

To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-76/54(P)
26 Ottobre 1976

M. Greco and H. Inagaki : COMMENTS ON RADIATIVE
DECAYS OF SCALAR MESONS. -

M. Greco and H. Inagaki^(x): COMMENTS ON RADIATIVE DECAYS
OF SCALAR MESONS. -

ABSTRACT. -

A previously proposed analysis of radiative decays of O^{++} mesons is reexamined. New estimates are given for the widths of $\chi \rightarrow \gamma\gamma$ and $\psi \rightarrow \chi\gamma$.

In a previous letter⁽¹⁾ we have applied to scalar meson decays a new treatment of radiative transitions⁽²⁾, where the basic currents of SU(4) are vector mesons dominated and exhibit the asymptotics implied by the quark current algebra. Predictions for the decays $\varepsilon \rightarrow \gamma\gamma$, $\chi \rightarrow \gamma\gamma$, $\psi' \rightarrow \chi\gamma$ and $\chi \rightarrow \psi\gamma$ have been given in detail, by constraining our dual-type vertices with the low energy theorem for $\sigma \rightarrow \gamma\gamma$ ⁽³⁾ and the appropriate large Q^2 behaviour of the free quark current algebra. More explicitly, given the vertex function^(1, 2)

$$(1) F_{\sigma}^{\chi}(q_1^2, q_2^2) = k \int_0^1 \int_0^1 dx dy x^{-\alpha(q_1^2)} y^{-\alpha(q_2^2)} (1-x)^{\gamma-1} (1-y)^{\gamma-1} (1-xy)^{\beta-2\gamma},$$

where $\alpha(q^2)$ are the vector meson Regge trajectories, the two parameters

(x) Address after November 1, 1976: International Centre for Theoretical Physics, Trieste, Italy.

2.

β and γ have to be fixed from the appropriate constraints. While β was unambiguously determined in ref. (1) from the large Q^2 behaviour ($\beta=2$), the value of γ ($\gamma = 1/2$) was more weakly fixed to previous FESR estimates of $\Gamma(\varepsilon \rightarrow \gamma\gamma)^{(4)}$ through the low energy theorem⁽³⁾

$$(2) \quad F'_\sigma(0, 0) = \frac{R}{6\pi^2 f_\sigma},$$

with the usual definition of the asymptotic ratio $R = \sigma(e^+e^- \rightarrow \text{hadrons}) / \sigma(e^+e^- \rightarrow \mu^+\mu^-)$.

In view of the results of a recent analysis of photon-photon scattering⁽⁵⁾, where arguments have been given against the naive application of Regge-resonance duality ideas to this process, we have reconsidered the problem of scalar meson radiative decays, in order to have a more reliable determination of the above parameter γ and therefore of our estimates of the decay widths. In particular we have reexamined the prescriptions in performing the large Q^2 limit of (1), to compare with the quark current algebra results, and concluded the right procedure being $q_1^2 \rightarrow \infty$ with q_2^2 fixed. Our final result is $\gamma = 5/2$, which in turn changes some of our previous estimates of the transition rates.

With the same notations of ref. (1) for the $\sigma\gamma\gamma$ vertex function, the asymptotic behaviour of the form factor $F'_\sigma(q_1^2, q_2^2)$ is given by the quark current algebra as

$$(3) \quad F'_\sigma(q_1^2, q_2^2) \xrightarrow{Q^2 \rightarrow \infty} \frac{8}{3} f_\varepsilon \frac{m_\varepsilon^2}{Q^4},$$

where $Q = \frac{1}{2}(q_2 - q_1)$. Defining $\xi = \frac{q_1^2 - q_2^2}{q_1^2 + q_2^2}$, and following Brandt and Preparata⁽⁶⁾ this limit has to be understood as $Q^2 \rightarrow \infty$ with ξ fixed and then $\xi \rightarrow 1$, which corresponds to $q_1^2 \rightarrow \infty$ and q_2^2 fixed. Such a limiting procedure for eq. (1) leads to the inequality $\beta < \gamma$, and

$$(4) \quad F'_\sigma(q_1^2, q_2^2) \xrightarrow[\substack{q_1^2 \rightarrow \infty \\ q_2^2 \text{ fixed}}]{} k \frac{\Gamma(\gamma) \Gamma(\beta) \Gamma(\gamma-\beta)}{\Gamma(2\gamma-\beta)} \left[-\alpha' q_1^2 \right]^{-\beta}.$$

Comparison with eq. (3) gives $\beta = 2$ and

$$(5) \quad k = \frac{32}{3} \frac{\Gamma(2\gamma - 2)}{\Gamma(\gamma) \Gamma(\gamma - 2)} \alpha'^2 f_\epsilon m_\epsilon^2 .$$

As in ref. (1), considering the ratio $F'_\sigma(q_1^2, q_2^2)/F'_\sigma(0, 0)$ from eqs. (2) and (3), we obtain in the same limit

$$(6) \quad \frac{F'_\sigma(q_1^2, q_2^2)}{F'_\sigma(0, 0)} \longrightarrow \frac{4 m_\rho^2 m_\epsilon^2}{q_1^4} \left(\frac{f_\epsilon}{f_\pi} \right)^2 ,$$

having used $R = 8\pi^2/f_\rho^2$ and the KSFR relation $2f_\rho^2 f_\pi^2 = m_\rho^2$. The comparison of eq. (6) with the corresponding ratio computed from eqs. (1) and (4) suggests the value $\gamma = 5/2$, which obviously satisfies the inequality $\gamma > \beta$, and gives $f_\epsilon = (\sqrt{3} m_\rho / 2 m_\epsilon) f_\pi$.

We give below an additional arguments in support of $\gamma = 5/2$. In ref. (5) it has been shown that, in virtual photon-photon scattering, the scalar and pseudoscalar contributions to the helicity amplitude T_{++} both scale in the limit $Q^2 \rightarrow \infty$ with $\xi^2 = 1$, and furthermore, there exists a precise duality relation between these particular resonance contributions and the scaling result as given by the quark parton model (box diagram). Similarly, demanding the same power behaviour in the q_1^2 fall-off of T_{++} for the vertices $\gamma(q_1^2) + \gamma(q_2^2 = m_{v_n}^2) \rightarrow$ scalar or pseudoscalar, for any vector meson v_n , one obtains $\gamma_s = \gamma_p + 1$. With $\gamma_p = 3/2^{(2)}$ one gets therefore $\gamma_s = 5/2$.

With this new value of γ , the previous estimate⁽¹⁾ for $\Gamma(\epsilon \rightarrow \gamma\gamma)$ is essentially unchanged (~ 6 keV), whereas we find now for the decays $\chi \rightarrow \gamma\gamma$ and $\chi \rightarrow \psi\gamma$, with $\chi(3, 41)$ the $c\bar{c}$ O^{++} meson,

$$(7) \quad g_{\chi\gamma\gamma} = k \frac{9\pi}{16} (22 - 7\pi) ,$$

$$(8) \quad g_{\psi\chi\gamma} = \frac{3\pi}{256} \frac{k}{\alpha'_\psi m_\psi^2} f_\psi \left\{ 1 + \frac{m_\chi^2 - m_\psi^2}{2m_\chi^2} \right\} ,$$

with

$$(9) \quad k = \frac{1}{\pi} \left(\frac{8}{3} \right)^3 \alpha'_\psi m_\chi^2 f_\chi .$$

4.

The transition $\psi' \rightarrow \gamma\chi$ deserves a special discussion. The coupling constant $g_{\psi'\chi\gamma}$ is proportional to

$$(10) \quad {}_3F_2 \left[-n, 1 - \alpha_{\psi}(0), 2\gamma - \beta; \gamma - n, \gamma + 1 - \alpha_{\psi}(0); 1 \right]$$

with $n=1$ which, taken literally, gives zero ($\alpha_{\psi}(0) \simeq -3/2$). There is however no dynamical reason for such a zero, and since it depends crucially upon the exact numerical value of the various parameters, we consider this result as a pure accident. We take then eq. (10) for various n , corresponding to the ψ and its radial excitations, and assign to the ψ' the best fitted value for $n=1$. Eq. (10) gives 1, $-1/3$, -2 , $-9/2$ and $-22/3$ for $n=0, 2, 3, 4$ and 5 respectively which is nicely fitted by the quadratic form $1 - n^2/3$. Instead of (10) we then assign to the ψ' a corresponding value of $2/3$, and finally have

$$(11) \quad g_{\psi'\chi\gamma} = - \frac{3\pi}{256} \frac{k}{\alpha_{\psi'} m_{\psi'}^2} f_{\psi'} \left\{ 1 + \frac{m_{\chi}^2 - m_{\psi'}^2}{2m_{\chi}^2} \right\},$$

with k given by (9).

Proceeding as in ref. (1) our estimates for the radiative decays involving the χ meson become

$$(12) \quad \begin{aligned} \Gamma(\chi \rightarrow \gamma\gamma) &\simeq 2.1 \text{ keV}, \\ \Gamma(\chi \rightarrow \psi\gamma) &\simeq 31 \text{ keV}, \\ \Gamma(\psi' \rightarrow \chi\gamma) &\simeq 11 \text{ keV}. \end{aligned}$$

Notice that the last decay width has not varied, due to accidental cancellation of the correction factors in eqs. (9) and (11), compared with the corresponding expressions in ref. (1). Our values (12) are a factor of two or three smaller than those obtained in non-relativistic calculations⁽⁷⁾.

REFERENCES. -

- (1) M. Greco and H. Inagaki, Frascati preprint LNF -76/41 (P) and Physics Letters B, to be published.
- (2) E. Etim and M. Greco, CERN preprint TH -2174 (1976)
- (3) M. S. Chanowitz and J. Ellis, Phys. Rev. D7, 2490 (1973);
J. Crewther, Phys. Rev. Letters 28, 1421 (1972)
- (4) A. Q. Sarker, Phys. Rev. Letters 25, 1527 (1970); A. Bramon and M. Greco, Lett Nuovo Cimento 2, 522 (1971) ; B. Schrempp - Otto, F. Schrempp and T. F. Walsh, Phys. Letters 36B, 463 (1971).
- (5) M. Greco and Y. Srivastava, to be published.
- (6) R. Brandt and G. Preparata, "Broken Scale Invariance and the Light Cone", Gordon and Breach 1971.
- (7) E. Eichten et al, Phys. Rev. Letters 36, 500 (1976); R. Barbieri R. Gatto and R. Kögerler, Phys. Letters 60B, 183 (1976).