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G. Matone and D. Prosperi: NUCLEON POLARIZABILITIES AND DEEP INELASTIC ELECTRON SCATTERING. -

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1. - INTRODUCTION. -

As it is well known, the interaction of photons with a system of charges may be described by a small amount of structure parameters (charge, magnetic moment and polarizabilities), provided that the wavelength of the photons suitable exceeds the dimension of the system and the photon frequency is suitably less than its resonance frequencies(1).

It has been shown(1, 4) that, in the low-energy limit, the spin-averaged nucleon Compton amplitude $(\varepsilon_f^\mu \hat{T}_{\mu\nu} \varepsilon_i^\nu)$ consists of two terms, the frequency independent Thomson amplitude, $-Z^2/M$, and the Rayleigh amplitude, quadratic in the photon frequency. In the laboratory frame we can write(5):

$$(\varepsilon_f^\mu \hat{T}_{\mu\nu} \varepsilon_i^\nu) = -(\vec{\varepsilon}_i \cdot \vec{\varepsilon}_f)\frac{Z^2}{M} + \omega_i \omega_f (\vec{\varepsilon}_i \cdot \vec{\varepsilon}_f)\left[\frac{1}{4M^3}(\mu^2 - Z^2) + G_1\right] + \tag{1}$$
$$+ (\vec{\varepsilon}_i \times \vec{k}_i)\cdot(\vec{\varepsilon}_f \times \vec{k}_f)\left[\frac{1}{4M^3}(Z^2 - \mu^2 \cos\theta) + G_2\right] ,$$

where $\underset{\sim}{k}_{i(f)} = (\omega_{i(f)}, \vec{k}_{i(f)})$ are the four-momenta of the incoming and outgoing photons, $\vec{\varepsilon}_{i(f)}$ are the corresponding polarization vectors, μ and M are the nucleon magnetic moment and its mass respectively. Finally, G_1 and G_2 are two structure constants proportional to the electric (α) and magnetic (β) polarizabilities through the relations $\alpha = (e^2/4\pi) G_1$, $\beta = (e^2/4\pi) G_2$(*). They are e-

(*) - We intend e to be the rationalized electron charge, $\hat{e} = e/\sqrt{4\pi}$, the non rationalized charge.

sentially determined by the virtual photoexcitations of the nucleon structure and can be measured by studying the angular distribution of the scattered photons.

The knowledge of these parameters, of undoubted interest for hadron physics, could also have important implications in nuclear and astrophysical studies (see for ex. refs. (3) and (6)).

The first direct experimental determination of the nucleon polarizabilities was made about fifteen years ago by Goldanski et al.[7] who investigated Compton scattering on protons (p) at an average energy of 56 MeV. Aconsiderably more accurate experiment has been carried out more recently by Baranov et al.[8] in the energy range (80 - 110) MeV. A common results of these experiments is that $\alpha_p > \beta_p$ and this has to be considered as an extremely surprising conclusion[3]. In fact, one knows from experiments that all photoabsorption processes on nucleons are dominated by the Δ(1236) resonance and this excitation is of magnetic (M1) nature. So, one could expect the nucleon to be essentially a good paramagnetic object characterized by $\beta > \alpha$.

To understand this apparently strange situation, one is forced to go in two different directions :

1) To critically analyze the adequacy of the available experiments. In particular, one should quantitatively estimate the overall amount of higher order contributions to the scattering amplitude of eq. (1) coming from the presence of the first barion resonance.
2) To search for different sources of information on the nucleon polarizabilities ; an interesting possibility of this kind is offered by the existence of sum rules relating α and β to the cross sections for real and virtual photon processes.

Aim of the present work is to undertake a systematic analysis of sum rules involving nucleon polarizabilities. Sect. 2 is devoted to list some general properties of the spin-averaged nucleon Compton amplitude which are relevant in the derivation of such rules. In Sect. 3 two previously known sum rules[9,10] are briefly derived starting from forward dispersion relationships for off-mass shell photons (see eqs. (23)). Moreover, a third sum rule, different in some aspects from the analogous one of ref. (11), is derived starting from backward dispersion relationships for on-shell photons in Sect. 4 (see eq. (30)). Finally, Sect. 5 is devoted to present a numerical estimate of α_p (β_p) by means of the above mentioned forward sum rules : in particular a numerical analysis of existing data on the longitudinal cross section σ_L for near on-shell photons is presented.

As a final remark let us note that the above estimate suggests $\alpha_p \gtrsim \beta_p$ and therefore supports the conclusion that the annihilation channel contribution to the backward sum rule cannot be neglected. As a consequence, also this last sum rule could be in agreement with the experiments. The same conclusions qualitatively hold for neutron as well.

2. - SPIN AVERAGED TWO-PHOTON AMPLITUDE. -

Let us label by i and f the initial and final states in the nucleon Compton scattering process. In the laboratory frame, the usual definition of T-matrix in terms of the scattering matrix S takes the form:

$$S_{fi} = \delta_{fi} + i(2\pi)^4 e^2 \left\{ \frac{M^2}{4\omega_i \omega_f M E_f} \right\}^{1/2} (\bar{u}_f T_{fi} u_i)\, \delta(\underset{\sim}{p}_f + \underset{\sim}{k}_f - \underset{\sim}{p}_i - \underset{\sim}{k}_i) \ , \tag{2}$$

where $\underset{\sim}{k}_{i(f)} \equiv (\omega_{i(f)}, \vec{k}_{i(f)})$ are the incident and outgoing photon four-momenta, and $\underset{\sim}{p}_{i(f)} \equiv (E_{i(f)}, \vec{p}_{i(f)})$ are the corresponding nucleon four-momenta. Moreover u_i and $\bar{u}_f$ are Dirac spinors ($\bar{u}u = 1$). We shall also introduce the following vectors(*):

$$\underset{\sim}{K} = \underset{\sim}{k}_i + \underset{\sim}{k}_f \ , \qquad \underset{\sim}{q} = \underset{\sim}{k}_i - \underset{\sim}{k}_f \ , \qquad \underset{\sim}{P} = \frac{1}{2}(\underset{\sim}{p}_i + \underset{\sim}{p}_f) \ . \tag{3}$$

The most general amplitude for off-mass shell photons can be separated in two parts as follows:

$$T_{fi} = \varepsilon_f^{\mu} T_{\mu\nu} \varepsilon_i^{\nu} = \varepsilon_f^{\mu} (T^B_{\mu\nu} + T^{NB}_{\mu\nu})\, \varepsilon_i^{\nu} \ , \tag{4}$$

$T^B_{\mu\nu}$ and $T^{NB}_{\mu\nu}$ being, respectively, the Born contribution and the continuum (or non-Born) contribution which depends on the details of the nucleon structure. As previously mentioned, in the quadratic approximation, $T^{NB}_{\mu\nu}$ depends only on two structure constants, namely the above mentioned polarizabilities.

Moreover, $T_{\mu\nu}$ must satisfy the gauge conditions:

$$k_f^{\lambda} T_{\lambda\nu} = T_{\mu\lambda} k_i^{\lambda} = 0 \ . \tag{5}$$

Other restrictions on $T_{\mu\nu}$ are due to the discrete symmetries (C, P and T invariances) and photon crossing. For a detailed discussion of this point, see ref. (12).

In the present work the amplitude $T^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$ will be expanded in terms of a complete set of independent tensors $T_i^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$. For photons equally off-mass shell ($\underset{\sim}{k}_i^2 = \underset{\sim}{k}_f^2 = -Q^2$) we can write:

$$T^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P}) = - \sum_{i}^{N} A_i(Q^2, \underset{\sim}{K}^2 - \underset{\sim}{q}^2, \underset{\sim}{P}\cdot\underset{\sim}{K})\, T_i^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P}) \ . \tag{6}$$

(*) - Our metric is defined by $\underset{\sim}{k} \equiv (k_o, \vec{k})$, $\underset{\sim}{k}^2 = k_o^2 - \vec{k}^2$. The γ_μ matrices satisfy the relation $(\gamma_\mu\gamma_\nu + \gamma_\nu\gamma_\mu) = 2 g_{\mu\nu}$, where $g_{oo} = +1$, $g_{ii} = -1$ (i = 1, 2, 3).

In the past many efforts have been devoted to find general methods allowing to construct amplitudes for two-photon processes which are free from kinematical zeros, constraints and singularities (see ref. (12, 13), and refs. quoted here). In this work we shall use results and definitions of ref. (12). The basis tensors $T_i^{\mu\nu}$ (N = 18) are all Lorentz-covariant and gauge-invariant, pole-free, and even or odd under crossing and discrete symmetries. As a consequence, the amplitudes $A_i(Q^2, \underset{\sim}{K}^2-\underset{\sim}{q}^2, \underset{\sim}{P}\cdot\underset{\sim}{K})$, besides being free of zeros and constraints, are not connected to each other by links due to the discrete symmetries and crossing.

Let us now fix our attention on the spin-averaged part of $T^{\mu\nu}$, namely $\hat{T}^{\mu\nu}$. More exactly, let us define spin-averaged tensors $\hat{T}_i^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$ through the relation:

$$\left\{\bar{u}_f T^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P}) u_i\right\}_{Av.} = \chi_f^+ \hat{T}^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})\chi_i =$$

$$= -\sum_i{}_1^{18} A_i(Q^2, \underset{\sim}{K}^2-\underset{\sim}{q}^2, \underset{\sim}{P}\cdot\underset{\sim}{K})\left[\chi_f^+\hat{T}_i^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})\chi_i\right], \qquad (9)$$

where χ_i and χ_f^+ are Pauli spinors $(\chi^+\chi = 1)$. A short description of the procedure followed to calculate the spin-averaged tensors $\hat{T}_i^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$ and their explicit form can be found in the Appendix.

Finally, $\hat{T}^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$ can be put in a particularly simple form by exploiting the fact that, for equally off-mass shell photons, time reversal invariance gives $A_i = 0$ for i = 5, 9, 11, 13, 14, 15, 20 (see ref. (12)). Moreover, $\hat{T}_i^{\mu\nu} = 0$ for i = 7, 8, 9, 11, 18. As a consequence (in the laboratory frame) $\hat{T}^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})$ can be expressed in terms of only seven elementary tensors such as:

$$\tau_1^{\mu\nu} = g^{\mu\nu}, \quad \tau_2^{\mu\nu} = K^\mu K^\nu, \quad \tau_3^{\mu\nu} = q^\mu q^\nu, \quad \tau_4^{\mu\nu} = P^\mu P^\nu,$$
$$\tau_5^{\mu\nu} = (K^\mu q^\nu - K^\nu q^\mu), \quad \tau_6^{\mu\nu} = (P^\nu K^\mu + P^\mu K^\nu), \quad \tau_7^{\mu\nu} = (P^\nu q^\mu - P^\mu q^\nu). \qquad (10)$$

In the following we shall put $\left\{\hat{T}^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})\right\}_{lab}$ in the form:

$$\left\{\hat{T}^{\mu\nu}(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})\right\}_{lab} = \sum_j{}_1^7 B_j(Q^2, \underset{\sim}{K}^2-\underset{\sim}{q}^2, MK_o)\left\{\tau_j(\underset{\sim}{K}, \underset{\sim}{q}, \underset{\sim}{P})\right\}_{lab}, \qquad (11)$$

where the amplitudes B_j (j = 1, ..., 7) are related to the amplitudes A_i (i = 1, ..., 18) of ref. (12) by the relationships:

$$B_1 = -\frac{1}{4}(\underset{\sim}{K}^2-\underset{\sim}{q}^2)A_1 - Q^4A_2 - \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})^2A_3 + Q^2(\underset{\sim}{P}\cdot\underset{\sim}{K})A_4 + 2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'A_{17},$$
$$B_2 = \frac{1}{4}A_1 - \frac{1}{16}(\underset{\sim}{K}^2-\underset{\sim}{q}^2+8Q^2)A_2 + \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})A_4 - \frac{1}{8}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'(A_6 + 2A_{12}), \qquad (12)$$

$$B_3 = -\frac{1}{4}A_1 + \frac{1}{16}(\underset{\sim}{K}^2 - \underset{\sim}{q}^2 - 8Q^2)A_2 + \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})A_4 + \frac{1}{8}(\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]A_6 -$$

$$-\frac{1}{2M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})^2\mathcal{F}A_{10} - \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{Q^2}{4M^2}\mathcal{F}\right]A_{12} + \frac{1}{4M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\left[\underset{\sim}{K}^2A_{16} + 2A_{17}\right],$$

$$B_4 = -\frac{1}{4}(\underset{\sim}{K}^2 - q^2)\left[A_3 - 8\mathcal{F}'A_{10}\right],$$

$$B_5 = -\frac{1}{4}A_1 - \frac{1}{16}(\underset{\sim}{K}^2 - \underset{\sim}{q}^2)A_2 - \frac{1}{8}(\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{Q^2}{2M^2}\mathcal{F}\right]A_6 - \frac{1}{8M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\left[2(\underset{\sim}{P}\cdot\underset{\sim}{K})A_{10} - \right.$$

$$\left. - Q^2A_{12} - \underset{\sim}{q}^2A_{16} - 2A_{17}\right], \qquad (12)$$

$$B_6 = \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})A_3 + \frac{1}{4}\underset{\sim}{q}^2A_4 - \frac{1}{8}\mathcal{F}'\left[2Q^2A_6 + 8(\underset{\sim}{P}\cdot\underset{\sim}{K})A_{10} - (\underset{\sim}{K}^2 - \underset{\sim}{q}^2)A_{12} - 4\underset{\sim}{q}^2A_{16} + 8A_{17}\right],$$

$$B_7 = \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})A_3 + \frac{1}{4}\underset{\sim}{K}^2A_4 + \frac{1}{4}Q^2\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]A_6 - (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' + (\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\frac{1}{4M^2}\mathcal{F}\right]A_{10} -$$

$$-\frac{1}{8}(\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]A_{12} + \frac{1}{2}\underset{\sim}{K}^2\mathcal{F}'A_{16} - \left[\mathcal{F}' - (\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\frac{1}{4M^2}\mathcal{F}\right]A_{17},$$

and $\mathcal{F} = \left[\frac{2M}{M + E_f}\right]^{1/2} \sim (1 - q^2/8M^2)$ and $\mathcal{F}' = (\mathcal{F}^{-1} - \mathcal{F}) = (P_o/M - 1)\mathcal{F} \sim q^2/4M^2$.

Eq. (11) represents a useful starting point to study different physical processes. The limit $Q^2 \to 0$ must give the spin-averaged Compton amplitude. On the other hand, when $\underset{\sim}{q} \to 0$, it must be related to the tensor $W^{\mu\nu}$ associated to the hadronic vertex in the inelastic electron scattering[(14)]. It offers a well defined connection between the amplitudes describing the two processes.

Before proceeding, let us report here the explicit form of the Born contribution to the amplitudes A_i (Fig. 1). By standard methods we have found that the only non vanishing amplitudes are:

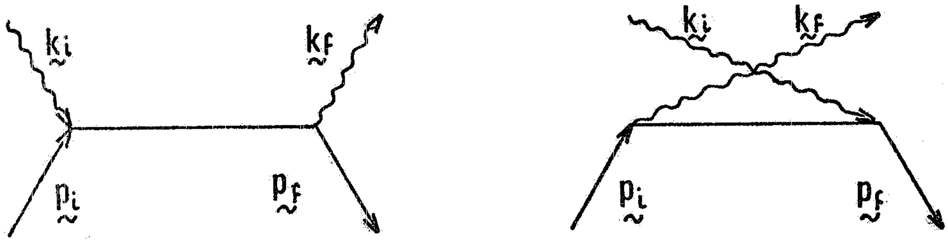

FIG. 1 - Born diagrams defining the nucleon pole.

$$A_1^B = -\frac{\lambda}{2M} F_2 (2F_1 + \lambda F_2) S_+ + \frac{\lambda^2}{8M^3} (\underset{\sim}{P}\cdot\underset{\sim}{K})(F_2)^2 S_- \; ,$$

$$A_3^B = \frac{\lambda^2}{2M^3} (F_2)^2 S_+ + \frac{2}{M(\underset{\sim}{P}\cdot\underset{\sim}{K})} (F_1)^2 S_- \; ,$$

$$A_6^B = \frac{\lambda^2}{4M^3} (F_2)^2 S_- \; , \tag{13}$$

$$A_{10}^B = \frac{\lambda^2}{16M^3} (F_2)^2 S_+ + \frac{1}{4M(\underset{\sim}{P}\cdot\underset{\sim}{K})} F_1 (F_1 + \lambda F_2) S_- \; ,$$

$$A_{16}^B = -\frac{\lambda^2}{16M^3} (F_2)^2 S_- \; ,$$

$$A_{17}^B = \frac{1}{4M} \left((F_1)^2 + 2\lambda F_1 F_2 + \lambda^2 (F_2)^2 \right) S_- \; ,$$

where λ is the anomalous nucleon magnetic moment, the functions $F_i \equiv F_i(Q^2)$ $(i = 1, 2)$ are the usual Pauli and Dirac nucleon form factors $(F_1(0) = Z,\; F_2(0) = 1)$; moreover $S_\pm = \left[(s - M^2)^{-1} \pm (u - M^2)^{-1} \right]$, where s and u are the usual Mandelstam variables.

We can finally identify the amplitudes contributing to the nucleon polarizabilities. For real transverse photons $(Q^2 = 0;\; \varepsilon_o = 0;\; \underset{\rightarrow}{\varepsilon}\cdot\underset{\rightarrow}{k} = 0)$ the amplitudes B_6 and B_7 do not contribute to $(\varepsilon_f^\mu \hat{T}_{\mu\nu} \varepsilon_i^\nu)$ and, in the low-energy limit, we have :

$$\mathcal{R}e \left\{ \varepsilon_f^\mu \hat{T}^{\mu\nu} \varepsilon_i^\nu \right\}_{Q^2=0} = \tag{14}$$

$$= \lim_{Q^2 \to 0} \left\{ -(\underset{\rightarrow}{\varepsilon}_i \cdot \underset{\rightarrow}{\varepsilon}_f) B_1 + \omega_i \omega_f \left[(\underset{\rightarrow}{\varepsilon}_i \cdot \underset{\rightarrow}{\varepsilon}_f) \cos\theta - (\underset{\rightarrow}{v}_i \cdot \underset{\rightarrow}{v}_f) \right] (B_2 - B_3 - \tfrac{1}{4} B_4 - 2B_5) + \theta(\omega_i^4) \right\}$$

where $\underset{\rightarrow}{v} = (\underset{\rightarrow}{\varepsilon} \wedge \underset{\rightarrow}{k})/\omega$ and $B_i = B_i^B + B_i^{NB}$. Moreover the Born contributions $B_i^B(Q^2, \underset{\sim}{K}^2 - \underset{\sim}{q}^2, \underset{\sim}{P}\cdot\underset{\sim}{K})$ can be immediately calculated by eqs. (12) and (13). By comparing eqs. (1) and (14) we find(*)

$$\alpha = -\hat{e}^2 \lim_{Q^2,\, \omega_i \to 0} (A_1^{NB} + M^2 A_3^{NB}) \; , \tag{15}$$

$$\beta = \hat{e}^2 \lim_{Q^2,\, \omega_i \to 0} (A_1^{NB}) \; .$$

(*) - Our convention is that, in expressions like $\lim_{a,b \to 0} F(a,b)$, the first limit to be calculated is for $a \to 0$.

3. - FORWARD SUM RULES. -

Forward sum rules can be obtained by connecting Compton scattering to inelastic electron scattering in the limit of real photons $(Q^2 \to 0)$. First of all we must identify convenient linear combinations of the amplitudes A_i whose non-Born contributions tend to expressions (15).

It is immediate to prove that, when $\underset{\sim}{q} \to 0$, eq. (11) leads to:

$$\lim_{\underset{\sim}{q} \to 0} \hat{T}^{\mu\nu} = (g^{\mu\nu} + \frac{k^\mu k^\nu}{Q^2}) \lim_{\underset{\sim}{q} \to 0} \left[Q^2(A_1 - Q^2 A_2) - \nu M(\nu M A_3 - 2Q^2 A_4) \right] + \\ + (p^\mu + \frac{\underset{\sim}{p} \cdot \underset{\sim}{k}}{Q^2} k^\mu)(p^\nu + \frac{\underset{\sim}{p} \cdot \underset{\sim}{k}}{Q^2} k^\nu) \lim_{\underset{\sim}{q} \to 0} \left[Q^2 A_3 \right] \quad , \tag{16}$$

where $\underset{\sim}{k}_i, \underset{\sim}{k}_f = \underset{\sim}{k} \equiv (\nu, \vec{k})$ and $\underset{\sim}{p}_i = \underset{\sim}{p}_f = \underset{\sim}{p}$. In the following we shall also adopt the notation $\lim_{q \to 0} A_i(Q^2, \underset{\sim}{K}^2 - \underset{\sim}{q}^2, \underset{\sim}{P} \cdot \underset{\sim}{K}) = A_i(Q^2, \nu)$. On the other hand the hadronic tensor $W^{\mu\nu}$ is usually written in the gauge invariant form(14, 15):

$$W^{\mu\nu} = -(g^{\mu\nu} + \frac{k^\mu k^\nu}{Q^2}) W_1(Q^2, \nu) + \frac{1}{M^2}(p^\mu + \frac{(\underset{\sim}{p} \cdot \underset{\sim}{k})}{Q^2} k^\mu)(p^\nu + \frac{(\underset{\sim}{p} \cdot \underset{\sim}{k})}{Q^2} k^\nu) W_2(Q^2, \nu) \quad , \tag{17}$$

where the two nucleon structure functions $W_i(Q^2, \nu)$ are subjected to direct experimental measurement with standard electron scattering experiments. As usual we shall also introduce the longitudinal function structure $W_L(Q^2, \nu)$ defined by:

$$W_L(Q^2, \nu) = W_2(Q^2, \nu)(1 + \frac{\nu^2}{Q^2}) - W_1(Q^2, \nu) \ . \tag{18}$$

By remembering that $W^{\mu\nu} = \frac{1}{\pi} \lim_{\underset{\sim}{q} \to 0} \mathrm{Im}\, \hat{T}^{\mu\nu}$, we are now able to conclude that:

$$\lim_{\underset{\sim}{q} \to 0} \mathrm{Im}(M^2 A_3) = \pi \frac{W_2(Q^2, \nu)}{Q^2} \quad , \qquad \lim_{\underset{\sim}{q} \to 0} \mathrm{Im}(A_L) = \pi \frac{W_L(Q^2, \nu)}{Q^2} \quad , \tag{19}$$

where $A_L = A_1 + M^2 A_3 - Q^2 A_2 + 2M\nu A_4$.

Moreover from eqs. (13) and (15) one obtains:

$$\lim_{\underset{\sim}{q}, Q^2 \to 0} \mathrm{Re}(M^2 A_3) = -\frac{Z^2}{M\nu^2} + \frac{1}{\hat{e}^2}(\alpha + \beta) + O(\nu^2) \ , \\ \lim_{\underset{\sim}{q}, Q^2 \to 0} \mathrm{Re}(A_L) = -\frac{Z^2}{M\nu^2} - \frac{\lambda^2}{4M^3} + \frac{1}{\hat{e}^2}\alpha + O(\nu^2) \ . \tag{20}$$

It should be noted, however, that in the previous expression the limits $\underset{\sim}{q} \to 0$, $Q^2 \to 0$ and $\nu/M \to 0$ cannot be arbitrarily commuted.

Let us now proceed to write the required forward dispersion relations for $A_3(Q^2, \nu)$ and $A_L(Q^2, \nu)$. The number of needed subtractions depends upon their high-energy behaviour which is known only in a model-dependent way. As a matter of fact, Regge models strongly suggest to handle A_3 by an unsubtracted dispersion relation(*), while the situation is less clear as far as A_L is concerned. One can hope that the high-energy dominant terms of $W_1(Q^2, \nu)$ and $W_2(Q^2, \nu)\left[\nu^2/Q^2\right]$ exactly cancell altogether, as in the naive parton model (see for ex. ref. (15) and refs. quoted here).

We shall write forward dispersion relations for the amplitudes $A_i(Q^2, \nu)$ even under crossing in the following non subtracted standard form at fixed Q^2 :

$$\mathrm{Re}\, A_i(Q^2, \nu) = \pi_i(Q^2, \nu) + \frac{2}{\pi} P \int_{\nu_{th}}^{\infty} \frac{\mathrm{Im}\, A_i(Q^2, \nu')}{(\nu'^2 - \nu^2)} \nu' d\nu' , \tag{21}$$

where π_i is the pole contribution and $\nu_{th} = \mu + (\mu^2 + Q^2)/2M$ is the threshold for inelastic excitation.

To calculate the pole contributions π_3 and π_L it is sufficient to know the elastic part $W_i^{el}(Q^2, \nu)$ of the two nucleon structure functions(15). By the pole definition we directly obtain :

$$\pi_3(Q^2, \nu) = \frac{1}{M^3} \left(\frac{1}{Q^2 f^2 - \nu^2} \right) \left(\frac{G_E^2 + f^2 G_M^2}{1 + f^2} \right) ,$$

$$\pi_L(Q^2, \nu) = \frac{1}{M} \left(\frac{1}{Q^2 f^2 - \nu^2} \right) \left(\frac{G_E^2 + f^2 G_M^2}{1 + f^2} \left[1 + f^2\right] - f^2 G_M^2 \right) , \tag{22}$$

where $f^2 = (Q^2/4M^2)$ and $G_E = F_1 - f^2(z + \lambda) F_2$, $G_M = F_1 + (z + \lambda) F_2$ are standard functions of the squared mass Q^2.

From eqs. (19) to (22) we finally obtain :

(a) $$(\alpha + \beta) = 2 \hat{e}^2 \lim_{Q^2 \to 0} \int_{\nu_{th}}^{\infty} \frac{W_2(Q^2, \nu')}{Q^2 \nu'} d\nu' ,$$

(b) $$\alpha = \frac{(\lambda \hat{e})^2}{4M^3} + 2 \hat{e}^2 \lim_{Q^2 \to 0} \int_{\nu_{th}}^{\infty} \frac{W_L(Q^2, \nu')}{Q^2 \nu'} d\nu' . \tag{23}$$

(*) - For real photons, we have $\lim_{\nu \to \infty} A_3 \sim \lim_{\nu \to \infty} \sigma_T/\nu \sim \nu^{-1}$, where σ_T is the total umpolarized cross section.

The validity of these sum rules evidently requires both functions W_2 and W_L to have a non vanishing and finite slope as $Q^2 \to 0$ and to vanish asymptotically as ν^{-n}, being $n > 0$. Let us stress again that the validity of sum rule (23b) must be considered as an indirect test of a good asymptotic behaviour of $W_L(Q^2, \nu)$.

Let us now introduce the usual definitions relating W_1 and W_2 to the transverse (σ_T) and longitudinal (σ_L) cross sections and to the $R(Q^2, \nu)$ function. We have:

$$\begin{aligned} \sigma_T(Q^2, \nu) &= \frac{\mathscr{C}}{F} W_1(Q^2, \nu) \quad , \\ \sigma_L(Q^2, \nu) &= \frac{\mathscr{C}}{F} W_L(Q^2, \nu) \quad , \\ R(Q^2, \nu) &= \sigma_L(Q^2, \nu)/\sigma_T(Q^2, \nu) \quad , \end{aligned} \tag{24}$$

where $\mathscr{C} = 4\pi^2 \hat{e}^2$ and $F = \nu + O(Q^2)$ is the incident photon flux[15]. Since $R(Q^2, \nu)$ vanishes as Q^2 goes to zero, eqs.(23) can be put in the final form[(*)]:

$$\text{(a)} \qquad \alpha + \beta = \lim_{Q^2 \to 0} \frac{1}{2\pi^2} \int_{\nu_{th}}^{\infty} \frac{\sigma_T(Q^2, \nu')}{(\nu')^2} d\nu' = \frac{1}{2\pi^2} \sigma_{-2} \quad ,$$

$$\text{(b)} \qquad \alpha = \frac{(\lambda \hat{e})^2}{4M^3} + \lim_{Q^2 \to 0} \frac{1}{2\pi^2} \int_{\nu_{th}}^{\infty} \sigma_T(Q^2, \nu') \frac{R(Q^2, \nu')}{Q^2} d\nu' \quad . \tag{25}$$

Sum rule (25a) was first explicitely derived by Baldin[1] without recourse to the structure functions $W_i(Q^2, \nu)$. Moreover, the validity of the sum rule (25b) was first hypothized by Bernabeu and Tarrach[10] who used the invariant amplitudes formalism of ref.(13). Its correctness requires $R(Q^2, \nu)$ to asymptotically vanish more rapidly than ν^{-1}. It is presently hard, to arrive to an accurate estimate of α through this way, due to the lack of reliable experimental determinations of $R(Q^2, \nu)$, particularly for small values of Q^2 and in the resonance region. A preliminary attempt will be reported in Sect. 4.

4. - SOME REMARKS ON A BACKWARD SUM RULE. -

Let us go back to eqs.(4), (9), (11), and let us calculate the amplitude $(\varepsilon_f^{\mu} \hat{T}_{\mu\nu} \varepsilon_i^{\nu})$ for real transverse photons in the backward direction. For $\vec{\varepsilon}_i = \vec{\varepsilon}_f$, eq.(14) gives:

(*) - Let us remind that:

$$\lim_{Q^2 \to 0} W_2(Q^2, \nu)/W_1(Q^2, \nu) = \lim_{Q^2 \to 0} \frac{(1 + R(Q^2, \nu))}{(1 + \nu^2/Q^2)} = \frac{Q^2}{\nu^2} \quad .$$

$$\lim_{Q^2 \to 0} \left\{ (\varepsilon_i^{\mu} \hat{T}_{\mu\nu} \varepsilon_i^{\nu}) \right\}_{\theta=\pi} = \left\{ \lim_{Q^2 \to 0} B_1 \right\}_{\theta=\pi} \quad . \tag{26}$$

Moreover, by recourse to eqs. (12), (13) we immediately obtain :

$$\left\{ \lim_{Q^2 \to 0} B_1 \right\}_{\theta=\pi} = -\frac{Z^2}{M} - \frac{Z^2}{2M^3} \nu^2 + \nu^2 \lim_{\nu \to 0} \left[2A_1^{NB} + M^2 A_3^{NB} \right]_{\theta=\pi} + O(\nu^4) \; . \tag{27}$$

where $\lim_{\nu \to 0} \left[2A_1^{NB} + M^2 A_3^{NB} \right] = (\alpha - \beta)/\hat{e}^2$ (see eqs. (15)).

On the other hand we have[11]:

$$\lim_{Q^2 \to 0} \; \mathrm{Im}\,(\varepsilon_i^{\mu} \hat{T}^{\mu\nu} \varepsilon_f^{\nu})_{\theta=\pi} = \frac{1}{e^2} \frac{(s - M^2)}{2M} \tilde{\sigma}(\nu) \quad , \tag{28}$$

where :

$$\tilde{\sigma}(\nu) = \left\{ \sigma^{Av}(\Delta\pi \text{ yes}) - \sigma^{Av}(\Delta\pi \text{ no}) \right\} \quad ,$$

and $\sigma(\Delta\pi$ yes) ($\sigma(\Delta\pi$ no)) is the spin-averaged cross section corresponding to parity flip multipoles (not parity flip).

To connect the real parts of the amplitudes A_i (or B_i) to their absorptive parts through dispersion relations when $t \neq 0$, we cannot use fixed-angle (one-dimension) dispersion relations. This is due to the fact that the dispersion relations for a fixed value of t contain subtraction functions which depend on t. Only for scattering angle $\theta = 0$, these functions can be determined through low-energy theorems, whereas for scattering angles different from $\theta = 0$, the subtraction functions remain unknown. This problem was solved with the help of double dispersion relations ; in this way it has been possible both to determine the subtraction functions and to include in a systematic way all the effects associated with the annihilation channel[16].

By carrying out a subtraction at the point $s_o = u_o = M^2$, $t_o = 0$, we can write for any amplitude $A_i(s,t)$, even under crossing, a dispersion relation of the form[16]:

$$\mathcal{R}e\left[A_i(s,t)\right] = \pi_i^s + \pi_i^u + \pi_i^t + \frac{1}{\pi} P \int \frac{\mathrm{Im}\left[A_i(x,t)\right]}{x - M^2} \left\{ \frac{s - M^2}{x - s} + \frac{u - M^2}{x - u} \right\} dx \; + \\ + \frac{t}{\pi} \int \frac{\Phi_i(y, M^2)}{y(y-t)} dy + A_i(M^2, 0) \quad , \tag{29}$$

where $\pi_i^{s,u,t}$ are the pole contributions in channels s, u and t respectively, $\mathrm{Im}\left[A_i(x,t)\right]$ is the absorptive part of A_i in channel s and, finally, Φ_i is a complicated function defined in terms of the absorptive parts of A_i in the channel t and of its absorptive parts associated to two-pion graphs[16]. The most relevant graphs contributing to Φ_i are shown in Fig. 2.

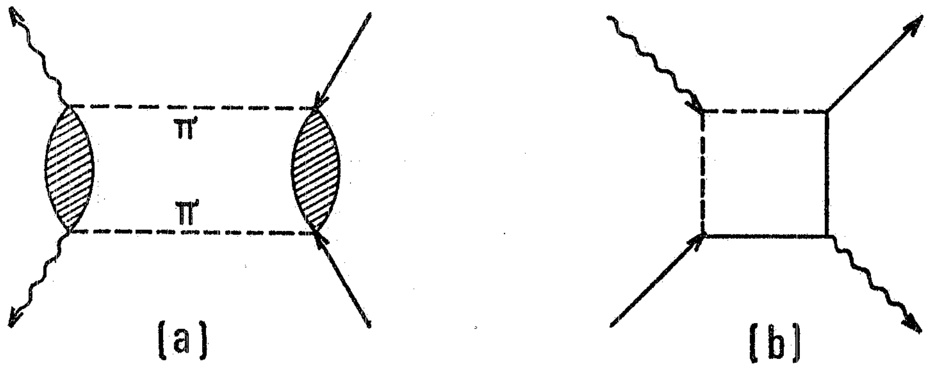

FIG. 2 - Some diagrams whose absorptive parts contribute to the function $\Phi(s, u, t)$ (see ref(16)). More exactly, Φ depends on A_t and A_{su}, where A_t is the absorptive part of the amplitude in the annihilation channel (a), A_{su} is the absorptive part of the amplitude associated with diagrams of the kind (b).

By applying eqs. (26) to (29) to the backward amplitude we obtain, in the low-energy limit, a sum rule of the kind(*):

$$\alpha - \beta = \frac{(Z\hat{e})^2}{2M^3} + \frac{1}{2\pi^2} \int_{\nu_{th}}^{\infty} \frac{\tilde{\sigma}(\nu')}{(\nu')^2} \left(1 + \frac{\nu'}{M}\right) d\nu' - \frac{1}{\pi^2} \int \frac{\Phi(y, M^2)}{y} \, dy \ . \qquad (30)$$

This rule differs in some essential aspects from that of ref. (11); in this work the constant $+(Z\hat{e})^2/2M^3$ and the factor $(1+\nu'/M)$ are replaced by $-\hat{e}^2\lambda(2Z+\lambda)/2M^3$ and by $(1+2\nu/M)^{1/2}$ respectively. Further, in ref. (11) the t-channel contribution was assumed to be negligeable.

It is an interesting question to state if the function Φ is dominated or not by a pole. There is some evidence that the S-wave π-π amplitude exhibits some enhancement just above threshold; however, it was not possible to obtain a firm proof of the existence of a pole (ε or σ meson, $I^{G\pi} = 0^{++}$). Limits $M_\varepsilon \lesssim$ 700 MeV and $\Gamma_\varepsilon \gtrsim$ 600 MeV were fixed for its mass and full width[17]. Another candidate is the f(1260) meson, but its coupling to the γ-γ channel is not well-known and its contribution should be further explored. Nevertheless it could be a useful exercise to estimate the last contribution of eq. (30) by introducing a "fictious pole" of strength G at $t = t_p$. In this case, proceeding as in ref. (18), we can put:

$$- \frac{1}{\pi^2} \int \frac{\Phi(y, M^2)}{y(y - t)} \, dy \sim 4 \frac{G\hat{e}^2}{M t_p} \ . \qquad (31)$$

Rough estimates of both parameters G and t_p will be briefly reported in Sect. 5.

(*) - Both π and η poles do not contribute to the spin-averaged amplitude. Moreover, the contribution of the nucleon pole to B_1 vanishes.

5. - NUMERICAL ESTIMATES. -

Table I illustrates the main characteristics and the results of the available experiments of photon scattering on hydrogen. The values of α_p and β_p quoted for the Goldanski's[7] experi-

TABLE I

Ref.	Energy range (MeV)	Angle range (d)	$\alpha_p \times 10^4$ (fm^3)	$\beta_p \times 10^4$ (fm^3)
(19)	60	70 - 150	-	-
(7)(11)	56	75 - 150	$10. \pm 5.$	$4. \pm 5.$
(8)	81 - 111	90 - 150	10.7 ± 1.1	-0.7 ± 1.6
Present fit	-	-	12.4 ± 0.6	1.8 ± 0.9

ment are those recalculated by Bernabeu et al.[11] taking into account both statistical and systematic errors.

Our analysis has been carried out by imposing the constraint :

$$\alpha_p + \beta_p = (14.2 \pm 0.3)\, 10^{-4} \ \mathrm{fm}^3 , \qquad (32)$$

resulting from sum rule (25a) with the value $\sigma_{-2}/2\pi^2 = (72 \pm 2)\ \mu b/\mathrm{GeV}$ quoted by Damashek and Gilman[9]. It should be noted that Baranov's experiment gives $(\alpha_p + \beta_p) = (10.0 \pm 2.3)\, 10^{-4}\ \mathrm{fm}^3$, namely a value lower than estimate (32) by about two standard deviations ; this small difference could be explained as due to the choice of an energy ($\sim$ 100 MeV) too close to the $\Delta(1236)$ resonance[(x)]. Our fit includes data of refs. (7, 8, 19) ; in addition to the statistical errors, it includes both the systematic errors and the error on the constraint. The errors of α and β are strongly correlated. The resulting fit is compared with experimental points in Fig. 3.

The main purpose of this section is to estimate sum rule (25b). Let us preliminary introduce some general considerations. Consistently with the light cone model we assume that, when $\nu \to \infty$, $\nu R(Q^2, \nu)$ scales as a function of the usual scaling variable $\omega = 2M\nu/Q^2$[20]. For illustrative purposes, let us briefly examine the simple (irrealistic) case :

$$\frac{W_2(Q^2,\nu)}{W_1(Q^2,\nu)} = \frac{Q^2}{\nu^2}\,\frac{1}{(1+\rho\, Q^2/\nu^2)} ,$$

$$R(Q^2,\nu) = (1-\rho)\,\frac{Q^2}{\nu^2}\,\frac{1}{(1+\rho Q^2/\nu^2)} , \qquad (33)$$

(x) - In ref. (6) it was suggested this experiment to be internally inconsistent. Data at our disposal did not allow to reach definite conclusions on this point.

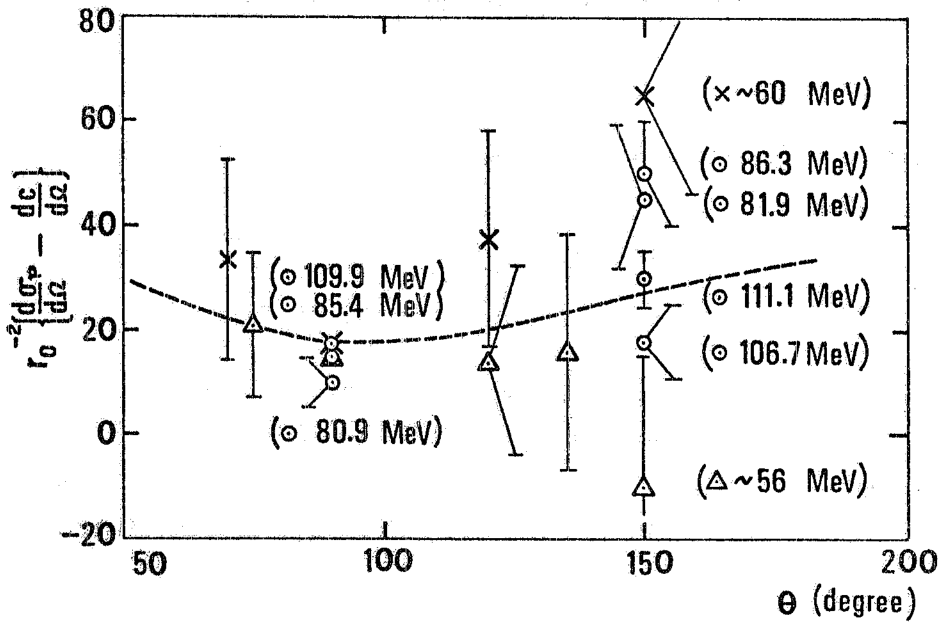

FIG. 3 - Fit of proton Compton cross section $r_o^{-2}\left\{\frac{d\sigma_p}{d\Omega} - \frac{d\sigma}{d\Omega}\right\} = \frac{m^2}{e^2}\left\{\alpha_p(1+\cos^2\theta) + 2\beta_p\cos\theta\right)$, where $(\frac{d\sigma}{d\Omega})^p$ is the theoretical cross section for point-like particles and r_o is the classical electron radius. Symbols x, ▲, o shows, respectively data of refs. (19, 7, 8). Points of ref. (8) refer to the energy explicitely indicated.

where $\rho = \rho(\omega) < 1$. The same model gives for the proton polarizabilities :

$$\alpha_p = \frac{(\lambda_p \hat{e})^2}{4M^3} + \frac{1}{2\pi^2}(1-\rho)\sigma_{-2} , \qquad \beta_p = -\frac{(\lambda_p \hat{e})^2}{4M^3} + \frac{1}{2\pi^2}\rho\,\sigma_{-2} . \tag{34}$$

If we use for $\rho(\omega)$ the naive parton model prediction $\rho = 0$, we obtain $\beta_p = -(\lambda_p\hat{e})^2/4M^3 = -0.54 \times 10^{-4}\ \mathrm{fm}^3$, that is $\alpha_p \gg \beta_p$. As it was recently proved[21], choice $\rho = 0$ is not incompatible with the generalized vector dominance model. Of some interest is also choice $\rho = 1/2$ giving finite mass shift for nucleons[22]; in this case we obtain $\alpha_p \approx \beta_p \approx \sigma_{-2}/4\pi^2$. Moreover, let us stress that the recent Sakurai's result[22] suggesting R to be ν-independent is not compatible with the sum rule (25b).

To obtain a more reliable determination of $R(Q^2, \nu)$, we have devoted our attention to the SLAC experimental data quoted by Rjordan et al.[24], selecting those in the ranges :

$$3\ \mathrm{GeV} \leq \nu \leq 11\ \mathrm{GeV} , \qquad 1\ \mathrm{GeV} \leq Q^2 \leq 11\ \mathrm{GeV} .$$

Various kinds of fit formulas were taken into account and our final choice was the formula :

$$\frac{R_p(Q^2, \nu)}{Q^2} = \frac{a_1}{1 + a_2\nu^2}\left(1 + a_3\omega^{-1} + a_4\omega^{-2}\right) , \tag{35}$$

scaling for $a_2 \nu^2 \gg 1$. Data of ref. (24) were averaged over ν in the three ranges :

(3 - 5) GeV , (6 - 8) GeV , (9 - 11) GeV ,

and then fitted by eq. (35). Systematic errors where simply added to the statistical ones. We obtained :

$$a_1 = (0.57 \pm 0.11)\ \mathrm{GeV}^{-2}, \qquad a_2 = (2.3 \pm 1.5) \times 10^{-2}\ \mathrm{GeV}^{-2},$$

$$a_3 = (-0.18 \pm 0.13)\, 2M, \qquad a_4 = (0.84 \pm 0.11)(2M)^2,$$

with $\chi^2/N_D = 0.48$. The errors on all coefficients are strongly correlated.

The resulting fits are shown in Fig. 4. Very similar results were obtained by treating the experimental data with different procedures, for ex. by first fitting $R(Q^2, \nu)/Q^2$ as a function of Q^2 at fixed ν values and then fitting $\lim_{Q^2 \to 0} R(Q^2, \nu)/Q^2$ as a function of ν.

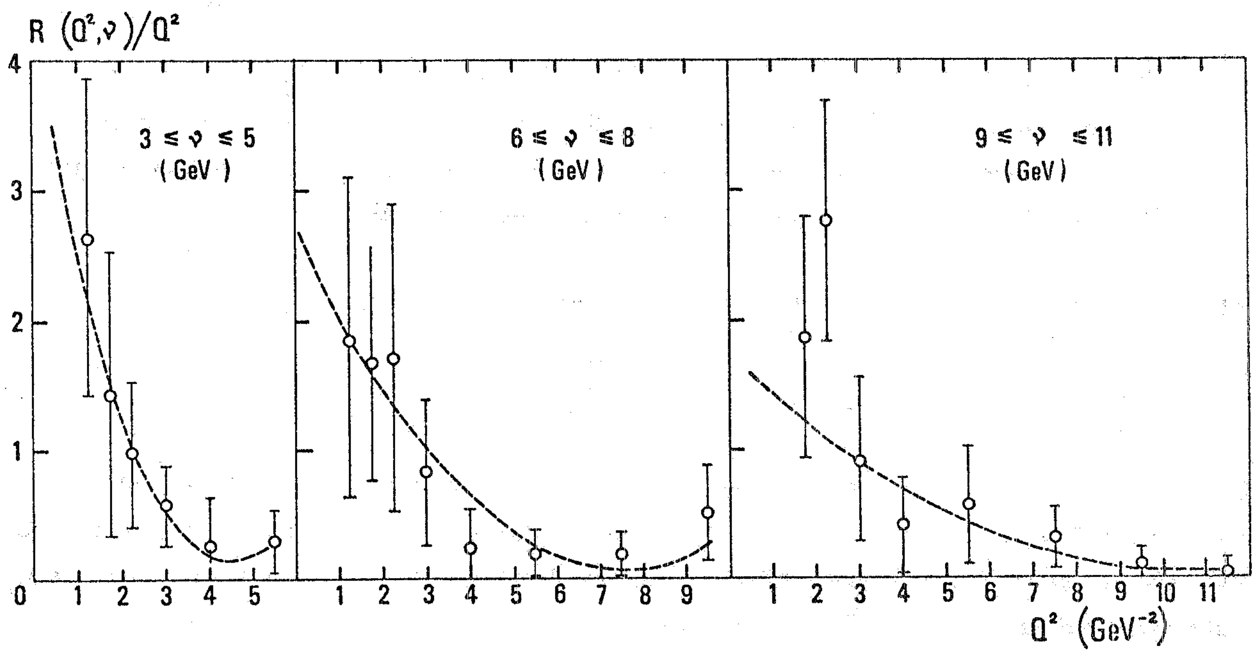

FIG. 4 - Comparison between experimental values of $R(Q^2, \nu)/Q^2$ (23) averaged over ν and fit result with eq. (35). Systematic errors have been added to the statistical ones.

All previously mentioned data refer to the high energy continuum. On the other hand, at the peak of the first resonance(25) we have :

$$\lim_{Q^2 \to 0} \frac{R_p(Q^2)}{Q^2} = (1.52 \pm 0.66)\ \mathrm{GeV}^{-2},$$

that is a value remarkably higher (a factor ~ 2.6) than the background extrapolation at the same energy.

By these fits and the analysis of the total cross section due to Damashek and Gilman[9], we were able to obtain the following estimate :

$$\alpha_p = (8.8 \pm 2.0) \times 10^{-4} \ \mathrm{fm}^3 \ .$$

Furthermore, the effect of the first resonance raises α_p of about $(0.7 \times 10^{-4})\ \mathrm{fm}^3$. Our estimate, giving α_p in the range $(7.5 - 11.5) \times 10^{-4}\ \mathrm{fm}^3$, suggest that $\alpha_p \gtrsim \beta_p$ as in the fit of Table I, in evident disagreement with the result $\alpha_p < \beta_p$ of ref. (11).

This last conclusion was reached by the use of the backward sum rule giving $(\alpha_p - \beta_p)$. By the use of the same data employed in ref. (11), eqs. (30), (31), give :

$$\alpha_p - \beta_p \simeq -4.8 \times 10^{-4}\ \mathrm{fm}^3 + 4 \left(\frac{G}{t_r}\right) \frac{\hat{e}^2}{M} \ . \tag{36}$$

If we assume $G \approx 1$ as suggested in ref. (18) and $t_r \approx (3\mu)^2$, we obtain $\alpha_p \simeq 11 \times 10^{-4}\ \mathrm{fm}^3$, $\beta_p \simeq 3 \times 10^{-4}\ \mathrm{fm}^3$. It would be very lucky if this crude estimate could give quantitatively correct values. However, it clearly shows the need for the inclusion of the t-channel contribution before arriving to definite conclusions (a coupling constant as low as $G = 0.35$ allows to have $\alpha_p \approx \beta_p$).

As far as neutron is concerned, the forward sum rule (25a) was evaluated by deuteron data[25], obtaining :

$$\alpha_n + \beta_n \approx 14 \times 10^{-4}\ \mathrm{fm}^3 \ .$$

In addition, as shown in ref. (24), R_p and R_n seem to be consistent within the statistical errors. We are so foreed to conclude that isovector contributions to both α and β cannot exceed about 20 %.

APPENDIX. -

We shall now briefly present the calculation line allowing to obtain an explicit form of the tensor $\hat{T}_i^{\mu\nu}$ defined in eq. (9). The matrix elements $(\bar{u}_f \theta u_i)$ of any Dirac operator θ can be put in the form $(\chi_f^+ \theta^{eff} \chi_i)$, where θ^{eff} is an effective operator acting in the space of the Pauli matrices. By standard techniques, neglecting spin-dependent terms, we obtain for the operator θ contributing to the $T_i^{\mu\nu}$ tensors the following Pauli form valid in the laboratory frame :

$$1 \to \mathcal{F}^{-1}, \qquad K\!\!\!/ \to \frac{(\underset{\sim}{P}\cdot \underset{\sim}{K})}{M}\mathcal{F}, \qquad \gamma_\mu \to \frac{P_\mu}{M}\mathcal{F},$$

$$\gamma_\mu K\!\!\!/ \to (K_\mu \frac{P_o}{M} + q_\mu \frac{K_o}{2M})\mathcal{F}, \qquad K\!\!\!/\gamma_\mu \to (K_\mu \frac{P_o}{M} - q_\mu \frac{K_o}{2M})\mathcal{F},$$

$$\gamma_\mu\gamma_\nu \to (g_{\mu\nu}\frac{P_o}{M} - \frac{1}{2M^2}\left[P_\mu q_\nu - P_\nu q_\mu\right])\mathcal{F}, \tag{A1}$$

$$\gamma_\mu\gamma_\nu K\!\!\!/ \to (g_{\mu\nu}\frac{(\underset{\sim}{P}\cdot \underset{\sim}{K})}{M} - \left[K_\mu P_\nu - K_\nu P_\mu\right]\frac{1}{M})\mathcal{F},$$

$$K\!\!\!/\gamma_\mu\gamma_\nu \to (g_{\mu\nu}\frac{(\underset{\sim}{P}\cdot \underset{\sim}{K})}{M} + \left[K_\mu P_\nu - K_\nu P_\mu\right]\frac{1}{M})\mathcal{F},$$

where $\mathcal{F} = (2M/(M+E_f))^{1/2}$ and $\mathcal{F}^{-1} = (P_o/M)\mathcal{F}$. The factors $\mathcal{F}^{-1}$ derive from the normalization of the Pauli spinors.

So the spin-averaged parts of the tensors $T_i^{\mu\nu}$ of ref. (12) can be written in the form :

$$\hat{T}_1^{\mu\nu} = \frac{1}{4}\mathcal{F}^{-1}\left[(\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\tau_1^{\mu\nu} - \tau_2^{\mu\nu} + \tau_3^{\mu\nu} + \tau_5^{\mu\nu}\right],$$

$$\hat{T}_2^{\mu\nu} = \frac{1}{16}\mathcal{F}^{-1}\left[16Q^4\tau_1^{\mu\nu} + (\underset{\sim}{K}^2 - \underset{\sim}{q}^2 + 8Q^2)\tau_2^{\mu\nu} - (\underset{\sim}{K}^2 - \underset{\sim}{q}^2 - 8Q^2)\tau_3^{\mu\nu} + (\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\tau_5^{\mu\nu}\right],$$

$$\hat{T}_3^{\mu\nu} = \frac{1}{4}\mathcal{F}^{-1}\left[(\underset{\sim}{P}\cdot\underset{\sim}{K})^2\tau_1^{\mu\nu} + (\underset{\sim}{K}^2 - \underset{\sim}{q}^2)\tau_4^{\mu\nu} - \frac{1}{4}(\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\tau_6^{\mu\nu} + \tau_7^{\mu\nu}\right]\right],$$

$$\hat{T}_4^{\mu\nu} = \frac{1}{4}\mathcal{F}^{-1}\left[-4Q^2(\underset{\sim}{P}\cdot\underset{\sim}{K})\tau_1^{\mu\nu} - (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\tau_2^{\mu\nu} + \tau_3^{\mu\nu}\right] - \underset{\sim}{q}^2\tau_6^{\mu\nu} - \underset{\sim}{K}^2\tau_7^{\mu\nu}\right], \tag{A2}$$

$$\hat{T}_5^{\mu\nu} = \frac{1}{4}\mathcal{F}^{-1}\left[(\underset{\sim}{P}\cdot\underset{\sim}{K})\tau_8^{\mu\nu} - \underset{\sim}{q}^2\tau_9^{\mu\nu} - \underset{\sim}{K}^2\tau_{10}^{\mu\nu}\right],$$

$$\hat{T}_6^{\mu\nu} = \frac{1}{8}\Big[(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'\tau_2^{\mu\nu} - (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]\tau_3^{\mu\nu} + (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{Q^2}{2M^2}\mathcal{F}\right]\tau_5^{\mu\nu} - 4Q^2\mathcal{F}'\tau_6^{\mu\nu} - 2Q^2\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]\tau_9^{\mu\nu}\Big],$$

$$\hat{T}_7^{\mu\nu} = \hat{T}_8^{\mu\nu} = \hat{T}_9^{\mu\nu} = 0 ,$$

$$\hat{T}_{10}^{\mu\nu} = -2(\underset{\sim}{K}^2-\underset{\sim}{q}^2)\mathcal{F}'\tau_4^{\mu\nu} + \frac{1}{2M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})^2\mathcal{F}\tau_3^{\mu\nu} + (\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'\tau_8^{\mu\nu} +$$

$$+ (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' + \frac{1}{4M^2}(\underset{\sim}{K}^2-\underset{\sim}{q}^2)\mathcal{F}\right]\tau_7^{\mu\nu} + \frac{1}{4M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})^2\mathcal{F}\tau_5^{\mu\nu} ,$$

$$\hat{T}_{11}^{\mu\nu} = 0 ,$$

$$\hat{T}_{12}^{\mu\nu} = \frac{1}{8}\left[2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'\left[\tau_2^{\mu\nu} + \tau_3^{\mu\nu}\right] - \mathcal{F}'(\underset{\sim}{K}^2-\underset{\sim}{q}^2)\tau_6^{\mu\nu} + (\underset{\sim}{K}^2-\underset{\sim}{q}^2)\left[\mathcal{F}' - \frac{Q^2}{M^2}\mathcal{F}\right]\tau_7^{\mu\nu} -\right.$$

$$\left. - \frac{Q^2}{M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\left[\tau_5^{\mu\nu} + 2\tau_3^{\mu\nu}\right]\right] ,$$

$$\hat{T}_{13}^{\mu\nu} = \frac{1}{4}\mathcal{F}'\left[(\underset{\sim}{P}\cdot\underset{\sim}{K})\tau_8^{\mu\nu} - \underset{\sim}{q}^2\tau_9^{\mu\nu} - \underset{\sim}{K}^2\tau_{10}^{\mu\nu}\right] ,$$

$$\hat{T}_{14}^{\mu\nu} = \frac{1}{2}\left[\mathcal{F}'(\underset{\sim}{P}\cdot\underset{\sim}{K})\tau_9^{\mu\nu} + (\underset{\sim}{P}\cdot\underset{\sim}{K})\left[\mathcal{F}' - \frac{1}{4M^2}(\underset{\sim}{K}^2-\underset{\sim}{q}^2)\mathcal{F}\right]\tau_{10}^{\mu\nu} + \frac{1}{4M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})^2\mathcal{F}\tau_8^{\mu\nu}\right] ,$$

$$\hat{T}_{15}^{\mu\nu} = \frac{1}{2}\left[\underset{\sim}{q}^2\mathcal{F}'\tau_9^{\mu\nu} + \underset{\sim}{K}^2\mathcal{F}'\tau_{10}^{\mu\nu} + \frac{1}{4M^2}\underset{\sim}{q}^2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\tau_8^{\mu\nu}\right] ,$$

$$\hat{T}_{16}^{\mu\nu} = -\frac{1}{2}\left[\underset{\sim}{q}^2\mathcal{F}'\tau_6^{\mu\nu} + \underset{\sim}{K}^2\mathcal{F}'\tau_7^{\mu\nu} + \frac{1}{4M^2}\underset{\sim}{q}^2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\tau_5^{\mu\nu} + \frac{1}{2M^2}\underset{\sim}{K}^2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\tau_3^{\mu\nu}\right] ,$$

$$\hat{T}_{17}^{\mu\nu} = -2(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}'\tau_1^{\mu\nu} + \mathcal{F}'\tau_8^{\mu\nu} + \left[\mathcal{F}' - \frac{1}{4M^2}(\underset{\sim}{K}^2-\underset{\sim}{q}^2)\mathcal{F}\right]\tau_7^{\mu\nu} - \frac{1}{4M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\tau_5^{\mu\nu} - \frac{1}{2M^2}(\underset{\sim}{P}\cdot\underset{\sim}{K})\mathcal{F}\tau_3^{\mu\nu} ,$$

$$\hat{T}_{18}^{\mu\nu} = 0 ,$$

where $\mathcal{F}' = (\mathcal{F}^{-1} - \mathcal{F})$, the tensors $\tau_i^{\mu\nu}$ (i = 1 to 7) are defined in eq. (10) and:

$$\tau_8^{\mu\nu} = (K^\mu q^\nu + K^\nu q^\mu) , \quad \tau_9^{\mu\nu} = (P^\nu K^\mu - P^\mu K^\nu) , \quad \tau_{10}^{\mu\nu} = (P^\nu q^\mu + P^\mu q^\nu) . \qquad \text{(A3)}$$

Eqs. (11) and (12) immediately follow from the definitions.

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