

To be submitted to
Physics Letters

ISTITUTO NAZIONALE DI FISICA NUCLEARE
Laboratori Nazionali di Frascati

LNF-76/41(P)
16 Luglio 1976

M. Greco and H. Inagaki: RADIATIVE DECAYS OF
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ABSTRACT. -

A recently proposed scheme for radiative decays of mesons with vector meson dominated vertices and the asymptotics implied by quark current algebra is applied to scalar meson decays. The reactions $\varepsilon \rightarrow \gamma\gamma$, $\chi \rightarrow \gamma\gamma$, $\psi' \rightarrow \chi\gamma$ and $\chi \rightarrow \psi\gamma$ are considered in detail. Results are in good agreement with experiments.

A new treatment of radiative decays of mesons has been recently proposed⁽¹⁾, where the basic currents of SU(4) are dominated by vector mesons with appropriate quantum numbers and exhibit a quark structure asymptotically. Applied to pseudo scalar (P) and vector (V) meson decays $P \rightarrow \gamma\gamma$, $P \rightarrow V\gamma$ and $V \rightarrow P\gamma$ this approach gives an excellent description of SU(3) meson decays, including those for which the naive quark model failed, and predicts in addition for the new mesons considerably smaller widths than obtained in the non-relativistic bound state picture⁽²⁾.

Encouraged by those results we apply in this paper the same scheme to study the radiative decays which involve a scalar (σ) meson, namely $\sigma \rightarrow \gamma\gamma$, $V \rightarrow \sigma\gamma$ and $\sigma \rightarrow V\gamma$, having particularly in mind the case $\sigma \equiv \chi$, where χ is the newly discovered scalar meson of mass 3.41 GeV. Start-

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ing with a vertex function $F_\sigma(q_1^2, q_2^2)$ of the type proposed in ref. (1), which has the analyticity in q_1^2 and q_2^2 of a strong interaction vertex, we constrain it using the canonical trace anomalies at low energy⁽⁴⁾ and the appropriate large q^2 behaviour of the quark current algebra. Applied to the ϵ meson this scheme gives $\Gamma(\epsilon \rightarrow \gamma\gamma) \simeq 6 \text{ keV}$, in closer agreement with the finite energy sum rules estimates⁽⁵⁾. On the other hand our results involving the χ meson, differ by about a factor of three from non-relativistic calculations⁽²⁾ and are in good agreement with experiments.

Let us define the $\sigma\gamma\gamma$ vertex :

$$(1) \quad T_{\mu\nu}(q_1, q_2) = i \int d^4x e^{iQx} \langle \sigma(P) | T \left[J_\mu\left(\frac{x}{2}\right) J_\nu\left(-\frac{x}{2}\right) \right] | 0 \rangle,$$

where $Q = \frac{1}{2}(q_2 - q_1)$, $P = q_2 + q_1$ with $q_{1,2}$ the momenta of the two photons. $T_{\mu\nu}(q_1, q_2)$ can be decomposed⁽⁶⁾ as :

$$(2) \quad T_{\mu\nu}(q_1, q_2) = i A_{\mu\nu} F_\sigma(q_1^2, q_2^2) + i A'_{\mu\nu} F'_\sigma(q_1^2, q_2^2),$$

with :

$$(3) \quad A_{\mu\nu} = Q^2 P_\mu P_\nu + P^2 Q_\mu Q_\nu - (P \cdot Q)(P_\mu Q_\nu + P_\nu Q_\mu) + [(P \cdot Q)^2 - Q^2 P^2] g_{\mu\nu},$$

$$A'_{\mu\nu} = -\frac{1}{4} P_\mu P_\nu + Q_\mu Q_\nu + \frac{1}{2} (P_\mu Q_\nu - P_\nu Q_\mu) - (Q^2 - \frac{1}{4} P^2) g_{\mu\nu}.$$

The values $F_\sigma(q_1^2=0, q_2^2=0)$ and $F'_\sigma(q_1^2=0, q_2^2=0)$ are connected to the coupling constant $g_{\sigma\gamma\gamma}$ for the $\sigma \rightarrow \gamma\gamma$ decay as :

$$(4) \quad g_{\sigma\gamma\gamma} = \frac{1}{4} \left[m_\sigma^2 F_\sigma(0, 0) + 2 F'_\sigma(0, 0) \right],$$

where $g_{\sigma\gamma\gamma}$ is defined through the effective Lagrangian :

$$(5) \quad \mathcal{L}_{\text{int}} = \frac{1}{2} g_{\sigma\gamma\gamma} \sigma F_{\mu\nu} F^{\mu\nu}.$$

Let us restrict first to SU(3). The application of the canonical trace anomaly of the energy momentum tensor leads to the low energy theorem⁽⁴⁾:

$$(6) \quad F'_\sigma(0,0) = \frac{R}{6\pi^2 f_\sigma} ,$$

with the usual definition of the asymptotic ratio

$R \equiv \sigma(e^+e^- \rightarrow \text{hadrons})/\sigma(e^+e^- \rightarrow \mu^+\mu^-)$. In eq. (6) it has been assumed that the scalar meson σ dominates the trace of the energy momentum tensor, which defines f_σ as:

$$(7) \quad \langle \sigma(P) | \theta_{\mu\nu}(0) | 0 \rangle = \frac{i}{3} f_\sigma (m_\sigma^2 g_{\mu\nu} - P_\mu P_\nu) .$$

$F'_\sigma(0,0)$ is so far arbitrary. The authors of ref. (4) have neglected it in eq. (4) obtaining:

$$(8) \quad g_{\sigma\gamma\gamma} = \frac{1}{2} F'_\sigma(0,0) = \frac{R}{12\pi^2 f_\sigma} ,$$

and, using $f_\sigma \simeq 150$ MeV from the broken-scale invariance estimate $g_{\sigma\pi\pi} \simeq m_\sigma^2/2f_\sigma$, with $\Gamma(\sigma \equiv \varepsilon \rightarrow \pi\pi) \simeq 400$ MeV ($m_\varepsilon \simeq 700$ MeV),

$$(9) \quad \Gamma(\varepsilon \rightarrow \gamma\gamma) \simeq 0.2 R^2 \text{ keV} ,$$

which is much smaller ($R \simeq 2$) than other estimates⁽⁵⁾ based on finite energy sum rules. The above results (eqs. 8-9) for the $\sigma\gamma\gamma$ coupling constant deserve two comments. First one cannot generally neglect $F_\sigma(0,0)$ with respect to $F'_\sigma(0,0)/m_\sigma^2$. Rather, in the large q^2 limit we obtain (see eq. (17) below):

$$\lim_{q^2 \rightarrow \infty} F_\sigma(q^2, q^2) = \frac{1}{m_\sigma^2} F'_\sigma(q^2, q^2) .$$

Under the assumption of smooth extrapolation in q^2 this gives $F_\sigma(0,0) = F'_\sigma(0,0)/m_\sigma^2$ and therefore:

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$$(10) \quad g_{\sigma\gamma\gamma} = \frac{R}{8\pi^2 f_\sigma},$$

instead of eq. (8). Furthermore more recent analyses⁽⁷⁾ suggest $\Gamma(\varepsilon \rightarrow \pi\pi) \simeq 700$ MeV, with all possible question marks about a simple Breit-Wigner interpretation of the ε resonance. This brings f_σ to a value $f_\sigma \simeq 110$ MeV. Both facts would increase the theoretical estimate (9) by more than a factor of four, in better agreement with other predictions⁽⁵⁾.

Let us discuss now the large Q^2 behaviours of $F_\sigma(q_1^2, q_2^2)$ and $F'_\sigma(q_1^2, q_2^2)$ which, together with eq. (6), will finally constrain our dual-type vertex. The light cone expansion of the time ordered product of electromagnetic currents is^(8,9):

$$(11) \quad T \left\{ J_\mu \left(\frac{x}{2} \right) J_\nu \left(-\frac{x}{2} \right) \right\} = - \left\{ s_{\mu\rho\nu\sigma} \left[J_{Q^2}^\sigma \left(\frac{x}{2}, -\frac{x}{2} \right) - J_{Q^2}^\sigma \left(-\frac{x}{2}, \frac{x}{2} \right) \right] - i \varepsilon_{\mu\rho\nu\sigma} \left[J_{Q^2}^{5\sigma} \left(\frac{x}{2}, -\frac{x}{2} \right) + J_{Q^2}^{5\sigma} \left(-\frac{x}{2}, \frac{x}{2} \right) \right] \right\} \frac{\partial}{\partial x_\rho} D_F(x),$$

where $s_{\mu\rho\nu\sigma} = g_{\mu\rho} g_{\nu\sigma} + g_{\mu\sigma} g_{\nu\rho} - g_{\mu\nu} g_{\rho\sigma}$ and $J_{Q^2}^{(5)\sigma} \left(\frac{x}{2}, -\frac{x}{2} \right)$ are bilocal vector and axial-vector currents given in the free quark model by:

$$(12) \quad j_{Q^2}^{(5)\sigma} \left(\frac{x}{2}, -\frac{x}{2} \right) = \bar{q} \left(\frac{x}{2} \right) \gamma^\sigma (\gamma_5) Q^2 q \left(-\frac{x}{2} \right).$$

Our present discussion is only concerned with the $s_{\mu\rho\nu\sigma}$ part. The expansion of the bilocals near $x=0$ gives:

$$(13) \quad \langle \sigma(P) \left| J_{Q^2}^\sigma \left(\frac{x}{2}, -\frac{x}{2} \right) - J_{Q^2}^\sigma \left(-\frac{x}{2}, \frac{x}{2} \right) \right| 0 \rangle = 2ix_\rho \langle \sigma(P) \left| \theta_{Q^2}^{\sigma\rho}(0) \right| 0 \rangle,$$

where in the quark model:

$$(14) \quad \theta_{Q^2}^{\sigma\rho}(x) = \sum_q \frac{i}{2} \bar{q}(x) \gamma^{\sigma\leftrightarrow\rho} Q^2 q(x).$$

Extracting from $\theta^{\sigma\rho}(x)$ the singlet piece ($\theta^{\rho\sigma} \sim \frac{2}{9} \theta^{\rho\sigma}$) and assuming the ε meson an $SU(3)$ singlet we get, after use of eq. (7),

$$(15) \quad \langle \varepsilon(P) | \theta^{\sigma\rho}(0) | 0 \rangle = \frac{2i}{3} f_\varepsilon (m_\varepsilon^2 g^{\rho\sigma} - P^\rho P^\sigma).$$

A factor of 3 has been included to account for colour. By inserting eqs. (11), (13) and (15) into (1) we obtain in the large Q^2 limit⁽⁶⁾:

$$(16) \quad T_{\mu\nu}(q_1, q_2) \rightarrow \frac{1}{\pi} \frac{2}{3} f_\varepsilon s_{\mu\rho\nu\sigma} \int d^4x e^{iQx} \frac{x_\rho}{(x^2 - i\varepsilon)^2} \cdot \\ \cdot \left[m_\varepsilon^2 x_\sigma - (x \cdot P) P_\sigma \right] = \frac{8i}{3} \frac{1}{Q^4} f_\varepsilon (A_{\mu\nu} + m_\varepsilon^2 A'_{\mu\nu}),$$

after addition of suitable non leading terms. By comparing with (2) we finally obtain:

$$(17) \quad F_\varepsilon(q_1^2, q_2^2) = \frac{1}{m_\varepsilon} F'_\varepsilon(q_1^2, q_2^2) \rightarrow \frac{8}{3} f_\varepsilon \frac{1}{Q^4}.$$

Together with eq. (6) this gives:

$$(18) \quad \frac{F'_\varepsilon(q_1^2, q_2^2)}{F'_\varepsilon(0, 0)} \rightarrow \frac{m_\rho^4}{Q^4} \left(\frac{f_\varepsilon}{f_\pi} \right)^2 \left(\frac{m_\varepsilon}{m_\rho} \right)^2,$$

where we have used $R = 8\pi^2/f_\rho^2$ ⁽¹⁰⁾ and the KSFR relation⁽¹¹⁾ $2f_\rho^2 f_\pi^2 = m_\sigma^2$. We will assume in the following $F'_\sigma(q_1^2, q_2^2) = F'_\sigma(q_1^2, q_2^2)/m_\sigma^2$ for all q .

Let us consider now a vector meson dominated vertex, as in ref. (1). We have:

$$(19) \quad F'_\sigma(q_1^2, q_2^2) = k B \left[-a(q_1^2) + 1, \gamma \right] B \left[-a(q_2^2) + 1, \gamma \right] \cdot \\ \cdot {}_3F_2 \left[-a(q_1^2) + 1, -a(q_2^2) + 1, 2\gamma - \beta; \gamma - a(q_1^2) + 1, \gamma - a(q_2^2) + 1; 1 \right],$$

6.

where γ and β are two parameters which control the large q^2 behaviours, and $\alpha(q^2)$ are the vector meson Regge trajectories $\left[\alpha(0) = 1/2, \alpha' = 1/2m_\rho^2 \right]$. Using the limiting procedure of ref. (1), namely $q_1^2 \rightarrow -\infty$ with q_2^2 fixed, and next $q_2^2 \rightarrow -\infty$, we have:

$$(20) \quad F'_\sigma(q_1^2, q_2^2) \xrightarrow[\substack{q_{1,2}^2 \rightarrow -\infty \\ q_1^2/q_2^2 \rightarrow 1}]{\quad} \begin{cases} k \Gamma(\gamma) \Gamma(\beta - \gamma) [-\alpha' q_1^2]^{-\beta}; & \beta > \gamma \\ k \frac{\Gamma(\gamma) \Gamma(\beta) \Gamma(\gamma - \beta)}{\Gamma(2\gamma - \beta)} [-\alpha' q_1^2]^{-\beta}; & \beta < \gamma. \end{cases}$$

From (17) we have $\beta = 2$. Furthermore the ratio (18) suggest the value $\gamma = 1/2$, for which we have $k = (4/3\pi) f_\varepsilon (m_\varepsilon^2/m_\rho^4)$ and

$$(21) \quad \frac{F'_\varepsilon(q_1^2, q_2^2)}{F'_\varepsilon(0, 0)} \rightarrow \frac{8}{3\pi} \frac{m_\rho^4}{Q^4}.$$

Comparison with eqs. (18) then leads to $f_\varepsilon \simeq 100$ MeV. For $\gamma = 3/2$ and $\gamma = 5/2$ we would have got $f_\varepsilon \simeq 150$ MeV and $f_\varepsilon \simeq 170$ MeV respectively. For the decay $\varepsilon \rightarrow \gamma\gamma$ we obtain now:

$$(22) \quad \Gamma(\varepsilon \rightarrow \gamma\gamma) \simeq 6 \text{ keV},$$

in more comfortable agreement with other theoretical estimates⁽⁵⁾ and with, albeit weak, experimental indication⁽¹²⁾ $\Gamma(\varepsilon \rightarrow \gamma\gamma) \simeq 10$ keV. Furthermore we can estimate from (19) the decay rate $\rho'' \rightarrow \rho\varepsilon$. A standard calculation obtains $\Gamma(\rho'' \rightarrow \rho\varepsilon) \sim 100$ MeV, which is of the right order of magnitude. This result however is not much reliable for the very crude narrow width approximations used and has to be taken only as an order of magnitude estimate.

Let us consider now the decays involving the $\chi(3.41)$ meson⁽³⁾, which we assume is a pure $c\bar{c}$ state. The calculation proceeds exactly on the same lines of ref. (1) for the $\eta_c(2.8)$ meson. By dominating the

e. m. currents with vector mesons of the ψ family $\left[a_{\psi}(0) \approx -3/2, (1/a_{\psi}^{\dagger}) \approx 4 \text{ GeV}^2 \right]$, and taking out only the $c\bar{c}$ piece of $\theta_{2}^{\sigma\rho}$ in eq. (14) we find instead of (17) :

$$(23) \quad F_{\chi}^{\dagger}(q_1^2, q_2^2) \rightarrow \frac{16}{9} \frac{m_{\chi}^2 f_{\chi}}{Q^4},$$

and similarly from eq. (19), with the same values of β and γ ,

$$(24) \quad F_{\chi}^{\dagger}(0, 0) = k \frac{11}{256} \pi^2,$$

with:

$$(25) \quad k = \frac{32}{9\pi} a_{\psi}^{\dagger 2} m_{\lambda}^2 f_{\chi}.$$

The coupling constants $g_{\chi\gamma\nu}$ for the decays $\psi^{\dagger} \rightarrow \chi\gamma$ and $\chi \rightarrow \psi\gamma$ are given by:

$$(26) \quad g_{\chi\gamma\nu} = \frac{1}{4} \frac{f_{\nu}}{m_{\nu}^2} \left\{ \frac{m_{\chi}^2 - m_{\nu}^2}{m_{\chi}^2} + 2 \right\} \left\{ \lim_{q^2 \rightarrow m_{\nu}^2} (m_{\nu}^2 - q^2) F_{\chi}^{\dagger}(q^2 \approx m_{\nu}^2, 0) \right\}.$$

Then a straightforward calculation leads to :

$$(27) \quad g_{\psi^{\dagger}\chi\gamma} = -\frac{\pi}{16} \frac{k}{a_{\psi}^{\dagger} m_{\psi^{\dagger}}^2} f_{\psi^{\dagger}} \left\{ 1 + \frac{m_{\chi}^2 - m_{\psi^{\dagger}}^2}{2m_{\chi}^2} \right\},$$

$$g_{\psi\chi\gamma} = \frac{3\pi}{16} \frac{k}{a_{\psi}^{\dagger} m_{\psi}^2} f_{\psi} \left\{ 1 + \frac{m_{\chi}^2 - m_{\psi}^2}{2m_{\chi}^2} \right\}.$$

The radiative decay widths are finally given by :

$$(28) \quad \Gamma(\chi \rightarrow \gamma\gamma) = \frac{e^4}{16\pi} m_{\chi}^3 g_{\chi\gamma\gamma}^2 \approx 15.2 r^2 \text{ keV},$$

$$\Gamma(\psi^{\dagger} \rightarrow \chi\gamma) = \frac{e^2 (m_{\psi^{\dagger}}^2 - m_{\chi}^2)^3}{24\pi m_{\psi^{\dagger}}^3} g_{\psi^{\dagger}\chi\gamma}^2 \approx 5.63 r^2 \text{ keV},$$

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$$\Gamma(\chi \rightarrow \psi \gamma) = \frac{e^2 (m_\chi^2 - m_\psi^2)^3}{8\pi m_\chi^3} g_{\psi\chi\gamma}^2 \simeq 137 r^2 \text{ keV},$$

where r is defined as $f_\chi = r f_{\eta_c}$, with $f_{\eta_c} = m_\psi f_\psi / \sqrt{2} \simeq 183 \text{ MeV}^{(1)}$. In the limit of chiral symmetry $r = 1$. A better estimate for r , taking into account the chiral symmetry breaking, can be obtained as follows.

The hadronic energy density θ_{00} is usually decomposed as⁽¹³⁾:

$$(29) \quad \theta_{00} = \bar{\theta}_{00} + u + \delta,$$

with

$$u = u'_0 + c_8 u_8 + c_{16} u_{16},$$

where $\bar{\theta}_{00}$ is both scale and $SU(4) \times SU(4)$ invariant, δ is chiral invariant but breaks scale invariance and u_i violate both chiral and scale invariance and together with v_i are members of $(4, 4^*) + (4^*, 4)$ and satisfy the algebra:

$$(30) \quad [Q_i^5, u_j] = i d_{ijk} v_k, \quad [Q_i^5, v_j] = -i d_{ijk} u_k,$$

being Q_i^5 the axial charges. In the quark model $u_i = \bar{q} \lambda_i q$ and $v_i = i \bar{q} \lambda_i \gamma_5 q$. We have introduced $\lambda'_0 = \frac{1}{2}(\sqrt{3} \lambda_0 + \lambda_{15})$ and $\lambda_{16} = \frac{1}{2}(\lambda_0 - \sqrt{3} \lambda_{15})$.

$$(31) \quad \Delta_\theta(q) = -i \int d^4x e^{-iqx} \langle 0 | T [\theta_\mu^\mu(x) \theta_\nu^\nu(0)] | 0 \rangle,$$

simple application of Ward identity, together with eqs. (30) and the relation:

$$(32) \quad \partial_\mu D_\mu = \theta_\mu^\mu = (4 - d_\delta) \delta + (4 - d_u) u,$$

leads to:

$$(33) \quad \Delta_{\theta}^{\overline{cc}}(0) = \frac{x}{2} c_{16} \langle u_{16} \rangle_0 ,$$

with $x = (d_u - 4)(d_\delta - d_u)$, d_u being the dimension of the operator u . Similarly, taking the divergences of the axial-vector currents :

$$(34) \quad \partial^\mu A_\mu^i(x) = \text{SU(3) - part} + c_{16} d_{16ij} v_j(x) ,$$

together with:

$$(35) \quad \Delta^i(q) = -i \int d^4x e^{-iqx} \langle 0 | T \left[\partial^\mu A_\mu^i(x) \partial^\nu A_\nu^i(0) \right] | 0 \rangle ,$$

one obtains :

$$(36) \quad \Delta^{\overline{cc}}(0) = \frac{1}{2} c_{16} \langle u_{16} \rangle_0 .$$

From eqs. (33) and (36), and the assumption of simple χ and η_c dominance in eqs. (31) and (35) respectively we finally get :

$$(37) \quad f_\chi = \sqrt{x} \frac{m_{\eta_c}}{m_\chi} f_{\eta_c} .$$

Thus r depends, as expected, on the dimension of the chiral symmetry breaking term. In the quark model $d_u = 3$ and with the plausible assumption of a c -number δ term^(9, 14) ($d_\delta = 0$) we get $x = 3$, and therefore $r = \sqrt{3}(m_{\eta_c}/m_\chi) \approx 1.42$.

Substituting this value of r in eqs. (28) we finally obtain:

$$(38a) \quad \Gamma(\chi \rightarrow \gamma\gamma) \approx 31 \text{ keV} ,$$

$$(38b) \quad \Gamma(\psi' \rightarrow \gamma\chi) \approx 11 \text{ keV} ,$$

$$(38c) \quad \Gamma(\chi \rightarrow \psi\gamma) \approx 280 \text{ keV} .$$

Let us compare these results with previous theoretical estimates as well as experiments. This is done in detail in Table I. As one can see, the theoretical situation is rather contradictory. Compared with

TABLE I

	Eichten et al. ⁽²⁾	Chaichian and K�ogerler ⁽¹⁵⁾	Barbieri et al. ⁽¹⁶⁾	This work	Experimental data
$\Gamma(\psi' \rightarrow \chi \gamma)$ (keV)	36	1.3	---	11	15 ± 5 ⁽¹⁷⁾
$\Gamma(\chi \rightarrow \psi \gamma)$ (keV)	90	39.3	---	280	$\text{Br}(\psi' \rightarrow \chi \gamma) \cdot \text{Br}(\chi \rightarrow \psi \gamma) \approx 0.008$ ^(17, 18)
$\Gamma(\chi \rightarrow \gamma \gamma)$ (keV)	--	---	3.5	31	---

the conventional non-relativistic calculations by Eichten et al.⁽²⁾, our estimate of $\Gamma(\psi' \rightarrow \chi \gamma)$ is roughly a factor of three smaller, while that of $\Gamma(\chi \rightarrow \psi \gamma)$ is on the contrary bigger by the same factor. The scheme proposed by Chaichian and K ogerler⁽¹⁵⁾, where the photon is dominated by vector mesons and the hadronic couplings are calculated through the overlaps of their wave functions, gives on the other hand both radiative widths considerably reduced (by about a factor of 7) with respect to ours. The relative ratio however roughly coincides. Furthermore our two-photon decay width is an order of magnitude larger than that obtained by Barbieri et al.⁽¹⁶⁾ in the framework of gauge theory of strong interactions.

On the other hand our value of $\Gamma(\psi' \rightarrow \chi \gamma)$ is in good agreement with experiments. As far as the $\Gamma(\chi \rightarrow \psi \gamma)$ decay width is concerned, in absence of any experimental information on $\Gamma(\chi \rightarrow \text{all})$, one is forced to introduce a theoretical input. Using the asymptotic freedom estimate⁽¹⁶⁾ $\Gamma(\chi \rightarrow \text{all}) \approx 2.4 \text{ MeV}$ the experimental indications^(17, 18) lead to $\Gamma(\chi \rightarrow \psi \gamma) \approx 300 \text{ keV}$, in good agreement with our value (eq. 38c).

To summarize we have applied to scalar mesons a recently proposed treatment for radiative decays of mesons. Low energy theorems and the asymptotics of the quark current algebra are used to constrain our vector mesons dominated vertices. Our results are in good agreement with experiments.

We are grateful to G. Parisi for discussions, and to Y. Srivastava for a critical reading of the manuscript.

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