

To be submitted to
~~Physics Letters B~~
Nuovo Liment

COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-76/36(P)
24 Giugno 1976

A. Majecki and L. Satta: THE MICROSCOPIC ANALYSIS OF THE
 $^4\text{He} - ^4\text{He}$ ELASTIC SCATTERING AT INTERMEDIATE ENERGIES.

A. Malecki^(x) and L. Satta: THE MICROSCOPIC ANALYSIS OF THE
 $^4\text{He}-^4\text{He}$ ELASTIC SCATTERING AT INTERMEDIATE ENERGIES.

ABSTRACT. -

The Glauber model of nucleus-nucleus scattering is discussed, the emphasis being put on an approximate formula obtained by neglecting virtual excitations of the colliding nuclei. The numerical calculations are performed for $^4\text{He}-^4\text{He}$ elastic scattering using three models of the nuclear ground state wave function; a considerable effect of short-range correlations has been found.

In the recent ten years there has been a considerable progress in studying nuclear structure by means of intermediate energy protons^(1, 2). The store of experimental data should now be supplemented by the measurements performed with beams of fast ions. It is expected that some effects of nuclear structure will be enhanced in nucleus-nucleus collisions due to mechanisms which are absent in particle-nucleus scattering; in particular, in nucleus-nucleus scattering are involved multi-nucleon collisions⁽⁵⁾ which should be sensitive to the short-range nuclear correlations.

Among the nuclei studied with beams of energetic protons

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particular attention has been paid to the α -particle⁽²⁾. Recently the measurements of p-⁴He elastic scattering has been extended to large momentum transfers by the use of accelerated alphas hitting a hydrogen target⁽³⁾. A natural continuation of this work is the experiment on the ⁴He-⁴He scattering, in phase of realization at Saclay⁽³⁾.

The proton-nucleus scattering experiments at intermediate energies have successfully been interpreted in terms of the Glauber model⁽⁴⁾ of multiple collisions. The model may straightforwardly be generalized to scattering of two nuclei^(5, 6).

In this letter we describe a new approximate and relatively simple version of the Glauber formula for nucleus-nucleus scattering. We give physical and numerical arguments for this approximation and discuss its relation to other works. Our approach is then applied to α - α elastic scattering in order to provide a theoretical framework to the forthcoming experimental results⁽³⁾.

The Glauber model is based on the two assumptions :

- i) the eikonal approximation⁽⁴⁾ for high-energy, small-angle scattering ;
- ii) the approximation of closure over the intermediate states⁽⁷⁾ of colliding nuclei.

The Glauber amplitude for scattering of two nuclei A and B is written in the form of the impact parameter integral :

$$F_{AB} = \frac{ip}{2\pi} \int d^2b e^{i\vec{q} \cdot \vec{b}} \Gamma_{AB}(b) , \quad (1)$$

p, q being the incident c. m. momentum and the momentum transfer respectively. The nucleus-nucleus profile function Γ_{AB} for elastic scattering is^(4, 5) :

$$\Gamma_{AB}(b) = 1 - \left\langle \psi_A \psi_B \left| \prod_{j=1}^A \prod_{k=1}^B \left[1 - \gamma_{jk} \left(\vec{b} - \vec{s}_j^A + \vec{s}_k^B \right) \right] \right| \psi_A \psi_B \right\rangle \quad (2)$$

The elementary profile functions γ_{jk} depend on the transverse (i. e. in the plane perpendicular to the bisetrix of the c. m. scattering angle) coordinates \vec{s}_j^A, \vec{s}_k^B of the constituent nucleons and are to be related to the nucleon-nucleon elastic scattering amplitudes. ψ_A and ψ_B are the nuclear ground state wave functions which depend both on the transverse and longitudinal intrinsic nucleon coordinates.

If one multiplies out the AB factors in Eq. (2) one obtains terms, each of which is a product of certain number n of the profiles γ ; $1 \leq n \leq AB$. In particular there are many terms in which a given nucleon coordinate appears more than once. The terms with repetition of nucleon variables represent a peculiarity of nucleus-nucleus collisions; for particle-nucleus scattering the Glauber model does not contain such terms. A large variety of the repetition terms in scattering of two complex nuclei makes the evaluation of ground state expectation value in Eq. (2) a very difficult task, even for simple model wave functions.

An enormous simplification of the Glauber formula may, however, be obtained by relaxing the closure approximation. Closure implies the possibility of excitations and deexcitations (connected between one another in such a way that the final result is elastic scattering) of the two nuclei during scattering. The effect of virtual nuclear excitations which we will call quasi-elastic shadowing^(x) should not, however, be large. In fact, at high energies the inelastic transitions will prevalently lead to the break-up channels (for ^4He besides this is, because of the lack of discrete excitation, the unique possibility) and it is hardly probable that disintegrated products could recombine to the initial state. The neglect of quasi-elastic shadowing would reduce Eq. (2) to the following simple form :

(x) - The analogous virtual excitations of the elementary projectile in particle-nucleus scattering are usually referred to as inelastic shadowing or the Gribov effect⁽⁸⁾.

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$$\Gamma_{AB}(b) = 1 - (1 - S_{AB})^{AB} \quad (3)$$

$$S_{AB} = \langle \psi_A \psi_B | \gamma(\vec{b} - \vec{s}^A + \vec{s}^B) | \psi_A \psi_B \rangle.$$

Eq. (3) is obtained by inserting between the profile functions γ_{jk} in Eq. (2) the operators $\sum_{n_A} |n_A\rangle \langle n_A|$, $\sum_{n_B} |n_B\rangle \langle n_B|$, $|n_A\rangle$, $|n_B\rangle$ being the complete sets of states for the two nuclei, and then by deleting the inelastic states. For sake of simplicity we have assumed in Eq. (3) that the elementary profiles for the p-p and p-n interaction are identical.

An expression similar to Eq. (3) has been obtained by Czyż and Maximon⁽⁵⁾ in the discussion of the optical limit of the multiple scattering amplitude. In contrast to their work (see also Ref. (6)) we do not, however, make neither any assumption on the nuclear ground state wave functions, nor need any estimate of the repetition terms. Our Eq. (3) is obtained only by ignoring quasi-elastic shadowing. The exponentiation of Eq. (3) in the limit $AB \rightarrow \infty$ would give the well-known optical limit of the nucleus-nucleus profile^(4, 5, 6). Thus in our approach the optical limit acquires a new physical interpretation as being essentially equivalent to the neglect of quasi-elastic shadowing.

For light nuclei the exponentiation would, of course, be unjustified. Let us also notice that the effect of translational invariance^(5, 9) which is very important in this case is included in Eq. (3) since ψ_A and ψ_B are to be intrinsic wave functions. Therefore the correction connected with the c. m. constraint⁽⁹⁾ will not lead, unlike as in Ref. (5), to any inconsistency with the approximate form of the nucleus-nucleus profile.

The approximation of Eq. (3) has been checked by us, on the example of the ${}^4\text{He}-{}^4\text{He}$ elastic scattering, by a term by term comparison with Eq. (2) up to the fifth order in the nucleon-nucleon interaction - see Fig. 1. To this end the Independent Particle Model (IPM) has

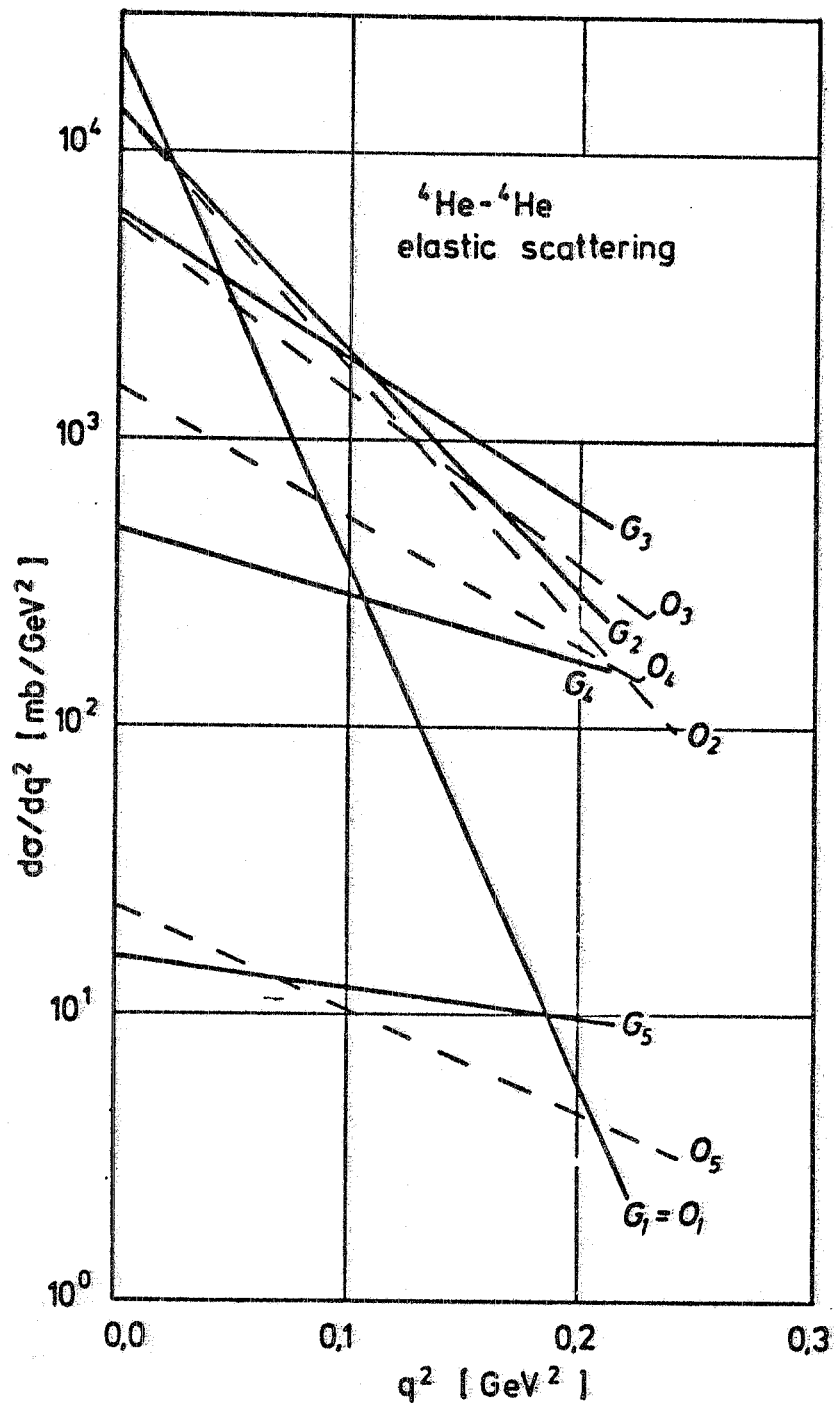


FIG. 1 - Comparison of the Glauber multiple scattering amplitude for the $\alpha - \alpha$ scattering with its approximated form, obtained by neglecting quasi-elastic shadowing. The curves correspond to contributions from the single, double, etc. scattering which result either from Eq. (2) (full lines denoted G_n) or Eq. (3) (broken lines denoted O_n). The double Gaussian nuclear density (Eq. (5)) with the parameters $R_1 = 1.25$ fm, $R_2 = 0.77$ fm, $\delta = 1.0$, providing a good fit to the ${}^4\text{He}$ charge form factor⁽¹⁰⁾, has been used. The N-N parameters are appropriate for scattering at $P_{\text{lab}} = 1.25$ GeV/c per nucleon: $\sigma = 40$ mb, $\alpha = -0.2$, $a = 3$ GeV⁻².

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been used:

$$|\psi_A|^2 = \prod_{j=1}^A \rho_A(r_j^A), \quad |\psi_B|^2 = \prod_{k=1}^B \rho_B(r_k^B). \quad (4)$$

The single-particle density has been chosen as a double Gaussian:

$$\rho(r) = \pi^{-3/2} \left[R_1^3 - \delta R_2^3 \right]^{-1} \left[\exp\left(-\frac{r^2}{R_1^2}\right) - \delta \exp\left(-\frac{r^2}{R_2^2}\right) \right], \quad (5)$$

which gives a chance, in contrast to a single Gaussian, of fitting a diffracted structure of the elastic charge form factor of ${}^4\text{He}$ ⁽¹⁰⁾. The c. m. constraint on the nuclear model of Eqs. (4) and (5) has been imposed using the Gartenhaus-Schwartz prescription⁽⁹⁾.

The elementary profiles have been assumed in the form:

$$\gamma(b) = \frac{\sigma(1-i\alpha)}{4\pi a} \exp\left(-\frac{b^2}{2a}\right), \quad (6)$$

which corresponds to a Gaussian q -dependence of the N-N elastic scattering amplitudes. The parameters σ (the total N-N cross-section), α (the Re/Im ratio of the forward amplitude) and a (the slope) are, in general, energy dependent.

A similar agreement, as in Fig. 1, between Eqs. (2) and (3) has been found for the single Gaussian density ($\delta = 0$). In this case our Eq. (3) has been also compared with the full calculation of the Glauber multiple scattering amplitude carried out in Ref. (5); the agreement up to the third diffraction maximum is very good^(x).

(x) - On the contrary, in Ref. (5) a considerable difference between the multiple scattering and the optical limit results in the case of 4×4 scattering has been found. It should, however, be stressed again that our Eq. (3) differs from the optical limit of Ref. (5) by the way of introducing the c. m. correction and by the lack of exponentiation.

The simplicity of Eq. (3) allows to perform an analysis of nucleus-nucleus scattering using more sophisticated forms of the nuclear wave functions than IPM. In addition to the product of single and double Gaussian densities we have also used the model of correlated pairs⁽¹¹⁾:

$$|\psi|^2 = \prod_{j=1}^A \rho(r_j) \left\{ 1 + \sum_{l=1}^{A/2} (2^{l-1} l!)^{-1} \sum_{j_1 \neq k_1 \neq \dots \neq j_l \neq k_l} [\Delta(j_1, k_1) \dots \Delta(j_l, k_l)] \right\} \quad (7)$$

$$\Delta(j, k) = G^2(\vec{r}_j - \vec{r}_k) - 1 ,$$

which is capable of accounting for the short-range correlations between the target nucleons. The successive terms of (7) correspond to an expansion in numbers of correlated pairs: independent particles, one correlated pair, etc. The two inputs of the model are the single-particle density $\rho(r)$ (taken by us as a single Gaussian with the size parameter R) and the correlation operator $G(r_{jk})$ assumed in the form⁽¹¹⁾:

$$G^2 = \frac{g^2 + (M-1)g}{M} , \quad g(r_{jk}) = 1 - \exp\left(-\lambda \frac{r_{jk}^2}{R^2}\right) , \quad (8)$$

the coefficient $M = -1 + (1+2\lambda)^{3/2} (1+4\lambda)^{-3/2}$ being determined by normalization.

The predictions of the three nuclear models for the ${}^4\text{He}-{}^4\text{He}$ elastic scattering are compared in Fig. 2. The parameters of IPM with a double Gaussian density and those of the model of correlated pairs provide good fits to the ${}^4\text{He}$ elastic charge form factor in a wide range of momentum transfer^(10, 11). The parameter of IPM with a single Gaussian has been fixed by the value of the root-mean square radius of the α -particle which in all three cases is the same: 1.64 fm in agreement with a low q experiment⁽¹²⁾. In Fig. 2 the Coulomb inter

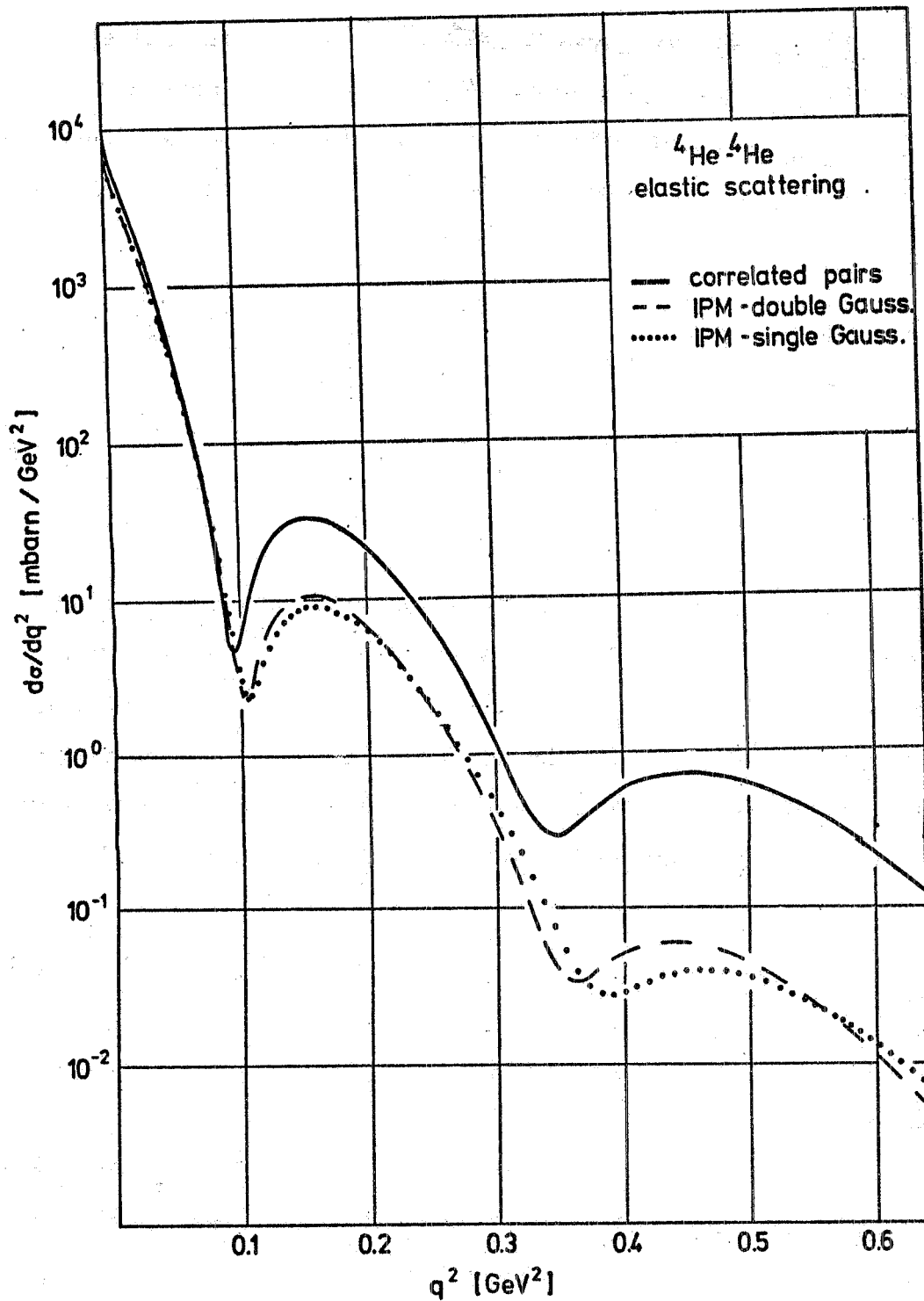


FIG. 2 - Comparison of the ${}^4\text{He}-{}^4\text{He}$ elastic cross-section in three nuclear models: dotted line - IPM with the single Gaussian density ($R_1 = 1.36$ fm); dashed line - IPM with the double Gaussian ($R_1 = 1.25$ fm, $R_2 = 0.77$ fm, $\delta = 1.0$); full line - the model of correlated pairs ($R = 1.265$, $\lambda = 0.652$). The N-N parameters are given in the caption of Fig. 1.

action of the α -particles has been included⁽¹³⁾ assuming charges extended over the volume of the nuclei.

The most striking feature of Fig. 2 is a considerable difference between the model of correlated pairs and that of independent particles. On the other hand the two curves of IPM, corresponding to various single-particle densities and a quite different behaviour of the form factors, appear to be rather similar.

Thus it can be stated that the effect of nucleon-nucleon correlations in nucleus-nucleus scattering is greatly enhanced as compared with the proton-nucleus scattering⁽¹⁴⁾. A detailed study of nucleus-nucleus scattering might therefore put a severe constraint on the model of nuclear structure and eliminate ambiguities arising in the analysis of the charge form factor.

One of the authors (A. M.) would like to thank Profs. G. Bellettini and P. Picchi for their hospitality at the Laboratori Nazionali di Frascati.

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