ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-76/43

G. Parisi and R. Petronzio:
ON THE BREAKI NG OF BJORKEN SCALING

Phys. Letters 62B, 331 (1976)

Servizio Documentazione
dei Laboratori Nazionali di Frascati
Cas. Postale 13 - Frascati (Roma)

# ON THE BREAKING OF BJORKEN SCALING

G. PARISI
*Istituto Nazionale di Fisica Nucleare, Frascati (Roma), Italy*

and

R. PETRONZIO
*Istituto di Fisica "G. Marconi", Roma, Italy*
*Istituto Nazionale di Fisica Nucleare, Sezione di Roma, Italy*

Received 25 March 1976

In the framework of coloured quark model we obtain detailed predictions for the $q^2$ dependence of the structure functions of the proton and the neutron and the $\sigma_{\bar{\nu}}/\sigma_\nu$ ratio.

In a gauge theory of strong interactions with colored quarks the Bjorken scaling law in deep inelastic scattering is not valid [1,2]. The deviations from this law can be exactly computed for the difference of the structure functions of the proton and the neutron ($I = 1$ in the $t$-channel) [3]. However specific assumptions are required to compute the deviations from Bjorken scaling in the sum of the structure functions of the proton and neutron ($I = 0$ in the $t$-channel). In this letter we try to obtain predictions also in the latter case.

It is well known that for each spin $N$ three different operators contribute to the light cone expansion of two electromagnetic currents in the $I = 0$ channels. (Only one operator is present in the $I = 1$ channel). Neglecting $\gamma$ matrices, covariant derivatives and indices, these three operators can be formally written as:

$$O^8 = \bar{n}n + \bar{p}p - 2\bar{\lambda}\lambda \; ; \quad O^{\mathrm{o}} = \bar{n}n + \bar{p}p + \bar{\lambda}\lambda \; ; \quad O_{\mathrm{V}} = VV \tag{1}$$

$p$, $n$, $\lambda$ and $V$ are respectively the three quark fields and the gluon field. We assume the presence of only three kinds of coloured quarks; the results can be extended to more complicated cases where 4, 6 to 7 quarks are present, provided one is able to treat correctly the consequences of large unitary symmetry breaking. Our results should remain valid in an intermediate energy region where the production of heavy quarks is negligible. The operators which transform multiplicatively under the action of the renormalization group are [4]

$$O_N^8 \; ; \quad O_N^{\mathrm{I}} = a_N O_N^{\mathrm{o}} + O_N^{\mathrm{V}} \; ; \quad O^{\mathrm{II}} = b_N O_N^{\mathrm{o}} - O_N^{\mathrm{V}} \, . \tag{2}$$

Using the eN scattering data at a fixed value of $q^2$ it is impossible to disentangle the contributions of these three operators, however each contribution has a different dependence from $q^2$.

In this letter we propose a minimal set of assumptions to fix the relative contributions of each of these operators. These assumptions allow us to compute the distributions of gluons and antiquarks inside the proton; these results have observable consequences for the $q^2$ dependence of the structure functions and on the ratio $\sigma_{\bar{\nu}}/\sigma_\nu$.

Roughly speaking, we suggest that the short range part of the interaction may be neglected; taking into account only the long range part the proton is composed in the $P_\infty$ frame of only three quarks, antiquarks and gluons being absent [5]. Turning on the short range part of the interaction we introduce gluons, antiquarks into the proton. We try to control this effect using perturbation theory and the renormalization group [6].

Technically speaking we propose that:

$$\langle \mathrm{p}|\bar{\lambda}\lambda|\mathrm{p}\rangle = \langle \mathrm{p}|VV|\mathrm{p}\rangle = 0 \, . \tag{3}$$

However the precise definition of the $O_N$ operators depends upon the choice of the renormalization point $\mu$: eq. (3) cannot be valid for any arbitrary $\mu$. We assume that there exists a value of $\mu$ such that eq. (3) is satisfied. We assume also that the $\gamma_N(g)$ functions which appear in the Callan Symanzik equation for the $O_N$ operators can be successfully approximated by neglecting terms of order $g^4$ ($g$ is the coupling constant of the coloured gauge field). In reality we need a weaker assumption: we have only to assume that the contributions to the $\gamma_N$ functions coming from graphs with many loops are proportional to the one loop contribution, i.e. the functions $\gamma_N(g)$ factorize into the product $A_N h(g)$ ($A_N$ being the "anomalous dimensions of the $O_N$ operators and $h(g)$ an $n$-independent function).

We understand that our assumptions are very strong and are definitively wrong for particular values of $N$ ($N \approx 1$); their validity for general $N$ critically depends upon the magnitude of the coupling constant at the renormalization point $\mu$. We consider them as a first rough approximation which can be improved after the construction of a reliable model of the nucleon. More general but less predictive results can be obtained studying the positivity properties of the matrix elements of the $O^N$ operators. This approach will not be pursued here.

Under these hypotheses we get [7,8]

$$F_N^{\mathrm{o}}(q^2) \equiv \int_0^1 F^{\mathrm{o}}(x, q^2) x^{N-2}\, \mathrm{d}x$$

$$= C_N[(1-\delta_N)\, G^{-A_N^{\mathrm{I}}} + \delta_N G^{-A_N^{\mathrm{II}}} + \tfrac{1}{4} G^{-A_N^{8}}] \,,$$

$$F^{\mathrm{o}}(x, q^2) = \tfrac{1}{2}[F_2^{\mathrm{p}}(x, q^2) + F_2^{\mathrm{n}}(x, q^2)] \,, \qquad (4)$$

$$\delta_N = b_N/(a_N + b_N) \,,$$

$$G = \exp\left[-\int_g^{\bar{g}(q^2)} \frac{h(g')}{\beta(g')} \mathrm{d}g'\right] ,$$

$G$ is defined in such a way that neglecting the high order contributions to $h$ and $\beta$ one gets

$$G = 1 + \frac{9\alpha}{4\pi} \ln(q^2/\mu^2) \,; \quad \alpha = \frac{g^2}{4\pi} \,. \qquad (5)$$

Eq. (4) is the main result of our work; the rest of this note is devoted to the extraction of observable consequences from it. $F_N(q^2)$ depends upon $q^2$ only via $G$. In the large $q^2$ region we get:

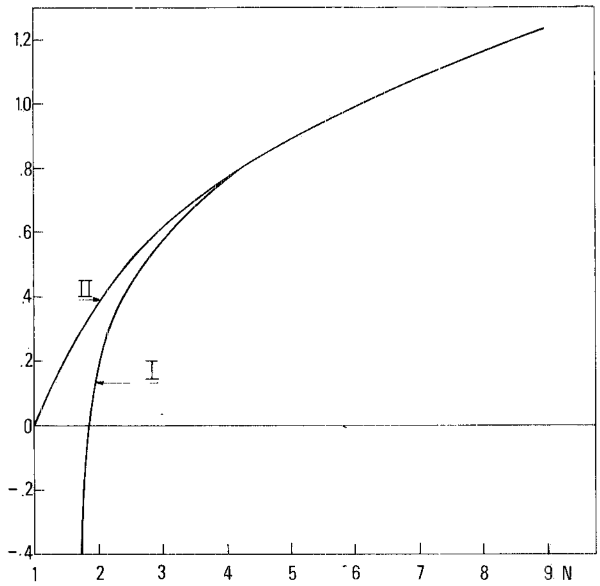


Fig. 1. Curves I and II represent respectively $A_N^{\mathrm{ef}}(G)$ and $A_N^8(G)$ for $G = 10$ as a function of the order $N$ of the moment to which they refer.

$$\frac{\mathrm{d} \ln G}{\mathrm{d} \ln q^2} = \frac{9}{4\pi} \alpha(q^2) \,,$$

$$\frac{\mathrm{d} \ln F_N^{\mathrm{o}}(q^2)}{\mathrm{d} \ln q^2} = -\frac{9}{4\pi} \alpha(q^2)[A_N^8 + \tilde{A}_N(G)]$$

$$= -\frac{9}{4\pi} \alpha(q^2) A_N^{\mathrm{ef}}(G) \,, \qquad (6)$$

$$\tilde{A}_N(G) = \frac{(1-\delta_N) G^{-A_N^{\mathrm{I}}} (A_N^{\mathrm{I}} - A_N^8) + \delta_N G^{-A_N^{\mathrm{I}}} (A_N^{\mathrm{II}} - A_N^8)}{(1-\delta_N) G^{-A_N^{\mathrm{I}}} + \delta_N G^{-A_N^{\mathrm{II}}} + \frac{1}{4} G^{-A_N^8}} \,.$$

It is clear that our assumptions fix the relative weight of the contributions of each operator in deep inelastic scattering and allow us to compute the derivative of each moment. It may be physically interesting to write the effective anomalous dimension as the sum of the octet part and of a correction due to the mixing of fermion and gluon operators. This correction is $G$ dependent: it is the only part of $A_N^{\mathrm{ef}}(G)$ which depends upon our assumptions. In fig. 1 we have plotted $A_N^{\mathrm{ef}}(G)$ and $A_N^8$ for a particular value of $G$ ($G = 10$).

If we analize the structure of the Feynman diagrams which contribute to eq. (4), we discover [6] that the right-hand side can be written as the sum of two pieces, the first being the contribution of the valence quarks, the second of the quark antiquark sea.

$$F_N^{\mathrm{o}}(q^2) = V_N(q^2) + S_N(q^2)\ ; \quad V_N(q^2) = \tfrac{5}{4} C_N G^{-A_N^8}\ ,$$

$$S_N(q^2) = C_N[(1-\delta_N)G^{-A_N^{\mathrm{I}}} + \delta_N G^{-A_N^{\mathrm{II}}} - G^{-A_N^8}]\ ,$$

$$S_N(q^2) = \frac{(1-\delta_N)G^{-A_N^{\mathrm{I}}} + \delta_N G^{-A_N^{\mathrm{II}}} - G^{-A_N^8}}{(1-\delta_N)G^{-A_N^{\mathrm{I}}} + \delta_N G^{-A_N^{\mathrm{II}}} + \frac{1}{4}G^{-A_N^8}}\, F_N^{\mathrm{o}}(q^2)$$

$$\equiv R_N(G) F_N^{\mathrm{o}}(q^2)\ . \qquad (7)$$

It is possible to separate the quark and the antiquark contributions to deep inelastic electron scattering and consequently compute the ratio of the antineutrino and neutrino cross sections.

In principle eq. (7) may be used to fix the value of $G$; however it can be done more accurately using the fact that $O_2^{\mathrm{I}}$ is the stress energy tensor and its matrix elements among states at rest are shown. For $N = 2$ eq. (4) can be written as:

$$F_2^{\mathrm{o}}(q^2) = \tfrac{2}{9}[\tfrac{9}{25} + \tfrac{16}{25}G^{-0.62}] + \tfrac{1}{18}G^{-0.4}\ . \qquad (8)$$

For $G = 1$ we recover the naive quark model sum rule, in the limit $G \to \infty$ we obtain the $G$ asymptotic prediction 2/25. Inserting the experimental value 0.14 we obtain $G = 10$.

Up to now we have analysed the behaviour of the moments of the structure functions. The results for the structure function itself may be obtained performing the antiMellin transform of eq. (4) and eq. (7) and using the Mellin convolution theorem [9]. One gets [3]:

$$\frac{\mathrm{d}F^{\mathrm{o}}(x,q^2)}{\mathrm{d}\ln q^2} = \frac{\alpha(q^2)}{3\pi}\Big\{[3 + 4\ln(1-x)]\ F^{\mathrm{o}}(x,q^2) + \int_x^1 \frac{\mathrm{d}y}{y}$$

$$\times\Big[\Big(2y(1-y) + \frac{4y^2}{1-y} + a(y)\Big)\ F(x/y,q^2) - \frac{4yF^{\mathrm{o}}(x,q^2)}{1-y}\Big]\Big\}$$

$$S(x,q^2) = \int_x^1 \frac{\mathrm{d}y}{y}\, r(y) F^{\mathrm{o}}(x/y,q^2)\ . \qquad (9)$$

The functions $a(y)$ and $r(y)$ are respectively the anti-Mellin transforms of $\widetilde{A}_N(G)$ and $R_N(G)$ and are $G$ dependent. They have been plotted in fig. 2 for $G = 10$.

Let us define by $q(x,q^2)$, $(\bar{q}(x,q^2))$ the number of non strange quarks (antiquarks) in an equal mixture of protons and neutrons. In our model the sea is SU(3) symmetric and therefore distribution functions of the other quarks can be easily obtained. These new functions can be directly extracted from the neutrino and the antineutrino data [10]; they are equal to:

$$xq(x,q^2) = \tfrac{18}{5}[F^{\mathrm{o}}(x,q^2) - S(x,q^2)] + \tfrac{3}{2}S(x,q^2)]\ ,$$

$$x\bar{q}(x,q^2) = \tfrac{3}{2}S(x,q^2)\ . \qquad (11)$$

Using as the input [11]

$$F_2^{\mathrm{p}}(x) = (1-x)^3[1.274 + 0.5989(1-x) + 1.675(1-x)^2]\ ,$$

$$F_2^{\mathrm{p}} - F_2^{\mathrm{n}} = x(1-x)^3\ ,$$

at $q^2 \approx 3$ GeV$^2$ (the reader may use his best fit to the data or the data themselves), one gets the results plotted in figs. 3 and 4, assuming $\alpha(q^2) = 0.4$ for $q^2$ in the few GeV range. This value is not very far from that obtained on the basis of the width of the J-$\psi$ particle [12,13]. We notice that our predictions are wrong in the limit $x$ goes to 0 or 1, because the $\gamma_N$ functions are divergent when $N$ goes to 1 or $\infty$.

In fig. 3 curve I is our prediction while curve II is obtained neglecting the term proportional to $\widetilde{A}_N(G)$ in eq. (6) [14]. The difference between the two curves is small for $x$ not too near to zero; an error in our assumptions would only affect proportionally this difference and would not change appreciably the final results.

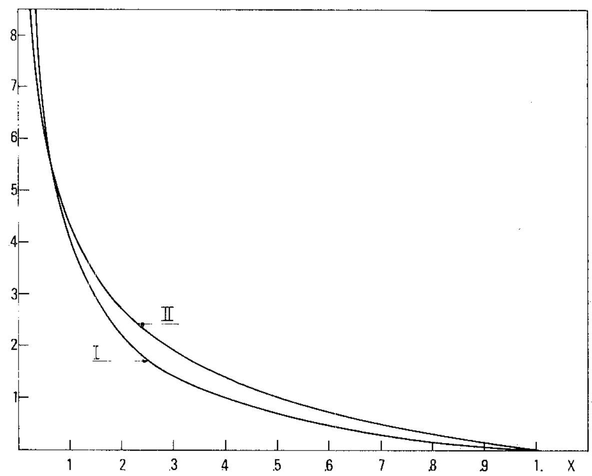


Fig. 2. Curves I and II represent respectively the behaviour of the kernels $a(y)$ and $r(y)$.

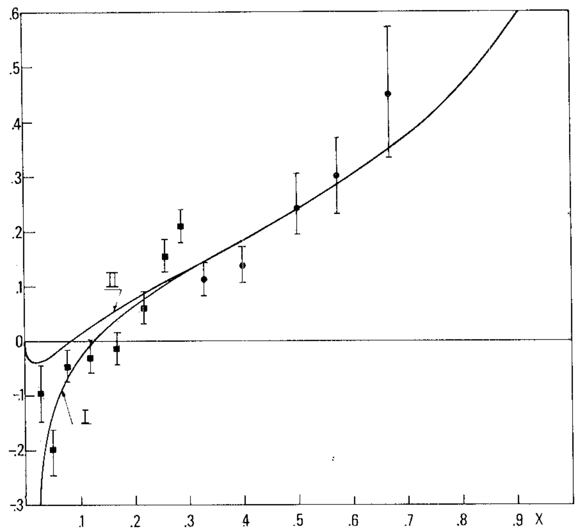


Fig. 3 Curve I is our prediction for $\mathrm{d}\ln F_2^{\mathrm{p}}(x, q^2)/\mathrm{d}\ln q^2$ compared with experimental data taken from ref. [15] (•) and ref. [16] (▪): the points taken from ref. [15] are $[\mathrm{d}\ln W_1/\mathrm{d}\ln q^2 + \mathrm{d}\ln W_2/\mathrm{d}\ln q^2]/2$ which correspond to $(c_1 + c_2)/2$ of their table 1. Curve II is obtained retaining only the octet operators in the operator expansion.

The experimental values for $\mathrm{d}\ln F_2^{\mathrm{p}}(x, q^2)/\mathrm{d}\ln q^2$ depend strongly upon the choice of the scaling variable ($x$ or $x'$). Moreover under our hypothesis the relation $xF_1 = 2F_2$ should be approximately valid for reasonably large $q^2$ and asymptotically true. Our predictions do not distinguish $2F_2(x, q^2)$ from $xF_1(x, q^2)$. We have resolved this ambiguity plotting in fig. 3 the experimental values for $\frac{1}{2}[\mathrm{d}\ln F_1/\mathrm{d}\ln q^2 + \mathrm{d}\ln F_2/\mathrm{d}\ln q^2]$ .

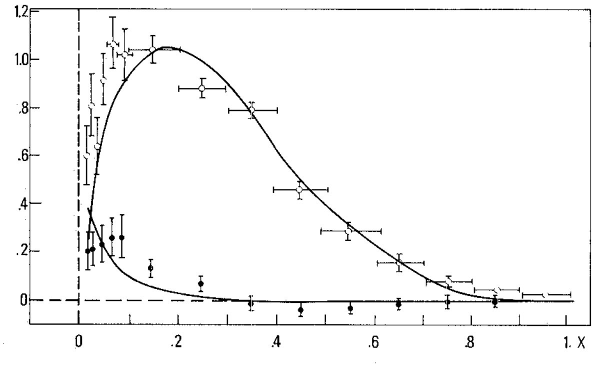


Fig. 4. Our predictions for the amount of quarks (○) and antiquarks (•) on an equal mixture of p and n nucleons are presented together with experimental values extracted from neutrino and antineutrino scattering.

This problem arises only for the e-p data [15], the $\mu$-p [16] data have not been used to separate $F_2$ from $F_1$.

It should be clear that we need more precise data in a wide range of $q^2$ to understand if the Bjorken scaling law is violated asymptotically or its violations are a low energy phenomena which will finally disappear with increasing $q^2$. Moreover if the Bjorken scaling law is not valid, it is not simple to distinguish asymptotically free theories and scale invariant theories with anomalous dimensions: one should determine the $q^2$ dependence of the violations of the Bjorken scaling law. We remember that it was noticed long time ago [9] that the "observed" pattern of scaling violations (increase at large $x$ and decrease at small $x$) is a general feature of any scale invariant theory (both asymptotically and non asymptotically free): it stems from the conservation of the stress energy tensor [17].

At the present moment one can only compare the derivative of the structure function with the predictions coming from field theory at only one value of $q^2$; in this letter we have accomplished this task in the case of coloured quarks interacting via gluon exchange.

## References

[1] G. 't Hooft, Marseille Conf. June 1972, unpublished.
[2] H.D. Politzer, Phys. Rev. Letters 30 (1973) 1946; D. Gross and F. Wilczek, Phys. Rev. Letters 30 (1973) 1343.
[3] G. Parisi, Phys. Lett. 50B (1974) 367.
[4] D. Gross and F. Wilczek, Phys. Rev. D9 (1974) 980.
[5] G. Altarelli, N. Cabibbo, L. Maiani and R. Petronzio, Nucl. Phys. B69 (1974) 531.
[6] G. Parisi and R. Petronzio, A dynamical approach to deep inelastic scattering, submitted to Nuovo Cimento.
[7] N. Christ, B. Hasslacher and A. Mueller, Phys. Rev. D6 (1972) 3543.
[8] K. Symanzik, Nuovo Cimento Lettere 6 (1973) 420.
[9] G. Parisi, Phys. Lett. 43B (1973) 207.
[10] CERN – Gargamelle collaboration, Phys. Lett. 46B (1973) 274.
[11] G. Miller et al., Phys. Rev. D5 (1972) 528.
[12] T. Appelquist et al., Phys. Rev. Letters 34 (1975) 365.
[13] E. Eichten et al. Phys. Rev. Letters 34 (1975) 369.
[14] W. Tung, University of Chicago report FF/75/14.
[15] E.M. Riordan, Phys. Lett. 52B (1974) 249.
[16] C. Chang et al., Phys. Rev. Lett. 35 (1975) 901.
[17] S. Ciccariello et al., Phys. Lett. 30B (1969) 546.