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A. Turrin: QUANTUM-MECHANICAL FORM OF THE DAMPED BLOCH EQUATIONS.

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ABSTRACT. -

The damped optical Bloch equations have been used to derive the form of the "corresponding" two-level atomic dynamics. It turns out that the resulting equations involve non-linear damping terms.

Formally, collisional effects can be introduced into the two-level atomic dynamics by combining Eqs. 1)

1a) $\dot{X} = -\Delta Y$

1b) $\dot{Y} = \Delta X + \omega Z$

1c) $\dot{Z} = -\omega Y$

for the Bloch vector[1] (in the rotating wave approximation and in a reference frame rotating with the rotating wave) and Eqs. 2)

2a) $\dot{X} = -\Gamma_2 X$

2b) $\dot{Y} = -\Gamma_2 Y$

2c) $\dot{Z} = -\Gamma_1 (Z+1)$

for the relaxation. Thus, the damped Bloch equations are

3a) $\dot{X} = -\Delta Y - \Gamma_2 X$

3b) $\dot{Y} = \Delta X - \Gamma_2 Y + \omega Z$

3c) $\dot{Z} = -\omega Y - \Gamma_1 (Z+1)$,

where $\omega = p\mathcal{E}(t)/\hbar$; p is the dipole matrix element between the upper state and the lower state, and $\mathcal{E}(t)$ is the envelope of the optical pulse. $\Delta = \Delta(t)$ is the detuning.

The phenomenological constants Γ_1 and Γ_2 denote the population rate and the phase relaxation rate respectively.

In this letter we give a quantum-mechanical representation of Eqs. 3) by use of suitable transformations.

We will restrict our analysis to the case where $\Gamma_1 = 0$, since if $\Gamma_1 \neq 0$ the two-level system decays back to its ground state $X=0$, $Y=0, Z=-1$ as the pulse goes out (i. e. the solution at $t=+\infty$ is known in the case $\Gamma_1 \neq 0$).

From Eqs. 3) it follows that the absolute value of the Bloch vector $R = \sqrt{X^2+Y^2+Z^2}$ decays in length during the pulse. This decay law is governed by the equation

4) $\dot{R}/R = -\Gamma_2 \{1-(Z/R)^2\}$.

Introduce in Eqs. 3) and 4) the two complex functions σ and ε defined by the relationships

5a) $X+iY=-(R+Z)/(\varepsilon E)$

5b) $X-iY=(R+Z)\sigma^{*}E$,

where $E=\exp(\Gamma_2 t)$, and the star denotes complex conjugation. Transformations 5a) and 5b) are a generalization of the ones given by Darboux(2) for the undamped ($\dot{R}=0$, $R=1$) case.

It follows for Z/R the expression

6a) $Z/R=(\varepsilon\varepsilon^{*}-E^{-2})/(\varepsilon\varepsilon^{*}+E^{-2})$

or, alternatively,

6b) $Z/R=(E^{-2}-\sigma\sigma^{*})/(\sigma\sigma^{*}+E^{-2})$,

and the relationship

7) $\sigma\varepsilon=-E^{-2}$

holds.

A straightforward (although rather tedious) calculation leads to the following differential equations for σ and ε :

8a) $\dot{\varepsilon}=(i/2)\omega E\varepsilon^{2}-i\Delta\varepsilon-(i/2)\omega/E-\{2\Gamma_2 E^{-2}/(\varepsilon\varepsilon^{*}+E^{-2})\}\varepsilon$

8b) $\dot{\sigma}=-(i/2)\omega E\sigma^{2}+i\Delta\sigma+(i/2)\omega/E-\{2\Gamma_2 E^{-2}/(\sigma\sigma^{*}+E^{-2})\}\sigma$.

On introduction of two new functions f and g by the substitutions

9a) $\varepsilon=i(2/\omega)(\dot{f}/f)E^{-1}$

9b) $\sigma=-i(2/\omega)(\dot{g}/g)E^{-1}$

one obtains the equations for f and g:

10a) $\ddot{f}+(i\Delta-\dot{\omega}/\omega-\Gamma_2 Z/R)\dot{f}+(\omega/2)^{2}f=0$,

10b) $\ddot{g}-(i\Delta+\dot{\omega}/\omega-\Gamma_2 Z/R)\dot{g}+(\omega/2)^{2}g=0$,

where

$Z/R=\{(2/\omega)^{2}|\dot{f}/f|^{2}-1\}/\{(2/\omega)^{2}|\dot{f}/f|^{2}+1\}$,

or

$Z/R=-\{(2/\omega)^{2}|\dot{g}/g|^{2}-1\}/\{(2/\omega)^{2}|\dot{g}/g|^{2}+1\}$.

As a consequence of the relationship 7), Eqs. 10a) and 10b) are coupled equations, i.e.

$$11)\qquad (\dot{f}/f)(\dot{g}/g)=-(\omega/2)^2 \quad .$$

In the case $\Gamma_2=0$, Eqs. 10) reduce just to the equations encountered by Zener[3], Froissart and Stora[4] and Horwitz[5] as a consequence of their quantum-mechanical formulation of the problem.

Now, if the two Eqs. 10a) and 10b) are multiplied by $\dot{g}/(gf)$ and $\dot{f}/(gf)$ respectively, taking into account that the relationship 11) holds, one obtains the equations

$$12a)\qquad -\ddot{g}/\dot{g}+\dot{f}/f=-(i\Delta+\dot{\omega}/\omega-\Gamma_2 Z/R) \quad ,$$

$$12b)\qquad -\ddot{f}/\dot{f}+\dot{g}/g=(i\Delta-\dot{\omega}/\omega-\Gamma_2 Z/R) \quad .$$

Integrating once, one gets

$$13a)\quad \dot{f}=\{\dot{f}(0)/g(0)\}\{\omega/\omega(0)\}g\exp\left\{-\int_0^t i\Delta dt+\int_0^t(\Gamma_2 Z/R)dt\right\} \quad ,$$

$$13b)\quad \dot{g}=\{\dot{g}(0)/f(0)\}\{\omega/\omega(0)\}f\exp\left\{\int_0^t i\Delta dt-\int_0^t(\Gamma_2 Z/R)dt\right\} \quad .$$

According to Eq. 11) and in order to maintain symmetry between the functions f and g we write

$$\dot{f}(0)/g(0)=\dot{g}(0)/f(0)=-i\omega(0)/2$$

and get

$$14a)\quad \dot{f}=-i(\omega/2)g\exp\left\{-\int_0^t i\Delta dt+\int_0^t(\Gamma_2 Z/R)dt\right\} \quad ,$$

$$14b)\quad \dot{g}=-i(\omega/2)f\exp\left\{\int_0^t i\Delta dt-\int_0^t(\Gamma_2 Z/R)dt\right\} \quad ,$$

in the form obtained at once by the quoted Authors[3],[4],[5] in their quantum-mechanical treatment (with $\Gamma_2=0$).

Now, to facilitate comparison with other work[6],[7],[8], we convert Eqs. 14a) and 14b), using

15a) $$f = a \exp\left\{-(i/2)\int_0^t \Delta\, dt\right\} ,$$

15b) $$g = b \exp\left\{(i/2)\int_0^t \Delta\, dt\right\} ,$$

to

16a) $$\dot{a} - (i/2)\Delta a = -(i/2)\omega b \exp\left\{\int_0^t (\Gamma_2 Z/R) dt\right\} ,$$

16b) $$\dot{b} + (i/2)\Delta b = -(i/2)\omega a \exp\left\{-\int_0^t (\Gamma_2 Z/R) dt\right\} ,$$

where

$$Z/R = \{(2/\omega)^2 |\dot{a}/a - (i/2)\Delta|^2 - 1\} / \{(2/\omega)^2 |\dot{a}/a - (i/2)\Delta|^2 + 1\} ,$$

or

$$Z/R = -\{(2/\omega)^2 |\dot{b}/b + (i/2)\Delta|^2 - 1\} / \{(2/\omega)^2 |\dot{b}/b + (i/2)\Delta|^2 + 1\} ,$$

which agree, for $\Gamma_2 = 0$, with those given by Kroll and Watson[7] and Lau[6],[8].

It is worth draving attention to a point about the occupation numbers a and b, when $\Gamma_2 = 0$: with the normalization condition $aa^* + bb^* = 1$, X, Y and Z assume the expressions $X = ab^* + ba^*$, $Y = i(ab^* - ba^*)$, $Z = aa^* - bb^*$. These are, in fact, the three components of the Bloch vector defined by Feynman, Vernon and Hellwarth[1]. This can be very quickly derived by Eqs. 5), 9), 14) and 15), with E=1.

The conclusion can be drawn that the quantum-mechanical version of the damped Bloch equations puts us in a certain difficulty, because one cannot obtain insight into the meaning of the phase relaxation terms that come in. This is a consequence of the phenomenological nature of the damped Bloch equations and of the fact that the microscopic interpretation of dephasing processes is still a contumacious problem.

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