To be submitted to
Phys. Rev. Letters

COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-76/23(P)
2 Aprile 1976

R. Dymarz and A. Małecki: THE EXTENSION OF THE GLAUBER MODEL OF MULTIPLE SCATTERING BY MEANS OF THE OPTICAL POTENTIAL. -

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati
Servizio Documentazione

LNF-76/23(P)
2 Aprile 1976

R. Dymarz[(x)] and A. Małecki[(x)]: THE EXTENSION OF THE GLAUBER MODEL OF MULTIPLE SCATTERING BY MEANS OF THE OPTICAL POTENTIAL. -

The optical potential equivalent to the Glauber model of multiple collision is evaluated. The use of this potential in the Schroedinger equation, solved without any approximation, provides a successful description of the p-$^4$He elastic scattering at intermediate energies.

In the eikonal approximation[1,2] of high-energy, small-angle potential scattering the amplitude is expressed in the form of the impact parameter (Fourier-Bessel) integral:

$$F(q,p) = \frac{ip}{2\pi} \int d^2b \, e^{i\vec{q}\cdot\vec{b}} \, \Gamma(\vec{b}) , \tag{1}$$

p, q being the incident c.m. momentum and the momentum transfer, respectively. The scattering profile function $\Gamma(\vec{b})$ is related to the potential of interaction $V(\vec{r})$ as follows:

$$\Gamma(\vec{b}) = 1 - \exp\left[ - \frac{i}{v} \int_{-\infty}^{+\infty} dz \, V(\vec{b}, z) \right] , \tag{2}$$

(x) - On leave of absence from Instytut Fizyki Jądrowej, Krakow, Poland.

where the eikonal axis Oz is directed along the bisectrix of the c.m. scattering angle, v being the relative velocity of two interacting particles.

If the profile function $\Gamma(b)$ is known and for a spherically symmetric interaction V(r) Eq. (2), being essentially the Abel integral equation[2], may easily be solved for the potential:

$$V(r) = \frac{iv}{\pi} \int_{-\infty}^{+\infty} dz \frac{\Gamma'(\sqrt{r^2+z^2})}{1-\Gamma(\sqrt{r^2+z^2})} , \quad \Gamma' = \frac{d}{dr^2}\Gamma(\sqrt{r^2+z^2}) . \tag{3}$$

The eikonal approximation of potential scattering constitutes a basis for the Glauber model[2] of multiple scattering between composite particles. Another approximation involved in the model regards closure[3] over the intermediate eigenstates of colliding particles. The elastic scattering profile of an elementary projectile on the composite nucleus in the Glauber model is:

$$\Gamma(b) = 1 - \left\langle \psi \right| \prod_{j=1}^{A} \left[1 - \gamma_j(\vec{b} - \vec{s}_j)\right] \left. \psi \right\rangle \tag{4}$$

where the profiles $\gamma_j$ of the target nucleons ($\vec{s}_j$ being their positions in the plane of impact parameters) are to be related, analogously as in Eq. (1), to the projectile-nucleon elastic scattering amplitudes $f_j$, and $\psi$ is the nuclear ground state wave function. Eq. (4) may readily be generalized to scattering of two nuclei[4].

The Glauber model has been applied with success to the description of hadron-nucleus scattering at intermediate energies[5]. Nevertheless there remain some discrepancies between the theory and experiment[6], especially at diffraction minima and large scattering angles. The difficulties seem to result from the limitations of the eikonal approximation, viz. its restriction to small angle scattering and the insensitivity to the sign of the real part of interaction.

The aim of this work is to propose the following extension of the Glauber model. First, one evaluates the Glauber profile (4). Secondly, the corresponding optical potential is obtained from Eq. (3). Finally, the Schroedinger wave equation with this potential is solved for the scattering amplitude without any approximation.

For calculating the profile $\Gamma(b)$ the knowledge of the nuclear ground state density is required. We have used the model of correlated pairs[7]:

$$|\psi|^2 = \prod_{j=1}^{A} \rho(r_j) \left\{ 1 + \sum_{l=1}^{[A/2]} (2^l l!)^{-1} \sum_{j_1 \neq k_1 \neq .. \neq j_l \neq k_l}^{A} [\Delta(j_1,k_1)..\Delta(j_l,k_l)] \right\},$$

$$\Delta(j,k) = G^2(\vec{r}_j, \vec{r}_k) - 1 \tag{5}$$

which is capable of accounting for the short-range interactions between the target nucleons. The two inputs of the model are the single particle density $\rho(r)$ and the correlation operator $G(\vec{r}_j, \vec{r}_k)$ that serves to determine the form of the interaction. The successive terms of (5) correspond to the expansion in numbers of correlated pairs : independent particles, one correlated pair, etc. up to $A/2$ (or $(A-1)/2$) correlated pairs.

The nuclear density (5) has to be supplemented by the prescription for eliminating the centre-of-mass coordinate since the wave function should be translationally invariant[8]. It turns out that the c.m. constraint may easier be imposed on the scattering amplitude $F(q)$ than directly on the profile $\Gamma(b)$. Using the density (5) one obtains for the elastic scattering amplitude of the Glauber model:

$$F(q) = \frac{ip}{2\pi} \, \Theta_{CM}(q) \int_0^{\infty} d^2b \, e^{iq\cdot b} \left[ 1 - \sum_{l=1}^{[A/2]} \frac{A!(1-S)^{A-2l} T^l}{2^l l! \, (A-2l)!} \right] \tag{6}$$

where the functions S and T are defined as follows :

$$S(b) = \int d^3r \, \rho(r) \, \gamma(\vec{b} - \vec{s}) \, ,$$

$$T(b) = \int d^3r_1 d^3r_2 \rho(1)\rho(2)\Delta(1,2) \left[1-\gamma(\vec{b}-\vec{s}_1)\right]\left[1-\gamma(\vec{b}-\vec{s}_2)\right] , \tag{7}$$

$\Theta_{CM}$ being a correction resulting from the c.m. constraint ; in fact the correction is multiplicative only if the nuclear density (5) can be factorized into the c.m. and intrinsic parts. From Eq.(6) the nuclear profile $\Gamma(b)$ may readily be obtained by reversing the Fourier-Bessel transform.

In this letter we present the results of our analysis of the elastic p-$^4$He scattering. This scattering has been carefully studied in the last decade in a wide range of energy starting with the pioneering experiment at Brookhaven[9] up to recent, precise measurements at Saclay[10-12] and Berkeley[13].

In our calculations we have used the Gaussian single particle density:

$$\rho(r) = \pi^{-3/2} R^{-3} \exp(-r^2/R^2), \tag{8}$$

the c.m. correction to the scattering amplitude is then $\Theta_{CM} = \exp(q^2R^2/4A)$. It should be emphasized that for small A it is extremely important to include the effect of this correction in the evaluation of the Glauber profile $\Gamma(b)$ and the equivalent optical potential.

For the short-range correlation operator we have assumed[7]:

$$G^2 = \frac{g^2 + (M-1)g}{M}, \qquad g(r_{jk}) = 1 - \exp(-\frac{\lambda r_{jk}^2}{R^2}), \tag{9}$$

the coefficient $M = -1 + (1+2\lambda)^{3/2}(1+4\lambda)^{-3/2}$ being determined by the normalization. The form (9) and especially the unitarity of the correlations[7] assure the validity of the model nuclear density (5). Let us notice that the correlations do not affect the c.m. correction. The parameters R and $\lambda$ have been established[7] from the analysis of the $^4$He elastic charge form factor[14].

The nucleon profiles have been taken as spin independent and in the form

$$\gamma(b) = \frac{\sigma(1-i\alpha)}{4\pi a} \exp(-\frac{b^2}{2a}) \tag{10}$$

which corresponds to a Gaussian q-dependence of the proton-nucleon elastic scattering amplitude. The parameters $\sigma$ (the total cross-section), $\alpha$ (the Re/Im ratio of the forward amplitude) and a (the slope) are, in general, energy dependent and different for the p-p and p-n interactions.

Under assumptions (8)-(10) the functions S and T, the scattering amplitude F(q) and the nuclear profile $\Gamma(b)$ may be calculated analytically. For illustration we quote the result for $\Gamma(b)$ in the case when the short-range correlations are absent (G = 1) and the nucleon profiles equal each other (i.e. $\sigma_{pp} = \sigma_{pn} = \sigma$ etc):

$$\Gamma(b) = \sum_{j=1}^{A} \left\{ \binom{A}{j} \frac{(-1)^{j+1}}{j} \left[\frac{\sigma(1-i\alpha)}{2\pi}\right]^j \frac{(R^2+2a)^{1-j}}{R_{eff}^2} \exp(-\frac{b^2}{R_{eff}^2}) \right\},$$

$$R_{eff}^2 = \frac{R^2+2a}{j} - \frac{R^2}{A}, \tag{11}$$

the second term in $R_{eff}^2$ resulting from the c.m. correction.

The integration of the Schroedinger equation with the optical potential obtained from Eq. (3) has been performed numerically using the code SQUAR[15]. This code is particularly suitable for calculating the scattering phase-shifts at large values of the c.m. momentum.

In Fig. 1 our results for the elastic differential cross-section (full curves) are compared with the Glauber model (dashed curves) and the experimental data[10-13]. It should be said that our findings are neither "best" nor "no free parameter" fits. In fact the input N-N amplitudes (especially the parameter $\alpha$ and the slope a at lower energies) are, at present, very uncertain. We have tried to reach a compromise between the nucleon parameters quoted in the literature and often inconsistent between each other, and fitting to the nuclear data.

Despite of this it is evident from Fig. 1 that the use of the effective optical potential substantially improves the agreement with experiment.

In Figs. 2 and 3 we illustrate the dependence of the optical potential and of the elastic cross-section on the nuclear structure. The essential role of the correlations induced by the c.m. constraint should be pointed out. They give rise to a very sharp transition region in the p-$^4$He potential at the nuclear surface (Fig. 2); as the result the probability of diffractive scattering after the first minimum is enormously enhanced (Fig. 3).

The necessity of having a sharp and not diffuse surface has been known earlier from the phenomenological optical-model analysis of the p-$^4$He scattering[10,16]. In our microscopic approach this property follows naturally from a correct, translationally invariant nuclear wave function. Moreover our optical-model analysis is successful without getting in conflict with the e-$^4$He data[14], in contrast to ref. (16).

This is so because our optical potential, besides having incorporated the c.m. constraint, takes also into account all terms in the Glauber series of multiple scattering. In fact, the restriction e.g. to the terms linear in the N-N interactions (as it is a common practice[16], the corresponding potential being referred to as the first order Watson potential[17]) would be insufficient for the $^4$He nucleus - see Fig. 1.

The short range correlations considerably improve the fits in the region of the second maximum (Fig. 3). Their role seems, however, to diminish at larger values of $q^2$; in fact the higher orders of multiple scattering depending only weakly on nuclear structure become more and more sensitive to the para

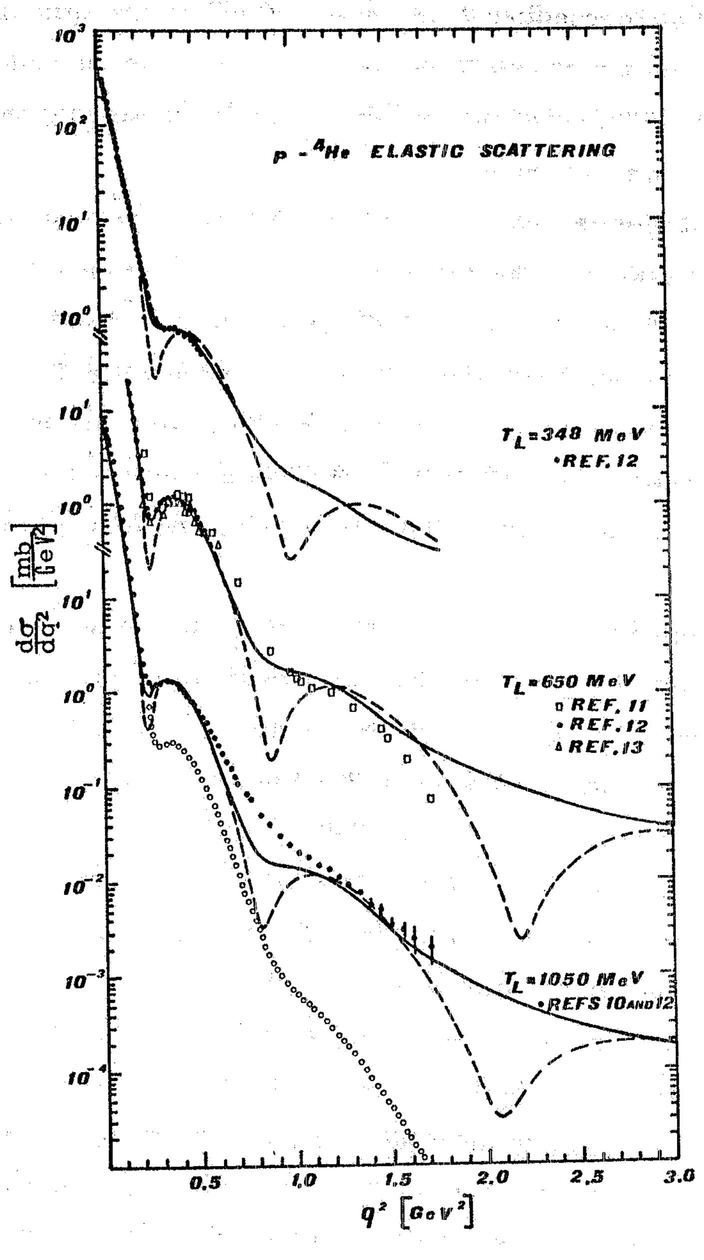


FIG. 1 - The differential cross-section of the p-$^4$He elastic scattering. The dashed curves correspond to the standard Glauber model, while the full curves to our method. The single dotted curve at 1 GeV results from the optical model analysis retaining only terms linear in the N-N amplitude. The experimental data are from Ref. (12) at 348 MeV, Refs. (11) and (12) at 650 MeV, and Ref. (10) (accounting for the renormalization of Ref. (12)) at 1050 MeV. In addition the data of Ref. (13) at 580 MeV are placed together with the 650 MeV data. The nuclear parameters are[7]: $R = 1.265$ fm, $\lambda = 0.652$. The nucleon parameters are: $\sigma_{pp} = 23.5$ mb, $\sigma_{pn} = 32.5$ mb, $\alpha_{pp} = \alpha_{pn} = -0.45$, $a_{pp} = a_{pn} = 1.5$ GeV$^{-2}$ at 348 MeV, $\sigma_{pp} = 41.0$ mb, $\sigma_{pn} = 37.0$ mb, $\alpha_{pp} = \alpha_{pn} = -0.25$, $a_{pp} = a_{pn} = 3.5$ GeV$^{-2}$ at 650 MeV, $\sigma_{pp} = 47.5$ mb, $\sigma_{pn} = 40.5$ mb, $\alpha_{pp} = \alpha_{pn} = -0.30$, $a_{pp} = a_{pn} = 5.0$ GeV$^{-2}$ at 1050 MeV.

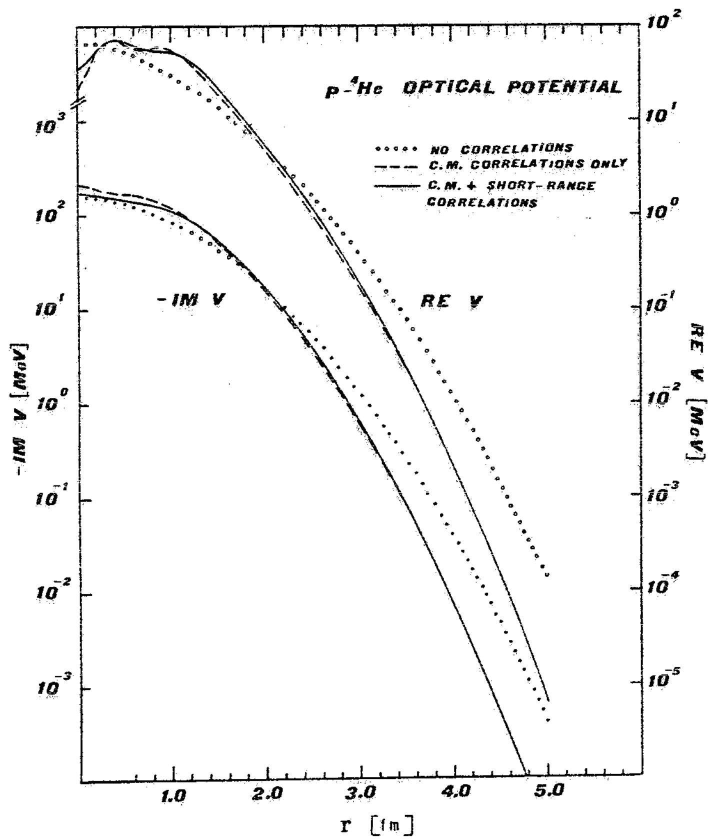


FIG. 2 - The optical potential of the p-$^4$He interaction at 1050 MeV. The dotted curve is obtained without the c.m. correction ($\Theta_{CM}$ = 1), nor the short-range correlations ($\lambda = \infty$). The dashed curve includes the c.m. constraint, and the full curve also the short-range correlations. The nuclear and nucleon parameters are given in the caption of Fig. 1.

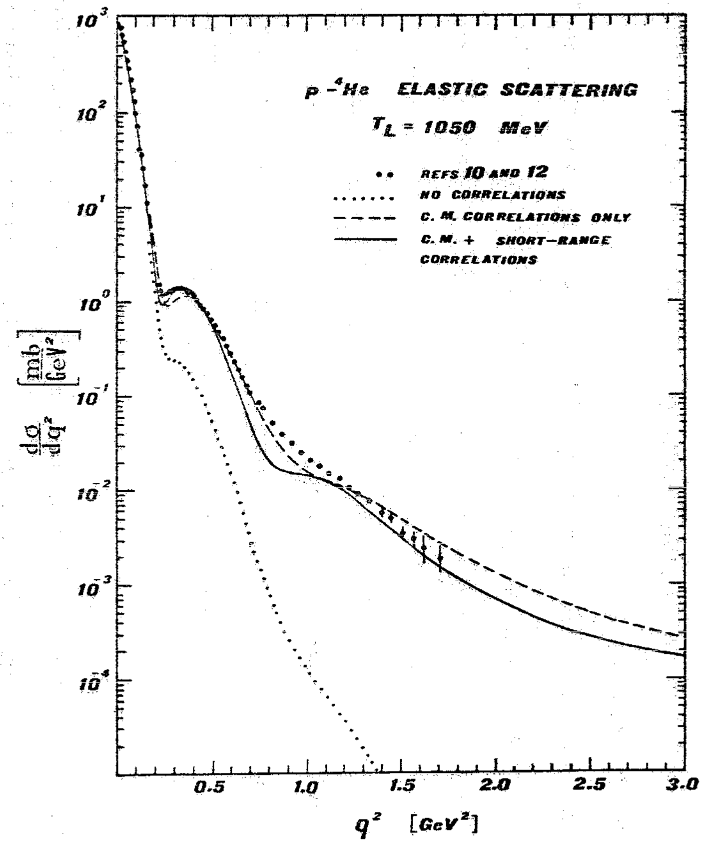


FIG. 3 - The dependence of the elastic cross-section (at 1 GeV) on nuclear structure. The description of the curves as in the caption of Fig. 2.

meters of the N-N interaction. Moreover, at large angles other effects, absent in the Glauber series, may be important; e.g. the "violent scattering" mechanism described in ref. (18) and, in the backward region, the exchange between the projectile and target protons.

In conclusion we would like to comment on the physical content that underlines the success of our approach. As it is well known the Glauber model follows from the rigorous Watson theory[17] of multiple collision as the result of cancellations, occuring in the eikonal limit, between the off-shell and rescattering terms[19]. By constructing the equivalent optical model we assure authomatically the correct treatment of the off-shell effects. Moreover, by exactly solving the Schroedinger equation we account, in a relatively simple way, for the non-eikonal propagation[20] between successive collisions as well as for the rescattering contributions.

We would like to thank Drs. D. Garreta, J. Oostens and P. Picozza for providing us with the experimental data obtained at Saclay. The hospitality of Profs. G. Bellettini and P. Picchi in the Laboratori Nazionali di Frascati is gratefully acknowledged.

REFERENCES -

(1) - G. Moliere, Z.f. Naturf. 2A, 133 (1947).

(2) - R.J. Glauber, in Lectures in Theoretical Physics, Ed. by W. E. Brittin and L.G. Dunham (Interscience, New York, 1959), Vol. 1, p. 315.

(3) - L.L. Foldy and J.D. Walecka, Ann. of Phys. 54, 447 (1969).

(4) - W. Czyż and L.C. Maximon, Ann. of Phys. 52, 59 (1969).

(5) - W. Czyż, Advan. Nucl. Phys. 4, 61 (1971); J. Saudinos and C. Wilkin, Ann. Rev. of Nucl. Sci., 24, 341 (1974).

(6) - G. Igo, in Sixth Intern. Conf. on High Energy Physics and Nuclear Structure, Santa Fe, 1975.

(7) - A. Małecki and P. Picchi, Riv. Nuovo Cimento 2, 119 (1970).

(8) - S. Gartenhaus and C. Schwartz, Phys. Rev. 108, 482 (1957); A. Małecki and P. Picchi, Lett. Nuovo Cimento 14, 390 (1975).

(9) - H. Palevsky et al., Phys. Rev. Lett. 18, 1200 (1967).

(10) - S.D. Baker et al., Phys. Rev. Lett. 32, 839 (1974).

(11) - J. Berger et al., Saclay-Caen-Frascati collaboration, private communication.

(12) - E. Aslanides et al., Saclay-Strasbourg collaboration, private communication.

(13) - S.L. Verbeck et al. Phys. Lett. 59 B, 339 (1975).

(14) -R.F. Frosch et al., Phys. Rev. 160, 874 (1967).

(15) - R. Dymarz and A. Małecki, Frascati report LNF-76/17, (1976).

(16) - B.C. Clark et al., Phys. Rev. C 7, 466 (1973); A.M. Saperstein, Phys. Rev. Lett. 30, 1257 (1973).

(17) - K.M. Watson, Phys. Rev. 89, 575 (1953); Phys. Rev. 105, 1388 (1957).

(18) - A. Małecki, P. Picchi and R. Dymarz, Lett. Nuovo Cimento 12, 101 (1975).

(19) - D. Harrington, Phys. Rev. 184, 1745 (1969); J.M. Eisenberg; Ann. of Phys. 71, 542 (1972).

(20) - M. Błeszyński and T. Jaroszewicz, Phys. Lett. 56 B, 427 (1975); S.A. Gurwitz, Y. Alexander and A.S. Rinat, Ann. of Phys., 93, 152 (1975).