S. Ferrara: ALGEBRAIC PROPERTIES OF EXTENDED
SUPERGRAVITY IN DE SITTER SPACE.

S. Ferrara: ALGEBRAIC PROPERTIES OF EXTENDED SUPER-
GRAVITY IN DE SITTER SPACE.

ABSTRACT:- General features of extended supergravity with a gauge in-
ternal symmetry are discussed from a group-theoretical point of view.

Recently a wide class of extended supergravity theories have been
constructed[1,2,3,4,5]. In these theories the spin $\frac{3}{2}$ Rarita - Schwinger
fields gauge a global supersymmetry algebra whose spinorial charges carry
an internal symmetry index.

A N-ple of Majorana spin $\frac{3}{2}$ gauge fields belong to the vector represen-
tation of O(N) which is the real invariance group of extended supergravity
models.

2.

These theories are particularly interesting because they combine internal symmetries with space-time symmetries in a non-trivial way and in fact it has been suggested[6] that they could be the possible candidates for building theories unifying gravitation with other fundamental interactions. On the other hand it has been recognized[2,7] that in presence of one coupling constant only, the gravitational one, the internal symmetry carried by the spinorial charges is not a local symmetry. This fact is merely due to the absence of a dimensionless coupling which would be required by the gauging of a non-abelian internal symmetry.

In view of future physical applications of local supersymmetry this is not very promising because of possible identifications of these internal symmetries with the gauge groups of ordinary weak, electromagnetic and strong interactions. Renormalizability properties of quantum supergravity are also in favour of such an identification[8].

Recently Freedman and Das[4], in several models, have shown that the gauging of internal symmetries carried by the spinorial charges is indeed possible provided a cosmological term and an apparent mass term for spin $\frac{3}{2}$ gauge field are introduced.

It is the aim of the present note to clarify the algebraic aspects of extended supergravity models and in particular the origin of the gauging of the internal symmetry O (N).

Let us first briefly summarize the algebraic properties of supergravity theories without a cosmological term. In this case the background (flat) metric is minkoskian and pure supergravity arises from the gauging of a global supersymmetry which is a grading of the Poincaré Algebra.

The relevant commutation relations are

$$\{Q^i_\alpha \, , \, \bar{Q}^j_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha} \, P_\mu \, \delta^{ij} \tag{1}$$

$$\left\{ Q_\alpha^i \ , \ Q_\beta^j \right\} = \varepsilon_{\alpha\beta} \, i \, Z^{ij} \qquad\qquad (2)$$

$$\left[Q_\alpha \ , \ P_\mu \right] = 0 \qquad\qquad (3)$$

one also has

$$\left[P_\mu \ , \ P_\nu \right] = 0 \qquad \left[Z^{ij} \ , \ Q_\alpha^k \right] = 0 \qquad \left[Z^{ij} \ , \ P_\mu \right] = 0 \quad , \ \bar{Q}_\alpha^i = (Q_\alpha^i)^*$$

and abvious commutation relations with the homogeneous Lorentz genera-tors $M_{\mu\nu}$. We use two-component Weyl spinors in van der Waerden nota -tions[9].

In eqs. 1, 2, 3, P_μ are four - dimensional displacements and Z^{ij} are the central charges of the Graded Lie Algebra.

As a consequence Z^{ij} generate an abelian group which is essentially isomorphic to $U(1)^p$ being p the dimension of the centre. As already dis-cussed in refs. 2 and 7 the spin $2, \frac{3}{2}$ and 1 gauge fields belonging to the gra-vitational supermultiplet are precisely the gauge fields related respectively to translations, spinorial charges and central charges. In particular the vector fields gauge an abelian group (the centre) rather than the (generally non-abe-lian) internal symmetry under which the spinorial charges transform[*]. This is the group-theoretical reason of the absence of a gauge coupling for the $\frac{N(N-1)}{2}$ vector fields belonging to the gravitational supermultiplet.

Let us now consider the pure supergravity theories investigated by Freedman and Das, in which the vector fields really gauge the internal sym-metry of the extended supersymmetry algebra. In this case, due to the pre-sence of the cosmological term, the background (flat) metric cannot be minkoskian and one must turn over de Sitter space[11].

(*) This result is natutally explained in the context of dual
string models where pure extended supergravity can be
obtained by dimensional reduction in the zero slope limit[10].

As pointed out by Deser and Zumino[12] the space-time symmetry group

of these theories is now the simple group O(3,2), the de Sitter group, rather

than the Poincaré group IO(3,1). The spin $\frac{3}{2}$ field is massless in de Sitter spa

ce and no breaking of supersymmetry occurs, contrary to the claims of ref (4).

We now study the modifications of the commutation rules given by

eqs. (1, 2 and 3) when the Lie Algebra part is O(3, 2) instead of IO(3, 1).

In order to obtain the grading of O(3, 2) we consider the grading of the

bigger group O(4, 2) which is more familiar being the conformal group of

space-time.

According to Haag, Lopuszanski and Sohnius[13] its related extended

supersymmetry algebra is

$$\{Q_\alpha^{1i}, \bar{Q}_{\dot\alpha}^{1j}\} = \delta^{ij} 2\sigma_{\alpha\dot\alpha}^{\mu} P_\mu \qquad\qquad i, j = 1 \dots N$$

$$\{Q_\alpha^{2i}, \bar{Q}_{\dot\alpha}^{2j}\} = -\delta^{ij} 2\sigma_{\alpha\dot\alpha}^{\mu} K_\mu$$

$$\{Q_\alpha^{1i}, Q_\beta^{1j}\} = \{Q_\alpha^{2i}, Q_\beta^{2j}\} = \{Q_\alpha^{1i}, \bar{Q}_{\dot\alpha}^{2j}\} = 0$$

$$\{Q_\alpha^{1i}, Q_\beta^{2j}\} = -2\delta^{ij}i(\sigma_{\alpha\beta}^{\mu\nu} M_{\mu\nu} - \varepsilon_{\alpha\beta} D) - 2i\varepsilon_{\alpha\beta} T^{ij}$$

$$[Q_\alpha^{1i}, P_\mu] = [Q_\alpha^{2i}, K_\mu] = 0 \qquad\qquad (4)$$

$$[Q_\alpha^{1i}, K^\mu] = \sigma_{\alpha\dot\alpha}^{\mu} \bar{Q}^{\dot\alpha 2i}$$

$$[Q_\alpha^{2i}, P^\mu] = -\sigma_{\alpha\dot\alpha}^{\mu} \bar{Q}^{\dot\alpha 1i}$$

$$(g_{\mu\nu}: g_{11} = g_{22} = g_{33} = -g_{00} = 1)$$

In (4) only the relevant commutators have been written. P_μ, K_μ, D, $M_{\mu\nu}$

are the usual generators of the O(4, 2) algebra and T^{ij} are the generators

of U(N) (SU(4) for N=4), i. e. they are an N^2 independent hermitean matri-

ces. The two Weyl spinors Q_α^{1i}, Q_α^{2i} just transform according to the vec-

tor representations of U(N).

In order to obtain the de Sitter supersymmetry algebra we consider the subalgebra obtained according to the definitions which follow.

Let us put

$$Q_\alpha^i = \frac{1}{2} (Q_\alpha^{1i} + Q_\alpha^{2\,i})$$

$$L_\mu = \frac{1}{2} (P_\mu - K_\mu)$$

(5)

In terms of Q_α and L_μ we get from the set of equations (4)

$$\left\{ Q_\alpha^i \, , \, \bar{Q}_{\dot{\alpha}}^j \right\} = \sigma_{\alpha\dot{\alpha}}^\mu \, L_\mu \, \delta^{ij}$$

(6)

$$\left\{ Q_\alpha^i , Q_\beta^j \right\} = -i\delta^{ij} \sigma_{\alpha\beta}^{\mu\nu} M_{\mu\nu} + \varepsilon_{\alpha\beta} A^{ij}$$

(7)

$$\left[Q_\alpha^i , L^\mu \right] = - \frac{1}{2} \sigma_{\alpha\dot{\alpha}}^\mu \, \bar{Q}^{\dot{\alpha}\,i}$$

(8)

$$\left[M_{\mu\nu} , L_\varrho \right] = i(g_{\nu\varrho} L_\mu - g_{\nu\varrho} L_\nu) \qquad \left[L_\mu , L_\nu \right] = iM_{\mu\nu}$$

(9)

In eq. (7) A^{ij} stand for $\frac{N\,(N-1)}{2}$ (real) antisymmetric independent (N x N) matrices. They correspond to the decomposition of the N^2 hermitean matrices in their real and imaginary part $T^{ij} = S^{ij} + i\,A^{ij}$.

Eq. (7) shows that, going from O(4,2) down to O(3,2) the symmetry group is reduced from U(N) to O(N)[+]. The definitions given by eqs. (5) explain also why supersymmetry makes the choise of O(3,2) rather than O(4,1) as de Sitter space. The latter choise would correspond to the complex combination $\frac{1}{2} (Q_\alpha^{1i} + i\,Q_\alpha^{2i})$ which therefore would not respect the Majo-

---

[+] From the point of view of the classification of GLA's (Graded Lie Algebra) the extended de Sitter supersymmetry corresponds to the grading of the Lie Algebra Sp (4) ⊗ O (N) rather than the SU (4) ⊗ U (N) of extended conformal supersymmetry[14].

rana constraint on the related 4-dimensional Dirac spinor.

Pure extended supergravity with a cosmological term and with a gauge internal symmetry correspond to the gauging of the algebra whose commutation relations are 6, 7, 8 and 9. If we compare eqs. (6,9) with the corresponding eqs. (1,3) in Minkoski space we see that the main change, apart the substitution of the Poincaré group with the de Sitter group, is in the internal symmetry part. The gauged internal symmetry is now $O(N)$ and not the abelian group given by the centre.

This general result, which is valid for any extended supergravity theory, is in complete agreement with the calculation of the commutator algebra in the model field theories considered by Freedman and Das[4]. This also explains why the gauged internal symmetry is $O(N)$ rather than $U(N)$ in pure supergravity[13].

The new dimensional constant λ, related to the cosmological term or equivalently to the curvature of the de Sitter space, is related in these models to the $O(N)$ gauge coupling by means of a formula of the type $\lambda \simeq e^2 k^{-2}$ being k the gravitational coupling[4].

REFERENCES. -

1. S. Ferrara and P.van Nieuwenhuizen, Phys. Rev. Letters 37, 1669 (1976).

2. S. Ferrara, J. Scherk and B. Zumino, Phys. Letters B66, 35 (1977) .

3. D. Z. Freedman, Phys. Rev. Lett. 38, 105 (1976).

4. D. Z. Freedman and A. Das,ITP - SB - 76 - 64.

5. A. Das,ITP -SB - 77 - 4 .

6. M. Gell-Mann,Talk given at the Coral-Gable conference January 1977.

7. S. Ferrara, J. Scherk and B. Zumino,PTENS 76/24 to be published in Nucl. Phys. B.

8. P.van Nieuwenhuizen, and J. A. M. Vermaseren,Phys. Lett. 65B, 263 (1976). M. T. Grisaru, P.van Nieuwenhuizen and J. A. M. Vermaseren,Phys. Rev. Lett. 37, 1662 (1976). P. van Nieuwenhuizen and J. A. M. Vermaseren,ITP-SB - 77 - 8 . S. Deser, J. H. Kay and K. S. Stelle,Phys. Rev. Lett. 38, 527 (1977) .

9. S. Ferrara, J. Wess and B. Zumino,Phys. Lett. 51B, 239 (1974).

10. F. Gliozzi, J. Scherk and D. Olive, Phys. Lett. 65B, 268 (1976) and CERN TH-2253 (1976) to be published in Nucl. Phys. B.

11. F. Gursey and T. D. Lee, Proc. Nat. Acad. Sci. 49, 179 (1963). G. Börner and H. P. Dürr, Nuovo Cimento A64 , 669 (1969). E. A. Tagirov , Ann. of Phys. 76 , 561 (1973).

12. S. Deser and B. Zumino,to be published.

13. R. Haag, J. T. Lopuszanski and M. Sohnius,Nucl. Phys. B88, 257 (1975)

14. W. Nahm, V. Rittenberg and M. Scheunert,Phys. Lett. 61B , 383 (1976).