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Hisakazu Inagaki : STRUCTURE FUNCTIONS IN DEEP INELASTIC SCATTERING AND IN DEEP INELASTIC ANNIHILATION.

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Hisakazu Inagaki[(o)]: STRUCTURE FUNCTIONS IN DEEP INELASTIC SCATTERING AND IN DEEP INELASTIC ANNIHILATION[(x)].

SUMMARY. -

Structure functions in $ep \rightarrow e + X$ are directly obtained by using the Wilson's short distance expansion and the conformal covariant light cone expansion. It is shown that Bjorken scaling is broken in general due to the existence of anomalous dimensions in the expansions. It is also shown that the Drell-Yan-West relation does not hold because of the anomalous dimension. The method is applied to $e^+e^- \rightarrow h + X$ and the one particle distribution is calculated to give

$$s \frac{d\sigma}{d\omega} \xrightarrow[\omega \to 1]{} (1-\omega)^{2l_\pi - 3},$$

where l_π is the dimension of pion field. The l_π is determined to be $l_\pi = 2.5$ from the experimental data of $e^+e^- \rightarrow \pi + X$.

1. - INTRODUCTION. -

The scaling behaviour of the deep inelastic lepton-hadron scattering structure functions has been one of the central points of much theoretical study in the past few years[(1)]. The deep inelastic region probes the light cone behaviour of the products of two current operators. Thus canonical light cone expansion of the products of currents or the light cone algebra abstracted from free quark model have been proposed. The

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structure functions are given by Fourier transforms of matrix elements of bilocal operators and truly exhibit Bjorken scaling. However we can not get the forms of structure functions themselves in this approach.

On the other hand, within the framework of renormalizable field theories, it has been shown that strict Bjorken scaling for the structure functions can not hold. Violation of scaling invariably arises as a consequence of the renormalization procedure. The rate of this scaling violation can, however, be different for different types of field theories. This scaling violation is established through the moment integral. By using the convolution theorem of Mellin transform, one can obtain the structure functions themselves from this moment integral. However one must use, as an input, the value of νW_2 at some q^2 and ω to predict the value at another q^2 and ω[(1)].

We shall propose in this paper a method of determining the structure function νW_2 itself at any value of ω and q^2. The method is based on the successive uses of operator product expansions of the short distance and of the light like distance. The results, therefore, do not depend on special models of the field theory.

In Sect. 2 the form of the structure function is obtained and the connection with the Drell-Yan-West relation[(2)] is examined. It is shown that Bjorken scaling is broken in general due to the existence of anomalous dimensions and that the Drell-Yan-West relation also does not hold because of the anomalous dimension. In Sect. 3 our method is applied to $e^+e^- \rightarrow h + X$, and the explicit form of one particle distribution function is obtained. The last section is devoted to discussions, in which the dimension of pion field is determined to be $l_\pi = 2.5$ from the experimental data of $e^+e^- \rightarrow \pi + X$.

2. - STRUCTURE FUNCTION IN THE DEEP INELASTIC SCATTERING.

2. 1. - The general form of νW_2. -

We shall first obtain the general form of the structure function by applying the conformal covariant light cone expansion[(3)]. The resulting form is very complicated but it reduces to a simple one when we take the limit $\omega \to 1$. Then we get the threshold behaviour of the structure function and examine the Drell-Yan-West relation.

Let us consider the retarded amplitude

$$T_{\mu\nu} = - \int dx \exp[iq\cdot x] \langle p | R(J_\mu(x); J_\nu(0)|p\rangle , \tag{1}$$

where R denotes the retarded commutator. We write the absorptive part of $T_{\mu\nu}$ as $W_{\mu\nu}$ which is decomposed into

$$W_{\mu\nu} = (q_\mu q_\nu - g_{\mu\nu} q^2)V_L + \left\{(p_\mu q_\nu + p_\nu q_\mu)p\cdot q - p_\mu p_\nu q^2 - g_{\mu\nu}(p\cdot q)^2\right\}V_2, \tag{2}$$

where V_L and V_2 are related to W_1 and W_2 by

$$V_L = \frac{1}{q^2}\left\{W_1 + \frac{\nu^2}{q^2} W_2\right\} , \qquad V_2 = - \frac{1}{m^2 q^2} W_2 , \tag{3}$$

and m is the mass of proton. If Bjorken scaling is exact, the structure functions scale as

$$W_1 \to \frac{1}{m} F_1(\omega) , \qquad \nu W_2 \to F_2(\omega) , \tag{4}$$

with $\omega = 2m\nu/(-q^2)$. To obtain the structure function itself, one more step is needed. By using the usual technique of the reduction formula, the connected part of $T_{\mu\nu}$ can be written as[(4)]

$$T_{\mu\nu} = \int d^4x \exp[iq\cdot x] A_{\mu\nu}(x, 0:p) , \tag{5}$$

and

$$A_{\mu\nu}(x, 0:p) = \int d^4z \exp[i p\cdot z] \langle 0|R(J_\mu(x); J_\nu(0)S(z))|p\rangle, \qquad (6)$$

where S(z) stands for a scalar source. In order not to complicate the discussions with inessential kinematics we consider the scalar proton.

A triple operator product $R(J_\mu(x); J_\nu(0)S(z))$ appears in eq. (6). We need a little care when we expand this triple operator product. Because the operator product expansion

$$J_\nu(0)S(z) = \sum_{L=1} \left(\frac{1}{z^2}\right)^{\frac{1}{2}\alpha_L} z_{\beta_1} \cdots z_{\beta_{L-1}} O^{(L)}_{\nu\beta_1\cdots\beta_{L-1}}(0), \qquad (7)$$

where $\alpha_L = 2 + D - l_L + L$, $D = \dim S$ and $l_L = \dim O^{(L)}$, converges weakly, the formula

$$R(J_\mu(x); J_\nu(0)S(z)) = \sum_{L=1} \left(\frac{1}{z^2}\right)^{\frac{1}{2}\alpha_L} z_{\beta_1} \cdots z_{\beta_{L-1}} R(J_\mu(x); O^{(L)}_{\nu\beta_1\cdots\beta_{L-1}}(0)) \qquad (8)$$

holds only if x remains independent of the limit $z \to 0$ (i.e. $z \ll x$)[5]. From eq. (5) we see that an important region of the integrand, when $\nu \to \infty$, $q^2 \to -\infty$ and ω = fixed, is in the neighbourhood of the light cone $x^2 \to 0$. Then the condition $z \ll x$ is really satisfied if we perform the light cone expansion for $R(J_\mu(x); O^{(L)}_{\nu\beta_1\cdots\beta_{L-1}}(0))$. Substituting eq. (7) into eq. (6), we have

$$A_{\mu\nu}(x, 0:p) = \sum_{L=1}^{\infty} \langle 0|R(J_\mu(x); O^{(L)}_{\nu\beta_1\cdots\beta_{L-1}}(0)|p\rangle c^{(L)} p_{\beta_1}\cdots p_{\beta_{L-1}} \qquad (9)$$

with $c^{(L)} = 2^{L-1}(m^2)^{\frac{1}{2}(D-l_L-L)}$. We apply the conformal covariant light cone expansion[3] to the product $R(J_\mu(x); O^{(L)}_{\nu\beta_1\cdots\beta_{L-1}}(0))$ in eq. (9). In our case, by preserving the manifest gauge invariance, it takes the form

$$R(J_\mu(x);O^{(L)}_{\nu\beta_1\cdot\cdot\beta_{L-1}}(0)) \underset{x^2\to 0}{=} \left\{-\partial_\mu\partial_\nu + g_{\mu\nu}\Box\right\}\sum_m \left(\frac{1}{x^2}\right)^{\frac{1}{2}(l_L+L-\tau_m)} \cdot$$

$$\cdot\, x_{\beta_1}\cdot\cdot x_{\beta_{L-1}} x_{\mu_1}\cdot\cdot x_{\mu_m}\int_0^1 du\, u^{A'}(1-u)^{B'} O^{(m)}_{\mu_1\cdot\cdot\mu_m}(ux) + \tag{10}$$

$$+\left\{-(g_{\mu\lambda}\partial_\nu + g_{\nu\lambda}\partial_\mu)\partial_\sigma + g_{\mu\lambda}g_{\nu\sigma}\Box + g_{\mu\nu}\partial_\lambda\partial_\sigma\right\}\sum_n \left(\frac{1}{x^2}\right)^{\frac{1}{2}(l_L+L-\tau_n-2)} \cdot$$

$$\cdot\, x_{\beta_1}\cdot\cdot x_{\beta_{L-1}} x_{\alpha_1}\cdot\cdot x_{\alpha_{n-2}}\int_0^1 u^A(1-u)^B O^{(n)}_{\lambda\sigma\alpha_1\cdot\cdot\alpha_{n-2}}(ux) ,$$

where $A' = (d_m - l_L + L)/2$, $B' = (d_m - l^* - L)/2$, $A = (d_n - l_L + L)/2$ and $B = (d_n - l^* - L)/2$. Here $d_n = l_n + n$, $l^* = 4 - l_L$, $l_n = \dim O^{(n)}$ and $\tau_n = l_n - n$. Inserting eq. (10) into eq. (9) we obtain for the second term which we are interested in,

$$A_{\mu\nu}(x,0;p) = \left\{-(p_\mu\partial_\nu + p_\nu\partial_\mu)p\cdot\partial + p_\mu p_\nu\Box + g_{\mu\nu}(p\cdot\partial)^2\right\} \cdot$$

$$\cdot \sum_{Ln} c^{(L)} b^{(n)} \left(\frac{1}{x^2}\right)^{\frac{1}{2}(l_L+L-\tau_n-2)} (x\cdot p)^{L-1+n-2} \cdot \tag{11}$$

$$\cdot \int_0^1 du\, u^A (1-u)^B e^{iux\cdot p} ,$$

where $b^{(n)}$ is defined by

$$\langle 0 | O^{(n)}_{\lambda\sigma\alpha_1\cdot\cdot\alpha_{n-2}}(0) | p\rangle = b^{(n)} p_\lambda p_\sigma p_{\alpha_1} \cdots p_{\alpha_{n-2}} .$$

Substituting eq. (11) into eq. (5), one can get, after taking the imaginary part,

$$W_{\mu\nu} = \left\{ (p_\mu q_\nu + p_\nu q_\mu) p\cdot q - p_\mu p_\nu q^2 - g_{\mu\nu}(p\cdot q)^2 \right\} \sum_{L\cdot n} a^{(L,n)} \cdot$$

$$\cdot\ (q^2)^{\frac{1}{2}(L+l_L-\tau_n)-3}\ \omega^{\frac{1}{2}(L+l_L-\tau_n)-3} \int_0^{1/\omega} du\ u^{\frac{1}{2}(d_n-l_L+L)} \cdot \qquad (12)$$

$$\cdot\ (1-u)^{\frac{1}{2}(d_n-l^*-L)}\ (u-\frac{1}{\omega})^{\frac{1}{2}(l_L-L-d_n)} .$$

If we use the integral representation of Gaussian hypergeometric function and an elementary formula of it

$$\int_0^x u^{\beta-1}(1-u)^{-\alpha}(u-x)^{\gamma-\beta-1} du \cong x^{\gamma-1} F(\alpha,\beta,\gamma;x) ,$$

we get

$$W_{\mu\nu} = \left\{ (p_\mu q_\nu + p_\nu q_\mu) p\cdot q - p_\mu p_\nu q^2 - g_{\mu\nu}(p\cdot q)^2 \right\} \sum_{L\cdot n} a^{(L,n)} \cdot$$

$$\cdot\ (q^2)^{\frac{1}{2}(L+l_L-\tau_n)-3}\ \omega^{\frac{1}{2}(L+l_L-\tau_n)-3+L-1}\ (\omega-1)^{l_L-L-1} \cdot \qquad (13)$$

$$\cdot F((l_L-d_n-L)/2+1,\ (d_n-l^*-L)/2+2, l_L-L;\ 1-\frac{1}{\omega}) .$$

Comparing eq. (13) with eqs. (2) and (3), we finally have

$$\nu W_2 = \sum_{L\cdot n} a^{(L\cdot n)} (q^2)^{\frac{1}{2}(l_L+L-\tau_n)-1}\ F^{(L\cdot n)}(\omega) , \qquad (14)$$

where

$$F^{(L\cdot n)}(\omega) = \omega^{\frac{1}{2}(3L-l_L-\tau_n)-2}(\omega-1)^{l_L-L-1}\,\theta(\omega-1)\,F((l_L-d_n-L)/2+1,$$

$$(d_n-l^*-L)/2+2l_L-L\,;\,1-\frac{1}{\omega})\,.$$

Equation (14) is the exact form of structure function itself at any value of q^2 and ω. It violates the Bjorken scaling in power of q^2. This fact will be discussed in more detail in the following subsection.

2.2. - Threshold behaviour and the Drell-Yan-West relation. -

The complicated form of the structure function in eq. (14) reduces to simple one near the threshold. To see this put the exponent of q^2 to be

$$(l_L + L - \tau_n)/2 - 1 = \delta_n^L\,, \tag{15}$$

then by cancelling out L, the exponent of $(\omega-1)$ becomes

$$l_L - L - 1 = 2l_L - 3 + (-2\delta_n^L - \tau_n)\,. \tag{16}$$

Let us consider the limit $\omega \to 1$. Kinematically the lowest pole dominates in the intermediate state and, therefore, lowest spin dominates in the expansion (7), i.e., $L = 1$. Then we get $l_L = l + 1$, where l is the dimension of the field of scalar target. Considering the fact that

$$F(\alpha, \beta, \gamma; 1-\frac{1}{\omega}) \xrightarrow[\omega \to 1]{} 1\,,$$

we obtain

$$\nu W_2 \xrightarrow[\omega\to 1]{} \sum_{n=2} a^{(n)}(-q^2)^{\delta_n^1}(\omega-1)^{2\Delta-1+(-2\delta_n^1+2-\tau_n)} \tag{17}$$

with $\Delta = l - 1$. From the definition of l_L and τ_n we get

$$l_L = L + 2 + \gamma_L\,, \qquad \tau_n = 2 + \delta_n\,,$$

where γ_L and δ_n are anomalous dimensions. Then δ^1_n becomes

$$\delta^1_n = \frac{1}{2}(\gamma_1 - \delta_n) , \tag{18}$$

which leads to

$$\nu W_2 \xrightarrow[\omega \to 1]{} (\omega - 1)^{2\Delta - 1 - \gamma_1} \sum_{n=2} a^{(n)} (-q^2)^{\frac{1}{2}(\gamma_1 - \delta_n)} . \tag{19}$$

We notice that this threshold behaviour has the similar functional form of q^2 with that obtained from the renormalization group method[6].

From general arguments, it has been shown[7] that the anomalous dimension of local operator $O^{(m)}$ in Wilson's operator product expansion is positive and monotonically increasing for $m \geq 2$. We do not know any properties of anomalous dimensions for $m < 2$. However if we exploit the result of perturbation theory, we may get for the anomalous dimension of $O^{(m)}$

$$\gamma_m = \frac{g^2}{8\pi^2} \left(\frac{1}{6} - \frac{1}{m(m+1)}\right) \tag{20}$$

in pseudoscalar theory[8]. Since this γ_m is regular for $m > 0$, we would continue analytically to obtain γ_1 as

$$\gamma_1 < 0 . \tag{21}$$

Considering the fact that $\delta_n \geq 0$ for $n \geq 2$, we can conclude from eq. (19) that νW_2 decreases as $(-q^2)$ becomes large. This q^2 dependence is consistent with recent experimental observation of scale noninvariance for small ω[9]. Further we must notice that the exponent of $(\omega - 1)$ becomes greater than $2\Delta - 1$ because of eq. (21). Let us examine what it means in the following passage.

It is interesting to see first whether the threshold behaviour (19) satisfies the Drell-Yan-West relation[2] or not. For this purpose, we have to see the asymptotic behaviour of the electromagnetic form factor.

The asymptotic behaviour of the on-shell electromagnetic form factor has been shown to have the form[10]

$$F(q^2) \longrightarrow \left(\frac{1}{q^2}\right)^{\Delta} \qquad (22)$$

for large q^2. Here $\Delta = l - 1$ or $\Delta = l - 3/2$ for scalar or spinor target, respectively, and l is the dimension of the field of the target hadron. This result has first been derived by Migdal from the heuristic argument as for conformal covariant 3-point function. Ferrara, Grillo and Parisi have obtained the above result for the scalar particle by applying the conformal covariant light cone expansion to the product of an electro magnetic current and a scalar field. The validity of the result have also been verified by Menotti through the field theoretical model calculation with conformal invariance. On the other hand Drell, Yan and West[2] have suggested the following relation in the framework of the parton model. If the electromagnetic form factor behaves as in eq. (22), then the structure function νW_2 for deep inelastic scattering behaves as

$$\nu W_2 \xrightarrow[\omega \to 1]{} (\omega - 1)^{2\Delta - 1} . \qquad (23)$$

Our result (19), therefore, says that the Drell-Yan-West relation does not hold because of the existence of the anomalous dimension γ_1 in the expansion (2). This anomalous dimension comes from the compositness of source function S(y) and reflects a strong interaction inside the target hadron. The parton model of Ref. (2), however, might lead to $\gamma_1 = 0$, because such a strong interaction seems to be switched off in the infinite momentum frame. In more realistic model of hadron, it is quite plausible to have $\gamma_1 \neq 0$. Finally we note that the exponent of $(\omega - 1)$ becomes greater than 3 when the dipole fit to the electromagnetic form factor is chosen[11].

3. - ONE PARTICLE DISTRIBUTION IN INCLUSIVE e^+e^- ANNIHILATION.

The differential cross section for the inclusive one hadron production in e^+e^- annihilation

$$e^+ + e^- \longrightarrow \text{hadron} + \text{anything}$$

is calculated, in one photon approximation, as[12]

$$\frac{d^2\sigma}{dE d\Omega} = \frac{\alpha^2}{q^4} \frac{m\nu}{\sqrt{q^2}} \left(1 - \frac{q^2}{\nu^2}\right)^{\frac{1}{2}} \left[2\overline{W}_1(\nu, q^2) + \frac{\nu^2}{q^2}\left(1 - \frac{q^2}{\nu^2}\right) \sin^2\theta\, \overline{W}_2(\nu, q^2)\right]. \tag{24}$$

Here E' and m is the c. m. s. energy and mass of the detected hadron (whose spin is averaged), q is the momentum of the time like photon and ν is its energy in the rest system of the hadron. By integrating with respect to the scattering angle eq. (24) becomes

$$\frac{d\sigma}{d\omega} = \frac{2\pi\alpha^2\omega}{q^2}\left(1 - \frac{4m^2}{q^2\omega^2}\right)^{\frac{1}{2}} \left[\overline{W}_1(\nu, q^2) + \frac{\omega}{6m}\left(1 - \frac{4m^2}{q^2\omega^2}\right)^{\frac{1}{2}} \nu \overline{W}_2(\nu, q^2)\right], \tag{25}$$

with $\omega = \frac{2p\cdot q}{q^2} = \frac{2m\nu}{s}$. If we do an approximation $\frac{m^2}{q^2} \ll 1$, it gives with $s = q^2$

$$s\frac{d\sigma}{d\omega} \simeq 2\pi\alpha^2\omega \left[\overline{W}_1(\nu, q^2) + \frac{\omega}{6} \frac{\nu\overline{W}_2(\nu, q^2)}{m}\right]. \tag{26}$$

$\overline{W}_1$ and $\overline{W}_2$ are given by the decomposition of $\overline{W}_{\mu\nu}$ in the similar manner to eq. (2), but in this case $\overline{W}_{\mu\nu}$ is written as

$$\overline{W}_{\mu\nu} = \frac{1}{4\pi}\int d^4x \exp\left[iq\cdot x\right] A_{\mu\nu}(x, 0{:}p) \tag{27}$$

with

$$A_{\mu\nu}(x, 0{:}p) = \int d^4y\, d^4z \exp\left[ip\cdot(z-y)\right] \langle 0 | R(J_\mu(x)S(y))R(J_\nu(0)S^+(z)) | 0 \rangle. \tag{28}$$

Here S(y) is the source of the observed hadron and R denotes retarted commutator.

If the Bjorken scaling holds when $\nu \to \infty$, $q^2 \to \infty$, structure functions are expected to scale as[12]

$$\overline{W}_1 \to F_1(\omega), \qquad \frac{\nu \overline{W}_2}{m} \to F_2(\omega). \tag{29}$$

In eq. (28) we have the product of four operators. First we make the operator product expansion at the short distance as

$$R(J_\nu(0)S^+(z)) = \sum_{L=1} \left(\frac{1}{z^2}\right)^{\frac{1}{2}(3+D-l_L+L-1)} z_{\beta_1}\cdot\cdot z_{\beta_{L-1}} O^{(L)}_{\nu\beta_1\cdot\cdot\beta_{L-1}}(0), \tag{30}$$

where D and l_L are the dimension of $S^+(z)$ and $O^{(L)}(0)$, respectively. Substituting eq. (30) into eq. (28) we have

$$A_{\mu\nu}(x, 0;p) = \sum_L c^{(L)} \int d^4y \exp\left[-ip\cdot y\right] \langle 0 | R(J_\mu(x)S(y)) \cdot$$
$$\cdot O^{(L)}_{\nu\beta_1\cdot\cdot\beta_{L-1}}(0)|0\rangle p_{\beta_1}\cdot\cdot p_{\beta_{L-1}} . \tag{31}$$

R denotes the retarded product

$$R(J_\mu(x)S(y)) = -i\,\theta(x_o-y_o)(J_\mu(x)S(z) - S(y)J_\mu(x)), \tag{32}$$

and the successive operator product $J_\mu(x)O^{(L)}_{\nu\beta_1\cdot\cdot\beta_{L-1}}(0)$ has to be expanded. In eq. (27) the light cone $x^2 \to 0$ is dominating when $\nu \to \infty$, $q^2 \to \infty$ and ω = fixed. Then we may safely have the condition $z \ll x$ for the convergence of triple operator product $J_\mu(x)J_\nu(0)S^+(z)$. We shall perform the conformal covariant light cone expansion for this product as

$$J_\mu(x)O^{(L)}_{\nu\beta_1\cdot\cdot\beta_{L-1}}(0) \underset{x^2\to 0}{=} \left\{-(\partial_\mu\partial_\nu + g_{\mu\nu}\Box)\right\} \sum_{m=0} \left(\frac{1}{x^2}\right)^{\frac{1}{2}(3+l_L-l_m+m-2)} \cdot$$
$$\cdot x_{\mu_1}\cdot\cdot x_{\mu_m} \int_0^1 du\, u^{A'}(1-u)^{B'} O^{(m)}_{\mu_1\cdot\cdot\mu_m}(ux) + \tag{33}$$

$$+\left\{-(g_{\mu\lambda}\partial_\nu+g_{\nu\lambda}\partial_\mu)\partial_\sigma+g_{\mu\lambda}g_{\nu\sigma}\Box+g_{\mu\nu}\partial_\lambda\partial_\sigma\right\}\cdot$$

$$\cdot\sum_{n=2}\left(\frac{1}{x^2}\right)^{\frac{1}{2}(3+l_L-l_n+L-1+n-2-2)} x_{\beta_1}\cdot\cdot x_{\beta_{L-1}} x_{\alpha_1}\cdot\cdot x_{\alpha_{n-2}}\cdot \tag{33}$$

$$\cdot\int_0^1 du\, u^A(1-u)^B\, O^{(n)}_{\lambda\sigma\alpha_1\cdot\cdot\alpha_{n-2}}(ux)\ ,$$

where $A=\frac{1}{2}(d_n-l_L+L)$, $B=\frac{1}{2}(d_n-l^*-L)$, $d_n=l_n+n$, $l^*=4-l_L$, $l_n=\dim O^{(n)}$ and $l_L=\dim O^{(L)}$. Finally we perform the short distance expansion again,

$$S(y)\,O^{(n)}_{\lambda\sigma\alpha_1\cdot\cdot\alpha_{n-2}}(ux)=\left(\frac{1}{(y-ux)^2}\right)^{\frac{1}{2}(D+l_n+n)}(y-ux)^\lambda\cdot$$

$$\cdot\,(y-ux)^\sigma\ldots\ (y-ux)^{\alpha_1}\ldots(y-ux)^{\alpha_{n-2}}I(ux)\ , \tag{34}$$

where only the unity operator is explicitly written, because we are only interested in the vacuum expectation value of these operators. Therefore there are no problems here as for convergence of the infinit series. Inserting eqs(33) and (34) into eq. (31) we have

$$A_{\mu\nu}(x,0;p)=\sum_{L=1}\sum_{n=2}c^{(L)}\left\{-(p_\mu\partial_\nu+p_\nu\partial_\mu)p\cdot\partial+p_\mu p_\nu\Box+\right.$$

$$\left.+g_{\mu\nu}(p\cdot\partial)^2\right\}\left(\frac{1}{x^2}\right)^{\frac{1}{2}(l_L+L-\tau_n-2)}(x\cdot p)^{L-1}(x\cdot p)^{n-2}\ \cdot \tag{35}$$

$$\cdot\int_0^1 du\, u^A(1-u)^B\, e^{-ip\cdot ux}+\ldots\ldots\ ,$$

where $\tau_n=l_n-n$ and dots stand for less singular terms. Substituting eq. (35) into eq. (27) and performing the similar calculations which lead to eqs. (13) and (14), we finally get

$$\frac{\nu \overline{W}_2}{m} = \sum_{L=1}^{\infty} \sum_{n=2}^{\infty} a^{(L,n)} (q^2)^{\frac{1}{2}(l_L+L-\tau_n)-1} \omega^{\frac{1}{2}(3L-l_L-\tau_n)-2} \cdot \tag{36}$$

$$\cdot (1-\omega)^{l_L-L-1} \theta(1-\omega) F((l_L-d_n-L)/2+1, (d_n-l^*-L)/2+2, l_L-L; 1-\frac{1}{\omega}).$$

This form is quite the same with eq. (14). Then we have the same threshold behaviour for $\omega \to 1$ with (19),

$$\frac{\nu \overline{W}_2}{m} \xrightarrow[\omega \to 1]{} (1-\omega)^{2\Delta-1-\gamma'_l} \sum_{n=2} a_n (q^2)^{\frac{1}{2}(\gamma'_l - \delta'_n)} , \tag{37}$$

with $\Delta = l - 1$, l being the dimension of observed hadron. Here γ'_l and δ'_n are anomalous dimensions of $O^{(L)}$ and $O^{(n)}$, respectively, in the expansion (30) and (33).

Provided that Bjorken scaling holds in $e^+e^- \to \pi$ + anything, then we may have $\delta'^L_n = \frac{1}{2}(l_L+L-\tau_n)-1 = 0$. Especially we have $\gamma'_1 = \delta'_n$ for any n, which says that δ'_n are same for all n. Therefore γ'_l becomes zero because we can have $\delta'_n = 0$ for $n = 2$. From eq. (37)

$$\frac{\nu \overline{W}_2}{m} \xrightarrow[\omega \to 1]{} (1-\omega)^{2l_\pi - 3} , \tag{38}$$

where l_π is the dimension of pion field. Further we obtain the structure function in the other limit $\omega \to 0$, for $\delta'^L_n = 0$, as

$$\frac{\nu \overline{W}_2}{m} \xrightarrow[\omega \to 0]{} \sum_{L=1} \sum_{n=2} a^{(L,n)} \omega^{-4+(2-n)-\frac{1}{2}(\gamma'_L+\delta'_n)} , \tag{39}$$

where we have used the formula

$$F(\alpha', \beta', \gamma', 1-\frac{1}{\omega}) \xrightarrow[\omega \to 0]{} \omega^{\alpha'} .$$

When the leading light cone singularity is taken out, i.e., n = 2 we get $\delta'_n = 0$, then

$$\frac{\nu \overline{W}_2}{m} \xrightarrow[\omega \to 0]{} \omega^{-4} \sum_{L=1} a^{(L)} \omega^{-\frac{1}{2}\gamma'_L} . \qquad (40)$$

Thus, when Bjorken scaling hold in $e^+e^- \to \pi$ + anything, both limits of structure function, that is to say, eqs. (38) and (40) reduce to simple forms. On the next section we shall determine l_π, by using eq. (38), from the recent experimental information in $e^+e^- \to \pi$ + anything.

4. - DISCUSSIONS.

In this section we shall discuss how to determine the dimension of pion field l_π[14]. The value of l_π is also the dimension of chiral symmetry breaking Hamiltonian[15]. It is, therefore, very interesting to determine l_π from experiments[16]. Formal arguments only tell us that $1 \lesssim l_\pi \lesssim 4$[15].

Recent experiments at SLAC[17] have reported evidence for the jet structure in $e^+e^- \to$ hadrons at the center-of-mass energies of 6.2 and 7.4 GeV. This jet structure strongly supports the quark-parton model[18]. The experiments, further, have decleared that the partons must have spin 1/2 from the angular distribution of the cross section. We can, therefore, conclude that $e^+e^- \to \pi + X$ will scale asymptotically according to the prediction of the quark-parton model, so eqs. (38) and (40) would really be correct statements. From spin 1/2 nature of partons, we have

$$\overline{W}_1 = \frac{\omega}{2} \frac{\nu \overline{W}_2}{m} , \qquad (41)$$

which leads to

$$s \frac{d\sigma}{d\omega} \xrightarrow[\omega \to 1]{} (1-\omega)^{2l_\pi - 3} , \qquad (42)$$

by using eqs. (26) and (38), and leads to

$$s\frac{d\sigma}{d\omega} \xrightarrow[\omega\to 0]{} \omega^{-2} \sum_{L=1} a^{(L)} \omega^{-\frac{1}{2}\gamma'_L} \quad , \tag{43}$$

by using eqs. (26) and (40). The present experiment of $e^+e^- \to \pi + X$ surely exhibit scaling for $1 > \omega \geq 0.5$[19] and we could parametrize $s\frac{d\sigma}{d\omega} \xrightarrow[\omega\to 1]{} (1-\omega)^2$ as for it[20]. We can, therefore, determine l_π to be 2.5 from eq. (42). Thus our method enables us to determine l_π with the help of experimental information. Such a value of l_π would play the very important role in study of the broken symmetry.

As a final remark we note that our main tools have been the operator product expansions with scale and conformal invariance. Therefore the results do not depend on special models of the field theory.

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