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## CHARMED QUARKS AND ASYMPTOTIC FREEDOM IN NEUTRINO SCATTERING

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Asymptotic freedom and charm production are both important ingredients for a theoretical analysis of neutrino cross sections. We study in detail the  $Q^2$  dependence of integrated quantities like cross sections,  $y$ -distributions and  $\langle x \rangle$  values. Deviations from scaling are quite substantial in the present energy range.

Deep inelastic neutrino scattering at the highest energies show a number of phenomena that indicate the existence of new quarks (besides  $p$ ,  $n$  and  $\lambda$ ). These phenomena include marked alterations of low energy scaling distributions and the presence of events with two leptons in the final state [1, 2]. While the existence of a new effect is clear, the question is whether all the observed features of the data can be explained in terms of only one new charmed quark, according to the simplest GIM scheme [3]. A related question is whether the data favor or not the existence of  $V+A$  couplings between the old quarks and the new ones [4, 5]. In this note we point out that the  $Q^2$  dependence predicted by asymptotically free gauge theories (AFGT) [6] is not all negligible in the rather large interval of  $Q^2$  that is being explored. Therefore these scaling breaking effects must necessarily be taken into account for a correct interpretation of the data. We find it useful to express in terms of partons the results of refs. [6, 7] on deep inelastic processes in AFGT and work with parton densities that include a  $Q^2$  dependence. In particular we construct simple expressions for the first two moments of valence, sea and charm distributions. These moments are relevant for total cross sections,  $y$  distributions and average  $x$  values. We find that the present experimental situation (including the behavior of the average value of  $y$  in  $\bar{\nu}$  [8] and the concentration of events at low  $x$ ) can in fact be accounted for by the GIM model because of the combined effect of AFGT and the charm threshold. We also predict a substantial rise of

the ratio  $R = \sigma^{\bar{\nu}}/\sigma^{\nu}$ .

We consider four types of quarks and adopt the simplifying assumption of decomposing parton densities into valence, sea and charm contributions according to:

$$\begin{aligned} p(x) &\equiv p_V(x) + s(x), & n(x) &\equiv n_V(x) + s(x), \\ \bar{n}(x) &\approx \bar{p}(x) \approx \lambda(x) \approx \bar{\lambda}(x) \equiv s(x), \\ p'(x) &\approx p'(x) \equiv c(x). \end{aligned} \quad (1)$$

We note that this is certainly compatible with AFGT where it amounts to specifying appropriate relations among a priori unknown matrix elements of local operators. In AFGT  $SU(4)$  singlet and non singlet operators acquire different logarithmic exponents [7]. We thus express the densities  $p_V$ ,  $n_V$ ,  $s$  and  $c$  in terms of an operator basis  $O_0, O_3, O_8, O_{15}$  transforming as  $\lambda_0, \lambda_3, \lambda_8$  and  $\lambda_{15}$  (with suitable normalizations). This leads to:

$$\begin{aligned} p_V &= \frac{1}{2}O_8 + \frac{1}{6}O_3, & n_V &= \frac{1}{2}O_8 - \frac{1}{6}O_3, \\ s &= \frac{1}{6}\left\{\frac{3}{4}O_0 + \frac{1}{4}O_{15} - O_8\right\}, & c &= \frac{1}{8}\{O_0 - O_{15}\}. \end{aligned} \quad (2)$$

The normalizations of  $O_i$  have been chosen in such a way that for  $O_i = 1$ ,  $p_V = 2n_V = 2/3$ ,  $s = c = 0$ .

The predictions of AFGT are simply expressed in terms of moments  $\langle q \rangle_N \equiv \int_0^1 dx x^N q(x)$ . Many useful results are directly obtained from moments with  $N=1$  and 2 which are relevant for cross sections ( $N=1$ ) and average values of  $x$  ( $N=2$ ). In this note, also in order to keep to a minimum the number of necessary inputs,

we restrict to these two moments. Moreover we are interested to the case of isoscalar targets, so that only the sum  $p_V + n_V \equiv V(x)$  is relevant.

In AFGT there are a tower of non singlet operators (containing  $O_3$ ,  $O_8$  and  $O_{15}$ ) and two towers of singlet operators [7]. Each tower has definite logarithmic exponents for each moment. One of the two singlet towers contains the energy momentum tensor (at the level  $N=1$ ) which has known matrix elements and zero logarithmic exponent. For each moment the admixture of the two singlet operators is fixed. For first moments one has:

$$\begin{aligned} \langle V \rangle_1 &= A_8^{(1)} L^{-32/75}, \\ \langle s \rangle_1 &= \frac{1}{8} \left[ \frac{3}{7} + \frac{4}{7} A_5^{(1)} L^{-56/75} + \left( \frac{1}{3} A_{15}^{(1)} - \frac{4}{3} A_8^{(1)} \right) L^{-32/75} \right], \\ \langle c \rangle_1 &= \frac{1}{8} \left[ \frac{3}{7} + \frac{4}{7} A_5^{(1)} L^{-56/75} - A_{15}^{(1)} L^{-32/75} \right]. \end{aligned} \quad (3)$$

Here

$$L = \frac{\alpha_s(\mu^2)}{\alpha_s(Q^2)} \approx \left[ 1 + \frac{25}{12\pi} \alpha_s(\mu^2) \ln \frac{Q^2}{\mu^2} \right], \quad (4)$$

with  $\mu^2$  being the normalization point and  $\alpha_s(Q^2)$  the strong interaction effective coupling constant:  $\alpha_s = g_s^2/4\pi$ . The expression for  $L$  corresponds to the leading logarithmic approximation<sup>†</sup>. In eqs. (3)  $A_8^{(1)}$  and  $A_{15}^{(1)}$  are the relevant matrix elements of  $O_8$  and  $O_{15}$  at the renormalization point. Similarly  $A_5^{(1)}$  is the matrix element of that singlet operator which is different from the energy momentum tensor.  $A_5^{(1)}$ ,  $A_8^{(1)}$  and  $A_{15}^{(1)}$  are to be fixed from experiment (at  $Q^2 \approx \mu^2$ ). Second moments are given by:

$$\begin{aligned} \langle V \rangle_2 &= A_8^{(2)} L^{-2/3}, \\ \langle s \rangle_2 &= \frac{1}{8} \left[ 0.925 A_8^{(2)} L^{-0.609} + 0.075 A_5^{(2)} L^{-1.39} \right. \\ &\quad \left. + \left( \frac{1}{3} A_{15}^{(2)} - \frac{4}{3} A_8^{(2)} \right) L^{-2/3} \right], \\ \langle c \rangle_2 &= \frac{1}{8} \left[ 0.925 A_8^{(2)} L^{-0.609} \right. \\ &\quad \left. + 0.075 A_5^{(2)} L^{-1.39} - A_{15}^{(2)} L^{-2/3} \right]. \end{aligned} \quad (5)$$

Note that in this case the number of unknown matrix elements has increased by one unit because both singlet matrix elements are to be determined.

We specify  $\mu^2 \approx 1 \text{ GeV}^2$  and fix  $A_i^{(1)}$  and  $A_i^{(2)}$

<sup>†</sup> The leading logarithmic approximation also allows to set (for each moment)  $F_2(x) \approx 2x F_1(x)$ . Neglected terms are of order  $\alpha_s(Q^2)/\pi$ .

from the experimental results in the Gargamelle energy region [9] also taking into account the SLAC electroproduction data [10] at lowest  $Q^2$ . In particular we assume a negligible contribution at  $Q^2 = 1 \text{ GeV}^2$  of charmed quarks to both moments and set:

$$\langle V \rangle_1 = 0.46; \quad \langle s \rangle_1 = 0.01; \quad \langle c \rangle_1 = 0 \quad \text{at } Q^2 \approx \mu^2, \quad (6)$$

which correspond to  $\sigma^p = G^2 M E_\nu 0.46/\pi$ ,  $R \equiv \sigma^{\bar{\nu}}/\sigma^p = 0.39$ ,  $\int dx \frac{1}{2} (F_2^p + F_2^N) = 0.14$  in agreement with experimental values at low energies within quoted errors. As for second moments we know from Gargamelle [9]  $\langle xy \rangle_\nu = (0.12 \pm 0.01)$  and  $\langle xy \rangle_{\bar{\nu}} = (0.07 \pm 0.01)$  which together with  $\langle c \rangle_2 = 0$  makes three independent inputs at  $Q^2 \approx \mu^2$ . We also take into account that an average of SLAC [10] data gives  $\int dx x \frac{1}{2} (F_2^p + F_2^N) \approx 0.036$ , but this is not an independent information in the parton model specified by eqs. (1). In order to disentangle the two singlet contributions  $A_8^{(2)}$  and  $A_5^{(2)}$  one more piece of information is needed which is physically equivalent to the knowledge of the  $N=2$  moment of the gluon distribution density in the nucleon  $G(x)$ . In fact  $\langle G \rangle_2 \equiv \int x^2 G(x) dx = 0.24 [A_8^{(2)} - A_5^{(2)}]$  at  $Q^2 \approx \mu^2$  (while  $\langle G \rangle_1 \equiv \int dx x G(x) = \frac{4}{7} (1 - A_5^{(1)}) = 0.48$  at  $Q^2 = \mu^2$ ). Thus as a fourth input we tentatively set  $\langle G \rangle_2 \approx 0.25 \langle G \rangle_1$  at  $Q^2 \approx \mu^2$  and:

$$\langle V \rangle_2 = 0.125; \quad \langle s \rangle_2 = 0.0015; \quad \langle c \rangle_2 = 0 \quad \text{at } Q^2 \approx \mu^2. \quad (7)$$

A crucial parameter is the value  $\alpha_s(\mu^2)$ . From the observed scaling violations in electroproduction at fixed  $x$ , studied in refs. [11, 12] and the widths of charmonium states [13] we find it reasonable to set roughly  $\alpha_s(1 \text{ GeV}^2) \approx 1/2$ .

While  $L$  depends on  $Q^2$  neutrino data are usually given in terms of the neutrino lab. energy  $E_\nu$ . Since  $L$  is a slowly varying function of  $Q^2$  we compute  $L$  from an effective  $Q^2$  value:

$$Q_{\text{EFF}}^2 \approx 2ME_\nu \langle xy \rangle, \quad (9)$$

where  $\langle xy \rangle$  is the ( $E_\nu$  dependent) average value, different for  $\nu$  and  $\bar{\nu}$ . At fixed  $E_\nu$  neutrino data involve larger values of  $Q_{\text{EFF}}^2$  than antineutrino data.

As for the effects of the charm threshold we approximate the threshold inequality by a condition on  $y$  only, according to ( $W_T$  is the threshold effective mass):

$$y \gtrsim \frac{W_T^2 - M^2}{2ME_\nu} \frac{1}{1 - x_0}, \quad (10)$$

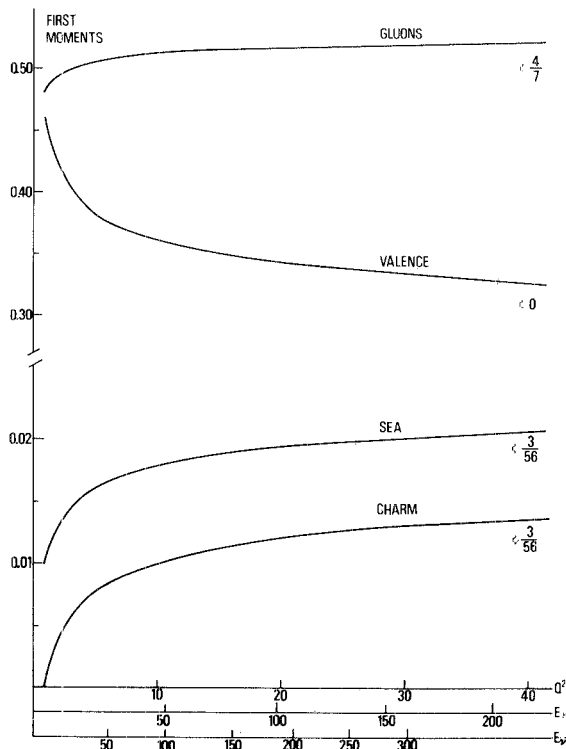


Fig. 1.  $\int_0^1 x f(x) dx$  for  $f(x) \equiv G(x)$  (gluons),  $V(x)$  (valence),  $s(x)$  (SU(3) symmetric sea),  $c(x)$  (charm) as functions of  $Q^2$  (or  $E_\nu$  and  $E_{\bar{\nu}}$ ). The arrows indicate the asymptotic values. The curves are computed from eqs. (3) with the initial values eq. (6) which correspond to  $A_8^{(1)} = 0.46$ ,  $A_{13}^{(1)} = 0.52$ ,  $A_S^{(1)} = 0.16$ .

with  $x_0 \approx 0.2$ . This approximation is not bad because only a few events are at large  $x$ ; it must be done in order to deal with complete moments in  $x$ .

We now describe our results. In fig. 1 first moments are plotted versus  $Q^2$  (or  $E_\nu$ ,  $E_{\bar{\nu}}$ ). The importance of sea and charm partons versus valence partons is rapidly increasing with  $Q^2$ . This peculiar feature of AFGT has several important consequences.

i) The ratio  $R = \sigma_{\bar{\nu}}/\sigma_\nu$  is predicted to rise with  $E_\nu$ . While for no  $Q^2$  dependence  $R$  would stay almost constant and insensitive to the charm threshold the increased proportion of momentum carried by the sea makes  $R$  to behave as in fig. 2. While this prediction is not supported by the data of the CALTECH group [14] (although not excluded within quoted errors) a sharp rise of  $R$  has been reported by the HPWE group [15].

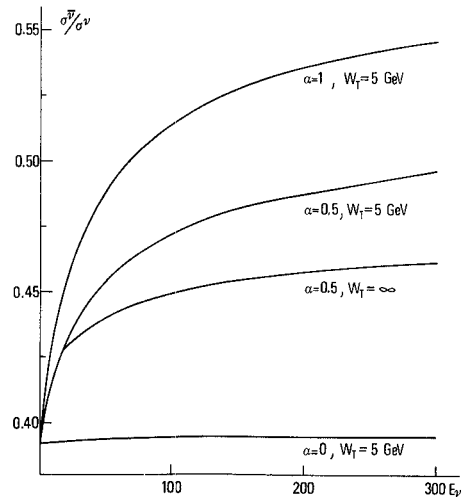


Fig. 2. The ratio  $\sigma_{\bar{\nu}}/\sigma_\nu$  for different values of  $\alpha \equiv \alpha_S(\mu^2)$  and  $W_T$ , the effective invariant mass for charm threshold.

ii) With no  $Q^2$  dependence the fractions  $\Delta\sigma/\sigma$  of charmed final states would not exceed 10% even at infinite energy. With the  $Q^2$  dependence included, at  $E_\nu = 50, 100, 300$  GeV we have  $(\Delta\sigma/\sigma)_\nu \approx 10\%, 13\%, 16\%$  and  $(\Delta\sigma/\sigma)_{\bar{\nu}} \approx 13\%, 17\%, 22\%$ , respectively. If charmed particles have an average branching ratio into muons of the order 5 to 10%, the observed yield [1] of events with oppositely charged muons is obtained.

iii) The raise with  $Q^2$  of the amount of nucleon momentum carried by charmed quarks (the theory is asymptotically SU(4) symmetric) may have important consequences, also for dimuon events with equal charges [1]. For example consider a purely V-A theory with the weak current given by [16]

$$J_\mu = (\bar{p}n_C) + [\bar{p}'(\cos\alpha\lambda_C + \sin\alpha b)] + [\bar{t}(-\sin\alpha\lambda_C + \cos\alpha b)], \quad (11)$$

where  $n_C$  and  $\lambda_C$  are the Cabibbo rotated quarks and  $t$  and  $b$  are new quarks with charges  $2/3$  and  $-1/3$  respectively. Then starting from  $\bar{\nu}(p)$  a  $b(\bar{b})$  quark can be excited from a  $p'$  quark ( $\bar{p}'$ ) in the nucleon. Then  $b(\bar{b})$  would decay into  $p'$  or  $t$  ( $\bar{p}'$  or  $\bar{t}$ ) which in turn may decay into another  $\mu^+(\mu)$ . The most favourable situation is above threshold for  $p'$  and  $b$  and below threshold for  $t$ . In this case:

$$\frac{R(\mu^+\mu^+ + \dots)}{R(\mu^+\mu^- + \dots)} \sim \tan^2\alpha \frac{\frac{1}{3}\langle c \rangle_1}{\langle s \rangle_1 + \frac{1}{3}\langle c \rangle_1} \sim \tan^2\alpha \cdot 1/7, \quad (12)$$

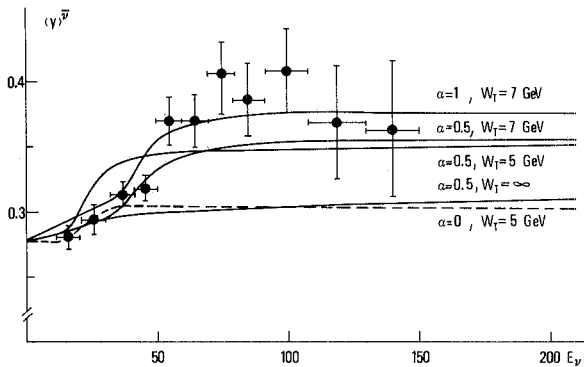


Fig. 3. Average value of  $y$  for antineutrinos for different values of  $\alpha \equiv \alpha_s(\mu^2)$  and  $W_T$ , the effective invariant mass for charm threshold.  $\alpha = 0$  corresponds to  $Q^2$  independent parton distributions.  $W_T \rightarrow \infty$  corresponds to neglecting effects from charm production. Both effects seem to be needed to reproduce the data.  $\alpha = 0.5$  is the value suggested in the text, while  $\alpha = 1$  is reported for comparison.

(and similarly  $\tan^2 \alpha \cdot 1/10$  for  $\nu$ ). If  $t$  also is produced  $\tan^2 \alpha$  is replaced by  $\sin^2 \alpha$ . Since in absence of V+A couplings  $\tan \alpha$  can be of order 1, the observed effect could be accounted for by this mechanism either entirely or in part.

iv) In fig. 3 the behavior of the average value of  $y$  for  $\bar{\nu}$  is plotted. It is apparent that the predicted  $Q^2$  dependence is enough to reconcile the GIM model with the data. We note that  $W_T$  appears larger than it could be expected. Note that in the model of eq. (11)  $\lambda_C$  is coupled to a combination of  $p'$  and  $t$ , and the entire new current is only effective after both  $p'$  and  $t$  can be produced. However the fact that  $W_T$  is so large is not necessarily significant since it is only an effective mass where rescaling after threshold is obtained. Indeed in  $\bar{\nu}$   $W_T$  could be expected to be larger than in  $\nu$  since the hadronic final state is exotic (a baryon with negative charm).

Fig. 4 refers to second moments. Valence and sea distributions slowly shrink toward zero. The behavior of  $\langle x \rangle_{\text{VALENCE}}$  as a function of  $Q^2$  is a rather firm prediction, while for  $\langle x \rangle_{\text{SEA}}$  the poor knowledge of the input values of  $\langle x \rangle_{\text{SEA}}$  and  $\langle x \rangle_{\text{GLUONS}}$  at  $Q^2 \approx \mu^2$  makes a much larger error quite possible.

In conclusion we find that the  $Q^2$  dependence as predicted by AFGT must be taken into account in an analysis of neutrino data over the present experimental range. The neglect of this important effect may

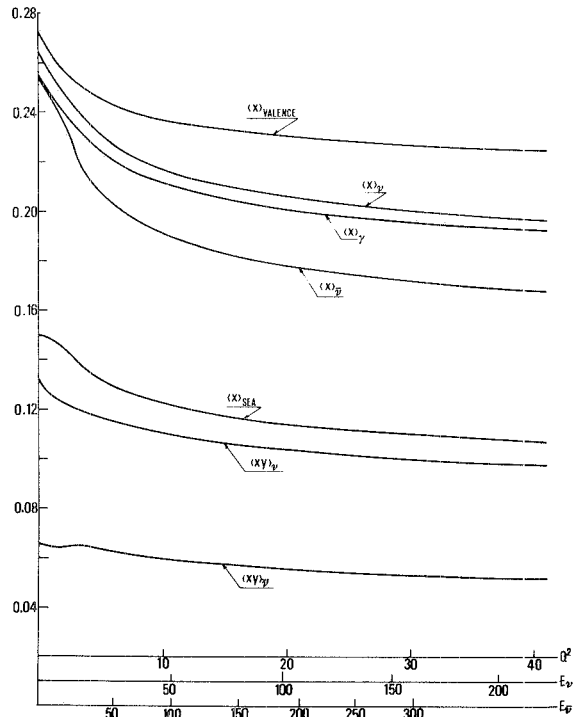


Fig. 4. Average  $x$  values for valence and sea partons ( $\langle x \rangle_{\text{VALENCE}} \equiv \int_0^1 dx x^2 V(x) / \int_0^1 dx x V(x)$ ;  $\langle x \rangle_{\text{SEA}} \equiv \int_0^1 dx x^2 s(x) / \int_0^1 dx x s(x)$ ) and for neutrino, antineutrino and electro-production (on isoscalar targets). Also shown are the average values of  $xy$  for neutrino and antineutrino. All curves are for  $\alpha_s(\mu^2) = 0.5$  and  $W_T \approx 5 \text{ GeV}$ . All curves are derived from eq. (5) with the initial values of eqs. (7) which correspond to  $A_8^{(2)} = 0.125$ ,  $A_{15}^{(2)} = 0.134$ ,  $A_0^{(2)} = 0.1715$ ,  $A_S^{(2)} = -0.3285$ .

otherwise lead to different qualitative conclusions on the structure of the weak current. With the  $Q^2$  dependence included we find the data to be compatible with four flavours.

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