3.41)$ meson, assumed a pure 0^{++} $c\bar{c}$ state. The calculation proceeds exactly as for the η_c meson. Eq. (4.9) is modified as

$$F_{\chi_o}(q_1^2, q_2^2) \rightarrow \frac{1}{m^2_{\chi_o}} F'_\chi(q_1^2,q_2^2) \longrightarrow \frac{16}{9}\frac{m^2_{\chi_o} f_\chi}{Q^4}, \qquad (4.10)$$

where f_χ plays same role of f_σ. In the limit of chiral symmetry $f_\chi = f_{\eta_c}$, however a better estimate can be achieved taking into account the chiral symmetry breaking. One finds $f_\chi \simeq \sqrt{3}(m_{\eta_c}/m_{\chi_0}) f_{\eta_c} \simeq \sqrt{2}\, f_{\eta_c}$.

The radiative widths involving the χ_0 meson finally become

$$\Gamma(\psi' \to \chi_0 \gamma) \simeq 11 \text{ keV}; \quad (\chi_0 \to \psi\gamma) \simeq 31 \text{ keV}; \quad (\chi_0 \to \gamma\gamma) \simeq 2.1 \text{ keV}. \qquad (4.11)$$

They are all linear in f_χ^2. The first one gives a $(\psi' \to \chi_0\gamma)$ branching fraction of 5% compared with the experimental value[22] of 7.5 ± 2.6 and the non-relativistic result[16] of 16%. An estimate of $\Gamma(\chi_0 \to \psi\gamma)$ is also a factor of three smaller than that of ref. (16), whereas a closer agreement is found with the value of $\Gamma(\chi_0 \to \gamma\gamma)$ = 3.5 keV of ref. (23).

Before closing this section let us discuss an interesting duality sum rule[24] connecting in the scaling limit the scalar and pseudoscalar resonance terms to the quark charges.

From eqs. (4.3), (4.8) and (4.10) are obtains in the limit $Q^2 \to \infty, \xi \to \pm 1$

$$T_\sigma^{++} \to i\, \frac{8}{3}\, f_\sigma \ ; \qquad T_{\chi_0}^{++} \to i\, \frac{16}{9}\, f_\chi \qquad (4.12)$$

and therefore

$$W_\sigma^{++--} \to \frac{64}{9}\, \pi f_\sigma^2 \ ; \qquad W^{++--} \to \frac{256}{81}\, \pi f_\chi^2, \qquad (4.13)$$

similary to the case of the pseudoscalars. More generally[25], and including also higher spins, the s-channel resonance contributions to the virtual photon-photon scattering survive in the scaling limit for those helicity amplitudes which scale in the quark parton model (box diagram).

Ithasbeen shown there that only two independent helicity amplitudes survive in that limit, namely

$$W_{box}^{++++}(\xi) \equiv \frac{1}{2}\left[g_s(\xi)+g_P(\xi_s)\right] \ , \quad W_{box}^{++--}(\xi) \equiv \frac{1}{2}\left[g_s(\xi) - g_P(\xi)\right] . \qquad (4.14)$$

Thus the full tensor $W^{\mu\nu\lambda\sigma}$ of eq. (2.3) in this limit is reduced to the simpler one in which scalars and pseudoscalars contribute. In particular the amplitude W_{box}^{++--} is finite for $\xi^2=1$ and is given by the fourth power of the quark charges,

$$W_{box}^{++--} \xrightarrow[\xi^2 = 1]{Q^2 \to \infty} -\frac{1}{2\pi}\, \Sigma_i\, Q_i^4 . \qquad (4.15)$$

A duality relation between the pseudoscalar and scalar resonance contributions (3.16) and (4.13) and the result of the algebra of bilocal currents (4.15) follows directly upon averaging the latter over a suitable range of s[24]:

$$\int ds \left[\sum_{\substack{\text{scalars}\\ \text{pseudo-scalars}}} W^{++--}(s, \xi^2=1) - W^{++--}_{box}(\xi^2=1) \right] \simeq 0 \tag{4.16}$$

For SU(3) mesons ($\Sigma_s W_s^{++--} \longrightarrow \frac{3}{2} W_\sigma^{++--}$, $\Sigma_P W_P^{++--} \longrightarrow 4 W_\pi^{++--}$, $f_\sigma \simeq f_\pi$) eq. (4.16) is satisfied for $\Delta s = 2m_\varrho^2$ with

$$\Sigma Q_i^4 = \frac{1}{3}\left(\frac{8\pi^2}{f_\varrho^2} \right) = \frac{1}{3} R, \tag{4.17}$$

where the last equality holds in EVMD model[13]. In the fractionally charged three triplet model one has for comparison $\Sigma Q_i^4 = 2/3 = R/3$.

For the charm sector, using $\Sigma Q_i^4 = 16/27$, $f_\chi \simeq \sqrt{2}\, f_{\eta_c}$ and $f_{\eta_c} = m_\psi/\sqrt{2}\, f_\psi$ the duality sum rule (4.16) holds provided $\Delta s \simeq 10\ \text{GeV}^2$, which is quite consistent with the masses of the η_c and χ mesons.

Higher spin resonance are also probably averaging the quark charges. The analysis carried out for 2^+ and 1^+ mesons and discussed below, is very suggestive in that sense. The situation here is much reminescent of e^+e^- duality[13], where the presence of only 1^- resonances however greatly simplifies the problem.

Finally the above results indicate that the estimate given for the various resonance parameters occurring in the radiative transitions are reasonably consistent.

5. - TENSOR MESONS[3]

This case is rather complicated because of the presence of five form factors. The complete kinematics is worked out in detail in ref. (3) and (25). The five independent helicity amplitudes can be expressed in terms of the form factors $F_i(q_1^2, q_2^2)$ (i = 1, ... 5) as follows

$$\begin{aligned}
\frac{T^{++}}{i} &= \frac{2}{\sqrt{6}} \frac{1}{m^2} \frac{(\nu^2 - m^2Q^2)^2}{\nu^2 - \frac{1}{4}m^4} F_1 - \frac{1}{\sqrt{6}}(\nu^2 - m^2Q^2)F_2 + \frac{2}{\sqrt{6}}\left(\frac{1}{4} - \frac{Q^2}{m^2}\right) \cdot \\
&\quad \cdot (\nu^2 - m^2Q^2)F_3 + \frac{1}{\sqrt{6}}\left[\nu^2\left(\frac{1}{4}m^2 + Q^2\right) - 2m^2Q^4\right] F_4 + \frac{1}{\sqrt{6}}\left[2\nu^2 - m^2\left(\frac{1}{4}m^2 + Q^2\right)\right] F_5, \\
T^{+-} &= (\nu^2 - m^2Q^2)F_2 + \left(\frac{1}{4}m^2 - Q^2\right)(\nu^2 F_4 - m^2 F_5), \\
\frac{T^{+o}}{i} &= -\frac{1}{\sqrt{2}} \frac{\sqrt{m^2}}{\sqrt{-q_2^2}} \left(-Q^2 - \nu - \frac{1}{4}m^2\right)\left[\left(\frac{1}{2}\nu - Q^2\right)\nu F_4 + \left(\nu - \frac{1}{2}m^2\right) F_5\right], \\
\frac{T^{o+}}{i} &= \frac{1}{\sqrt{2}} \frac{\sqrt{m^2}}{\sqrt{-q_1^2}} \left(-Q^2 + \nu - \frac{1}{4}m^2\right)\left[-\left(\frac{1}{2}\nu + Q^2\right)\nu F_4 + \left(\nu + \frac{1}{2}m^2\right) F_5\right],
\end{aligned} \tag{5.1}$$

$$\frac{T^{oo}}{i} = \frac{2}{\sqrt{6}} \frac{1}{\sqrt{q_1^2, q_2^2}} \left[(Q^2+\frac{1}{4}m^2)^2 - \nu^2 \right] \left[(Q^2 - \frac{\nu^2}{m^2})F_3 + \nu^2 F_4 - m^2 F_5 \right]. \qquad (5.1)$$

In the case of real photons the two surviving amplitudes T^{++} and T^{+-} have been shown(27) from various sum rules to satisfy the inequality

$$/T^{+-}/^2 \gg /T^{++}/^2. \qquad (5.2)$$

On the other hand in the light cone limit only T^{++} does scale, in complete analogy with the 0^+ case. The final set of form factors has to be consistent with both constraints.

As usual the light cone analysis and the expansion of bilocal around x=0 can be performed, starting from eqs. (2.1) and (2.4), in order to set the asymptotic behaviour of the form factors. Furthermore, the relations found in that limit among the various $F_i(q_1^2, q_2^2)$ are smoothly extended to all q^2, in order to reduce the dynamics to only one vertex function which, as in the previous cases, is assumed of the type (3.7).

As for 0^{++} mesons, only the $s_{\mu\varrho\nu\sigma}$ part in eq. (2.4) contributes at the leading order and one finds for an SU(3) singlet f_σ meson

$$T^{++} \longrightarrow \frac{16i}{3\sqrt{6}} \frac{\xi^2}{m\gamma} \; ; \quad T^{+-} \longrightarrow \frac{8i}{3} \frac{m}{\gamma Q^2} \; ; \quad T^{+o} \longrightarrow \frac{8i}{3\sqrt{2}} \frac{\xi}{\gamma\sqrt{-q_2^2}}$$

$$T^{o+} \longrightarrow \frac{-8i}{3\sqrt{2}} \frac{\xi}{\gamma\sqrt{-q_1^2}} \; ; \quad T^{oo} \longrightarrow \frac{-16i}{3\sqrt{6}} \frac{m}{\gamma} \frac{1}{\sqrt{q_1^2 q_2^2}} \qquad (5.3)$$

where γ is defined as $\langle f_o / \theta_{\varrho\sigma}(0)/0\rangle = i\, m^3 e^*_{\varrho\sigma} / \gamma$. Then the solution

$$F_1 = (m^2/4Q^2)F, \; F_2 = F/m^2, \; F_3 = F/Q^2, \; F_4 = F/m^2 Q^2, \; F_5 = -F/4Q^2. \qquad (5.4)$$

with $F \rightarrow c/Q^4$, can be shown to satisfy all the contraints of eqs. (5.2) and (5.3), giving as well the right k^3 dependence for the radiative transitions $1^- \rightleftarrows 2^+ \gamma$.

In contrast to the case of scalar and pseudoscalars, we don't have a low energy theorem as an additional constraint for the vertex function. We shall use, however, a sum rule which connects the tensor couplings to real photons to the corresponding ones of scalars and pseudoscalars.

The sum rule, over all the SU(3) low lying $0^+, 0^-, 2^+$ mesons reads(28)

$$\sum_P \frac{\Gamma i(P \rightarrow \gamma\gamma)}{m_i^3} + \sum_S \frac{\Gamma i(S \rightarrow \gamma\gamma)}{m_i^3} + 5 \sum_T \frac{\Gamma i(\Gamma \rightarrow \gamma_+\gamma_+)}{m_i^3} \simeq 5 \sum_i \frac{\Gamma i(\Gamma \rightarrow \gamma_+\gamma_-)}{m_i^3} \qquad (5.5)$$

Then, using our previous results in sections 3. and 4. we find for $F(q_1^2, q_2^2)$ $\beta=2$, $\gamma=5/2$, as in the scalar case. Furthermore, we obtain for the physical f meson

$$\sqrt{6}\, /T_{\gamma\gamma}^{++}/ = /T_{\gamma\gamma}^{+-}/ = 8\ (\frac{m_f}{\gamma}), \tag{5.6}$$

with

$$\gamma^2 = \frac{32}{m_f^2}\ \frac{1}{g^2_{\pi o \gamma\gamma}} \tag{5.7}$$

This leads to $\Gamma(f \to \gamma\gamma) \simeq 12.5$ keV, in good agreement with earlier estimates[27]. Eq. (5.7) also agrees with Renner's result[29] in the framework of tensor meson dominance. Furthermore inserting (5.7) in the first of eqs. (5.3) and comparing with the analogous expression for T_σ^{++} (eq. 4.12) we find in the scaling limit

$$T_\sigma^{++} \simeq T_{f_o}^{++} \tag{5.8}$$

This is consistent with the idea of a kind of duality between the light cone algebra and resonance saturation in virtual photon-photon scattering, as discussed in the previous section, the various resonances averaging the two scaling functions $g_S(\xi)$ and $g_P(\xi)$ of eq. (4..14).

The above analysis can be easily extended to the case of $c\bar{c}$ 2^{++} $\chi_2(3.55)$ meson, by inserting (5.4) into eqs. (5.1) and dominating the vertex function $F_{\chi_2}(q_1^2, q_2^2)$ by the ψ family, as in the case of the $\chi_o(3.14)$. With $\beta=2$, $\gamma=5/2$, the averall normalization is fixed by imposing in the scaling limit,

$$T_{\chi_2}^{++} = T_{\chi_o}^{++} \tag{5.9}$$

in complete analogy to the SU(3) case (eq. 5.8). Then for the decay $\chi_2 \to \gamma\gamma$ it is found

$$\sqrt{6}\, /T_{\chi_2}^{++}/ = /T_{\chi_2}^{+-}/ = \frac{1}{\sqrt{6}}\ (\frac{m_{\chi_2}}{m_{\chi_o}})^4 /T_{\chi_o}^{++}/$$

and therefore

$$\frac{\Gamma(\chi_2 \to \gamma\gamma)}{\Gamma(\chi_o \to \gamma\gamma)} = \frac{1}{5}\ \frac{7}{36}\ (\frac{m_{\chi_2}}{m_{\chi_o}})^7, \tag{5.11}$$

giving finally $\Gamma(\chi_2 \to \gamma\gamma) \simeq 0.1$ keV. Notice that, as in the case of the f meson, the main contribution comes from the helicity amplitude $T_{\chi_2}^{+-}$, in contrast to the $\chi_o(3.41)$ meson .

As far the cascade decays $\psi' \to \gamma\chi_2$ and $\chi_2 \to \gamma\psi$ are concerned, the following relations are found

$$\sqrt{6}\,/T_{\chi_2}^{++}/ \simeq \sqrt{2}\,/T_{\chi_2}^{+o}/ \simeq /T_{\chi_2}^{+-}/ = \simeq \frac{\sqrt{6}}{2}\;\frac{m_{\chi_2}^4 + m_{\psi(\psi')}^4}{m_{\psi(\psi')}^2\left[2m_{\psi(\psi')}^2 - m_{\chi_2}^2\right]}\;\frac{m_{\chi_2}^2 - m_{\psi(\psi')}^2}{m_{\chi_o}^2 - m_{\psi(\psi')}^2}\;/T_{\chi_o}^{++}/, \qquad (5.12)$$

obtaining[3]

$$Br\,(\psi' \rightarrow \gamma\chi_2) \simeq 0.07 \qquad \Gamma(\chi_2 \rightarrow \gamma\psi) \simeq 0.40 \text{ MeV}. \qquad (5.13)$$

The first of eqs. (5.13) in excellent agreement with the experimental value (0.08 ± 0.03)[15]. For comparison the non relativistic calculations by Eichten et al.[16] give $Br(\psi' \rightarrow \gamma\chi_2) = 0.045$ and $\Gamma(\chi_2 \rightarrow \gamma\psi) = .32$ MeV.

To summarize, the scheme proposed gives fairly good description also of tensor mesons decays, in agreement with earlier for $f \rightarrow \gamma\gamma$, as well as the new ψ spectroscopy. Furthermore our results consistently support the idea of duality in virtual photon-photon scattering.

6. - AXIAL MESONS[3]

There are three independent helicity amplitudes in the coupling of 1^{++} mesons to two (virtual) photons. By a suitable definition of the invariant amplitude are finds[25]:

$$\begin{aligned} T^{++} &= \frac{(\nu^2 - m^2Q^2)}{(\nu^2 - \frac{1}{4}m^4)}\;\nu\; F_1(q_1^2, q_2^2), \\ T^{+o} &= \frac{1}{\sqrt{-q_2^2}}\;(\nu + Q^2 + \frac{m^2}{4})(\nu^2 - m^2Q^2)\; F_2(q_1^2, q_2^2) \\ T^{o+} &= \frac{1}{\sqrt{-q_1^2}}\;(-\nu + Q^2 + \frac{m^2}{4})(\nu^2 - m^2Q^2)\; F_3(q_1^2, q_2^2). \end{aligned} \qquad (6.1)$$

The first amplitude correctly vanishes for $q_1^2 = q_2^2 = 0$, and is related to the analogous amplitude for pseudoscalars into two photons. (The corresponding invariant amplitude are both proportional to $\varepsilon^{\mu\nu\varrho\sigma}\, P_\varrho\, Q_\sigma$). The scaling properties are also similar, namely T_A^{++} and T_P^{++} both scale, the common leading behaviour coming form the $\varepsilon_{\mu\varrho\nu\sigma}$ part of (2.4). With the usual techniques one finds for the A_1 meson

$$T^{++} \longrightarrow 2\,\xi\,\frac{m_A}{f_A}, \qquad (6.2)$$

where f_A is defined as $\langle A_1/J_{5\sigma}(0)/0\rangle = e^*_\sigma\, m^2_A/f_A$. The validity of the Weinberg sum rules leads to $m^2_A/f^2_A = f^2_\pi$ and therefore to $/T^{++}_\pi/ = /T^{++}_A/$ in the scaling limit.

From these results one is naturally lead to identify the form factor $F_1(q^2_1, q^2_2)$ appearing in the first of eqs. (6. 1) with $F_\pi(q^2_1, q^2_2)$ as given in sect. 3 with the obvious substitution $f_\pi \longrightarrow m_A/f_A$.

The other two amplitudes T^{+o} and T^{o+} can be studied similarly in the light-cone limit. The situation however is slightly more complicated for the fact that T^{+o} and T^{o+} get contributions also from the $s^{\mu\varrho\nu\sigma}$ part of the light cone expansion of T(JJ) (eq. 2. 4). One finds in fact:

$$T^{+o} \longrightarrow -\frac{1}{2\sqrt{-q^2_2}}\frac{m_A}{f_A} - \frac{1}{\sqrt{-q^2_2}}\xi^2 g_A(0), \qquad (6.\ 3)$$

where $g_A(0)$ is defined as $\langle A(p)/\left[J_{\sigma Q^2}(\frac{x}{2}/\frac{x}{2}) - J_{\sigma Q^2}(-\frac{x}{2}/\frac{x}{2})\right]/0\rangle \underset{x\to 0}{=} \varepsilon_{\sigma\alpha\beta\gamma}\, p^\alpha x^\beta e^\gamma g_A(0)$ and similarly for T^{o+}.

The above results lead to a two-component structure for the form factors $F_{2,3}(q^2_1, q^2_2)$ in eqs. (6. 1), correspondingly to the $\varepsilon^{\mu\varrho\nu\sigma}$ and $s^{\mu\varrho\nu\sigma}$ components in the asymptotic expansion of eq. (2. 4). The former can be related then to $F_1(q^2_1, q^2_2)$ through eqs. (6. 2) and (6. 3), with the usual smoothness assumption in $q^2_{1,2}$, while the unknown quantity $g_A(0)$ on the contrary makes the latter component undetermined.

In pratice for the only case of interest, namely the radiative involving the $c\bar{c}$ state $\chi_1(3\ 51)$, an order of magnitude estimate can be obtained in the spirit of local duality by demanding asymptotically:

$$(T^{+o})_{1^+} \underset{\xi^2=1}{\approx} (T^{+o})_{2^+} \qquad (6.\ 4)$$

This lead to

$$B\ (\psi' \longrightarrow \gamma\chi_1) \simeq 0.\ 06, \qquad (6.\ 5)$$

and

$$\frac{\Gamma(\chi_1 \longrightarrow \gamma\psi)}{\Gamma(\chi_o \longrightarrow \gamma\psi)} \simeq 2.\ 3 \qquad (6.\ 6)$$

Notice that both T^{++} and the "$\varepsilon^{\mu\varrho\nu\sigma}$-component" of T^{+o} give negligeable contributions to the above transitions. This comes about due to the correlation between T^{++}_χ and $T^{++}_{\eta_c}$ and the corresponding rather weak decay $\psi' \longrightarrow \gamma\,\eta_c$. In addition, the much smaller phase-space available in the reaction $\psi' \longrightarrow \gamma\chi_1$ compared with that for $\psi' \longrightarrow \gamma\eta_c$, makes this contribution completely neglegeable.

The experimental ratio corresponding to (6. 5) is (0. 09 $\pm$0. 03). For comparison the non-relativistic bound state picture leads to B($\psi' \longrightarrow \gamma\chi_1$) = 0. 05 and $\Gamma(\chi_1 \longrightarrow \gamma\psi)/ \ \ (\chi_0 \longrightarrow \gamma\psi) = 2.6$. This shows once more the viability of duality ideas in virtual photon-photon scattering.

7. - CONCLUSIONS

We have presented a new treatment of radiative decays of mesons, where the basic currents of SU(4) are dominated by vector mesons with appropriate quantum numbers and exhibit a quark structure asymptotically. The scheme proposed gives a unified description of the radiative transitions involving pseudo-scalar, scalar, axial and tensor mesons, including the new particles. In the case of pseudoscalar mesons the model gives an excellent description of SU(3) meson decays, including those for which the quark model by itself failed, excepting the $\varrho \longrightarrow \pi\gamma$ decay. In addition, the suppression mechanism operating in the decays $\varphi \longrightarrow \eta\gamma$ and $k^* \rightarrow k\gamma$ is of the right strength to produce for the η_c meson considerably smaller widths than obtained in the non-relativistic bound state picture. For scalar and tensor SU(3) decays agreement is found with previous FESR estimates. The result for the new 0^{++}, 1^{++} and 2^{++} states are also in agreement with experiments and slightly smaller than predicted by non relativistic calculations. One of the most striking features emerging from this approach is the very good level of accuracy of the dual vertex in the extrapolation from the asymptotics of quark current algebra to the small q^2 region governed by the low energy theorems. Finally our results give strong support to the idea of a duality relation between the light cone algebra and resonance saturation in virtual photon-photon scattering.

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