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K. Zalewski: SUM RULES FOR VECTOR MESON-NUCLEON
SCATTERING FROM THE GENERALIZED VECTOR MESON
DOMINANCE MODEL. -

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K. Zalewski^(x): SUM RULES FOR VECTOR MESON-NUCLEON
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Three sum rules for vector meson-nucleon scattering are obtained. Under plausible assumptions they supply the necessary and sufficient condition for a realization of the generalized vector meson dominance model to fit simultaneously the data on $e^+e^- \rightarrow$ hadrons and $eN \rightarrow eX$ in the high energy (s or ω) scaling regions.

The generalized vector meson dominance model (GVD) yields very satisfactory predictions for total cross-sections of the processes $e^+e^- \rightarrow$ hadrons and $\gamma N \rightarrow$ hadrons, for the structure function $W_2(\nu, q^2)$ in deep inelastic eN scattering and for the photoproduction of vector mesons on nucleons. For results and references see e. g. refs. (1-5). As yet no significant contradiction with experiment has been reported. The price to pay, however, is that it is necessary to make guesses about the scattering of higher vector mesons and nucleons. To be sure, this information can be obtained from high energy photoproduction on nuclei, but at present it is not available and guesswork is necessary. In view of the successes of the GVD model, it seems justified to invert the usual procedure and to use the data from photon interactions and the model to learn about vector meson-nucleon

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scattering. As a byproduct, we should be able to understand why realizations of GVD, which make widely different assumptions about vector meson-nucleon scattering (compare e. g. refs. (1) and (2)), succeed in reproducing the same experimental data.

Following the usual practice, we assume that the vector mesons can be arranged into recurrences, with the squared mass of the n -th vector meson from recurrence v given by:

$$m_{nv}^2 = m_v^2 (1 + a_v n) \quad n = 0, 1, \dots \quad (1)$$

Since our argument is very general, we have to specify neither which recurrences are included, nor whether (nv) is a true resonance, or some set of particles with invariant mass close to m_{nv} .

The moduli of the photon-vector meson coupling constants can be obtained from data on the cross-section for the reaction $e^+e^- \rightarrow \rightarrow$ hadrons. Assuming that the contribution from each recurrence scales separately and denoting by R_v the high energy ratio of this contribution to $\sigma(e^+e^- \rightarrow \mu^+\mu^-)$, one obtains from a well-known argument (cf. e. g. ref. (1)):

$$f_{nv}^2 = \frac{12\pi^2}{a_v R_v} \left(\frac{m_{nv}}{m_v} \right)^2 = f_{ov}^2 (1 + a_v n) \quad (2)$$

Without loss of generality we may assume that all the f_{nv} are real positive, because the phase factors, if any, can be absorbed into the transition amplitudes (cf. formula (3)).

The relation between the vector meson-nucleon scattering amplitudes $T(V_m N \rightarrow V_n N)$ and $\sigma_T(\nu, q^2)$, the absorption cross-section on nucleons for transverse virtual photons of mass $-q^2$ and laboratory energy ν , is

$$\sigma_T(\nu, q^2) = 4\pi\alpha \sum_{v, v'} \frac{m_v^2 m_{v'}^2}{f_{ov}^2 f_{ov'}^2} \sum_{m, n} \frac{1}{q^2 + \bar{a}_v m + m_v^2} \cdot \quad (3)$$

$$\cdot \sum_{m, n} \frac{1}{q^2 + \bar{a}_{v'} n + m_{v'}^2}$$

where $\bar{a}_v = a_v m_v^2$ and $V_{mv, nv'} = V_{nv', mv} =$
 $= T(V_m N \rightarrow V_n N) \sqrt{(1 + a_v m)(1 + a_{v'} n)}$. We assume that there is a
 real finite b such that:

$$\lim_{\omega \rightarrow \infty} q^2 \sigma_T(\nu, q^2) \equiv \lim_{\omega \rightarrow \infty} 4\pi^2 a \nu W_{2T}(\nu, q^2) = b, \quad (4)$$

where as usual $\omega = 2m\nu/q^2$ with m denoting the nucleon mass. The
 dependence of the expressions on ω only is implied by Bjorken's scal-
 ing. The finite limit is suggested by pomeron exchange models (cf. e.
 g. refs. (1) and (2)).

In order to simplify the sum (3), we make the following assump-
 tions:

- i. The number of recurrences ν is finite.
- ii. There exists a finite integer K such that $V_{mv, nv'} \neq 0$ only if
 $|m - n| \leq K$.
- iii. There exists a finite real number A such that $|V_{mv, nv'}| < AN$,
 for all m, n, ν, ν' . N denotes here the smaller of the two indices
 m, n :

$$N = \frac{1}{2}(m+n - |m-n|). \quad (5)$$

The last assumption implies that all the amplitudes T are bound from
 above by a constant.

Formula (3) now reduces to:

$$\sigma_T(\nu, q^2) = 4\pi a \sum_{\nu, \nu'} \frac{m_\nu^2 m_{\nu'}^2}{f_{\nu} f_{\nu'}} \sum_{m, n} \frac{V_{mv, nv'}}{(q^2 + \bar{a}_\nu N)(q^2 + \bar{a}_{\nu'} N)} \cdot$$

$$\cdot \left\{ 1 - \left(\frac{a_\nu (m-N) + m_\nu^2}{q^2 + \bar{a}_\nu N} + \frac{a_{\nu'} (n-N) + m_{\nu'}^2}{q^2 + \bar{a}_{\nu'} N} \right) \right\} + NS, \quad (6)$$

where NS stands for terms, which in the scaling limit, when moreover
 ω is large, become negligible. To proceed further, we assume:

$$\lim_{N \rightarrow \infty} (V_{mv, nv'} - N\alpha_{vv'}^{(m-n)} - \beta_{vv'}^{(m-n)}) = 0 \quad (7)$$

where of course from time reversal invariance $\alpha_{vv'}(j) = \alpha_{v'v}(-j)$ and $\beta_{vv'}(j) = \beta_{v'v}(-j)$. This assumption generalizes the assumptions made in refs. (1-5).

Substituting (7) into (6), converting the summations over N into integrations and comparing the result with (4), one finds the following three sum rules:

$$\sum_{v, v'} \frac{m_v^2 m_{v'}^2}{f_{ov} f_{ov'}} \sum_{j=-K}^K \alpha_{vv'}^{(j)} f_i(\bar{a}_v, \bar{a}_{v'}) = 0; \quad i = 1, 2 \quad (8)$$

$$\sum_{v, v'} \frac{m_v^2 m_{v'}^2}{f_{ov} f_{ov'}} \sum_{j=-K}^K \left[\beta_{vv'}^{(j)} g(\bar{a}_v, \bar{a}_{v'}) - \alpha_{vv'}^{(j)} \left\{ (\bar{a}_v \frac{|j|+j}{2} + m_v^2) \cdot \right. \right. \quad (9)$$

$$\left. \left. \cdot h(\bar{a}_{v'}, \bar{a}_v) + (\bar{a}_{v'} \frac{|j|-j}{2} + m_{v'}^2) h(\bar{a}_v, \bar{a}_{v'}) \right\} \right] = \frac{b}{4\pi a} .$$

In these sum rules:

$$f_1(x, y) = \frac{1}{xy} \quad (10)$$

$$f_2(x, y) = \begin{cases} x^{-2}(1 - \ln x) & \text{if } x = y \\ (x-y)^{-1}(x^{-1} \ln x - y^{-1} \ln y) & \text{if } x \neq y \end{cases} \quad (11)$$

$$g(x, y) = \begin{cases} 1/x & \text{if } x = y \\ (x-y)^{-1} \ln(x/y) & \text{if } x \neq y \end{cases} \quad (12)$$

$$h(x, y) = \begin{cases} \frac{1}{2} x^{-2} & \text{if } x = y \\ (x-y)^{-2} \ln(y/x) + y^{-1} (x-y)^{-1} & \text{if } x \neq y . \end{cases} \quad (13)$$

The following two examples illustrate the use of the sum rules. For simplicity we neglect the transitions between vector mesons belonging to different recurrences. Then the contributions of the recurrences are additive :

$$R = \sum_v R_v ; \quad W_{2T}(\nu, q^2) = \sum_v W_{2T_v}(\nu, q^2) . \quad (14)$$

We assume moreover that each contribution separately scales and consequently satisfies the sum rules.

Putting $K = 0$, we obtain from the sum rules for each v :

$$\alpha_{vv}(0) = 0 ; \quad \beta_{vv}(0) = \frac{a_v f_{ov}^2}{4\pi a} \frac{b_v}{m_v^2} = \sigma_{vp}^D , \quad (15)$$

where in order to derive the last equality, the estimate for b_v from ref. (1) has been used. Formulae (15) correspond to the model from ref. (1) i. e. to :

$$T(V_m N \rightarrow V_n N) = \sigma_{vp}^D \left(\frac{m_v}{m_{nv}} \right)^2 \delta_{mn} . \quad (16)$$

The asymptotic predictions are :

$$\lim_{\omega \rightarrow \infty} \nu W_{2T_v}(\nu, q^2) = \frac{m_v^2}{a_v \pi f_{ov}^2} \sigma_{vp}^D = \frac{m_v^2}{12\pi^3} R_v \sigma_{vp}^D . \quad (17)$$

Assuming in contradiction to (16)

$$T(V_n N \rightarrow V_n N) = \sigma_{vp}^D , \quad (18)$$

it is longer possible to satisfy the sum rules with $K = 0$.

The most conservative extension is to put $K = 1$. Then the sum rules yield

$$\alpha_{vv}(0) = a_v \sigma_{vp}^D ; \quad \alpha_{vv}(1) = - \frac{a_v}{2} \sigma_{vp}^D ;$$

$$\beta_{\nu\nu}(0) = \sigma_{\nu p}^D ; \quad \beta_{\nu\nu}(1) = -\frac{1}{2}(1 - \delta a_{\nu}) \sigma_{\nu p}^D , \quad (19)$$

where $\delta = 0.28$ and the estimate for b_{ν} has been taken from ref. (2).

The predictions are:

$$\lim_{\omega \rightarrow \infty} \nu W_{2T\nu}(\nu, q^2) = \frac{m_{\nu}^2(1+2\delta)}{2\pi f_{\text{ov}}^2} \sigma_{\nu p}^D = \frac{m_{\nu}^2 a_{\nu}(1+2\delta)}{24\pi^3} R_{\nu} \sigma_{\nu p}^D \quad (20)$$

Formulae (19) give the high N form of the model from ref. (2).

It is instructive to compare the predictions (17) and (20) for given f_{ov} , m_{ν} and a_{ν} . The predictions for R_{ν} coincide. For $W_{2T\nu}$ however, formula (20) has the additional factor $a_{\nu}(1+2\delta)/2$. This results from different estimates of b in refs. (1) and (2). According to ref. (1), the contribution to b from the trajectories ρ , ω , φ , should be about 30 per cent lower than observed at present energies. According to ref. (2), on the other hand, we are already in the asymptotic region. For the ψ recurrence the discrepancy is even more pronounced. Taking $\sigma_{\psi p}^D = 1$ mb from ref. (6), and $f_{\psi}^2/4\pi = 11.7$ and $a = 0.41$ to fit the data from ref. (7), we find for $W_{2T\psi}$ the value 0.13 from formula (17) and 0.08 from formula (20). Thus data can decide between the two models. From the point of view of the sum rules, b_{ν} is just a given parameter and the problem how to extract it best from the data is not considered here.

We conclude with two remarks concerning the interpretation of our results.

- i. We have shown under plausible assumption that a realization of GVD yields scaling in both $e^+e^- \rightarrow$ hadrons and $eN \rightarrow eX$ at high s and ν , q^2 , ω respectively, if and only if the sum rules (8) and (9) are satisfied. Most of the additional assumptions are technical and could be relaxed at the expense of making the final result more complicated. We do not go into this problem, because the present state of GVD does not seem to require it.

ii. The sum rules can be interpreted as predictions abstracted from an infinite variety of possible realizations of GVD, all fitting the data for the two processes. Conversely, they show that a fit to these data is a rather incomplete test of the assumptions made concerning the amplitudes $T(V_m N \rightarrow V_n N)$ in a particular realization of GVD. It is interesting that the contributions of order 1 to the amplitudes T , i. e. the contributions $\alpha_{\nu\nu'}$, are not more important for the success of a fit than the contributions of order $(m_\nu/m_{\nu'})^2$, i. e. the contributions $\beta_{\nu\nu'}$. In the high N region, which is crucial for deep inelastic scattering, the later contributions to T are very small. Thus a small error in the estimates of the amplitudes T can decide about the success or failure of a fit. This difficulty was discussed in a different context in ref. (2). The new result here is that it is not limited to realizations of GVD, where the contributions $\alpha_{\nu\nu'}$ vanish.

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