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G. Matone and A. Tranquilli: LASER LIGHT MODULATION: THE ELECTROOPTIC EFFECT. -

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ABSTRACT. -

In the backward scattering of laser light against high energy electrons, the monochromaticity of the final photon beam is strongly dependent on the angular divergency of the electron beam. This point has been discussed in detail in the previous reports published by the LADON group.

In the middle part of the Adone straight section this crucial parameter can be extremely good ($\sim 2 \times 10^{-5}$) but in the quadrupoles region it cannot be better than $\sim 10^{-4}$. In order to avoid the interaction to take place in these regions the possibility to bunch the laser beam has been explored. This paper will present a discussion on the light modulation as based on the electrooptic effect: special design of modulators will be presented as well.

1. - INTRODUCTION. -

An electromagnetic wave propagating in a crystal causes an induced polarization $\overline{P}$ which is related to the wave electric field $\overline{A}$ by these general expressions:

$$P_x = \varepsilon_o(\chi_{11} A_x + \chi_{12} A_y + \chi_{13} A_z)$$
$$P_y = \varepsilon_o(\chi_{21} A_x + \chi_{22} A_y + \chi_{23} A_z)$$
$$P_z = \varepsilon_o(\chi_{31} A_x + \chi_{32} A_y + \chi_{33} A_z)$$

ε_o = vacuum electric permeability

The electric susceptibility tensor χ_{ij} and the electric permeability tensor $\varepsilon_{ij} = \varepsilon_o(1+\chi_{ij})$ depend upon the choice of the x, y, z axes and in particular when they coincide with the principal dielectric axes of the crystal, the off-diagonal elements vanish and one has:

$$P_x = \varepsilon_o \chi_{11} A_x, \qquad P_y = \varepsilon_o \chi_{22} A_y, \qquad P_z = \varepsilon_o \chi_{33} A_z$$

In an isotropic medium $\chi_{11} = \chi_{22} = \chi_{33}$ and the phase velocity of the electromagnetic wave $c = \sqrt{\mu\varepsilon}$ does not depend on its polarization direction relative to the crystal axes. On the contrary, for a wave propagating along the z-axis in an anisotropic medium, the two x and y-components will travel in general with different phase velocities: $c_x = \sqrt{\mu\,\varepsilon_{11}}$ and $c_y = \sqrt{\mu\,\varepsilon_{22}}$ respectively. It is a standard procedure to introduce the usual index ellipsoid equation (or indicatrix):

$$(1) \qquad \frac{x^2}{n_x^2} + \frac{y^2}{n_y^2} + \frac{z^2}{n_z^2} = 1 \quad \text{where} \quad \begin{cases} n_x = (\varepsilon_{11}/\varepsilon_o)^{1/2} \\ n_y = (\varepsilon_{22}/\varepsilon_o)^{1/2} \\ n_z = (\varepsilon_{33}/\varepsilon_o)^{1/2} \end{cases}$$

which defines a general ellipsoid with axes parallel to the principal dielectric axes and whose respective semi-lenghts are the principal refractive indices n_x, n_y, n_z. A plane passing through the origin and normal to an arbitrary wave propagation direction, intersects such an allipsoid along an allipse. The two semi-axes give the refraction indices n_1 and n_2 along the correspondent directions and therefore the phase velocities are determined as c_o/n_1 and c_o/n_2 respectively ($c_o = \sqrt{\varepsilon_o\,\mu_o}$ is the phase velocity in vacuum).

2. - THE ELECTROOPTIC EFFECT. -

The linear electrooptic effect (known as "Pockel effect") is the change in the refraction indices caused by an applied electric field $\bar{E}(E_x, E_y, E_z)$. In this case, following convention[1], the equation (1) can be generalized as follows:

$$(2) \qquad \left(\frac{1}{n^2}\right)_1 x^2 + \left(\frac{1}{n^2}\right)_2 y^2 + \left(\frac{1}{n^2}\right)_3 z^2 + 2\left(\frac{1}{n^2}\right)_4 yz + 2\left(\frac{1}{n^2}\right)_5 xz + 2\left(\frac{1}{n^2}\right)_6 xy = 1$$

where the coefficients $\left(\frac{1}{n^2}\right)_i$ are given by:

$$\left(\frac{1}{n^2}\right)_1 = \frac{1}{n_x^2} + \Delta\left(\frac{1}{n^2}\right)_1 = \frac{1}{n_x^2} + \sum_1^3{}_j\, r_{1j} E_j$$

$$\left(\frac{1}{n^2}\right)_2 = \left(\frac{1}{n_y^2}\right) + \Delta\left(\frac{1}{n^2}\right)_2 = \frac{1}{n_y^2} + \sum_1^3{}_j\, r_{2j} E_j$$

$$\left(\frac{1}{n^2}\right)_3 = \left(\frac{1}{n_z^2}\right) + \Delta\left(\frac{1}{n^2}\right)_3 = \frac{1}{n_z^2} + \sum_1^3{}_j\, r_{3j} E_j$$

$$\left(\frac{1}{n^2}\right)_i = \Delta\left(\frac{1}{n^2}\right)_i = \sum_1^3{}_i\, r_{ij} E_j \qquad i = 4, 5, 6$$

In the summation over j we used the convention $x = 1, y = 2, z = 3$. The linear change $\Delta(1/n^2)_i$ $i = 1, 2, \ldots, 6$ can be expressed in a matrix form as follows:

$$\begin{vmatrix} \Delta\left(\frac{1}{n^2}\right)_1 \\ \\ \\ \Delta\left(\frac{1}{n^2}\right)_6 \end{vmatrix} = \begin{vmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ \\ r_{61} & r_{62} & r_{63} \end{vmatrix} \begin{vmatrix} E_x \\ E_y \\ E_z \\ \end{vmatrix}$$

The (6 x 3) matrix with elements r_{ij} is called the electrooptic tensor.

Let us consider the following examples of uniaxial crystals:

2.1.- Crystal KH_2PO_4 (KDP).-

The electrooptic tensor in this case is[1]

$$r_{ij} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ r_{41} & 0 & 0 \\ 0 & r_{41} & 0 \\ 0 & 0 & r_{63} \end{vmatrix}$$

and the equation (2) becomes:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\, r_{41} E_x yz + 2 r_{41} E_y xz + 2\, r_{63} E_z xy = 1$$

where the constants involved in the first three terms do not depend on the applied field and, since the crystal is uniaxial, are taken as $n_x = n_y = n_o$, $n_z = n_e$.

When $E_x = E_y = 0$ one gets:

$$\frac{x^2}{n_o^2} + \frac{y^2}{n_o^2} + \frac{z^2}{n_e^2} + 2\, r_{63}\, E_z\, xy = 1$$

and this quadratic expression may be reduced in the canonical form (1) by rotating of $\pi/4$ the reference system along the z-direction as shown in Fig. 1a:

$$\frac{x'^2}{(n_o + \Delta n)^2} + \frac{y'^2}{(n_o - \Delta n)^2} + \frac{z^2}{n_e^2} = 1$$

where:

$$\Delta n = \frac{r_{63} E_z n_o^3}{2}$$

and

$$r_{63} E_z \ll \frac{1}{n_o^2}$$

2.2. - Crystal $LiTaO_3$. -

The electrooptic tensor in this case is[1]:

$$r_{ij} = \begin{vmatrix} r_{11} & 0 & r_{13} \\ -r_{11} & 0 & r_{13} \\ 0 & 0 & r_{33} \\ 0 & r_{42} & 0 \\ r_{42} & 0 & 0 \\ 0 & -r_{11} & 0 \end{vmatrix}$$

and when $E_x = E_y = 0$, the equation (2) becomes

$$\left(\frac{1}{n_o^2} + r_{13} E_z\right) x^2 + \left(\frac{1}{n_o^2} + r_{13} E_z\right) y^2 + \left(\frac{1}{n_e^2} + r_{33} E_z\right) z^2 = 1$$

Note that this is already in its canonical form (see Fig. 2a, 2b)

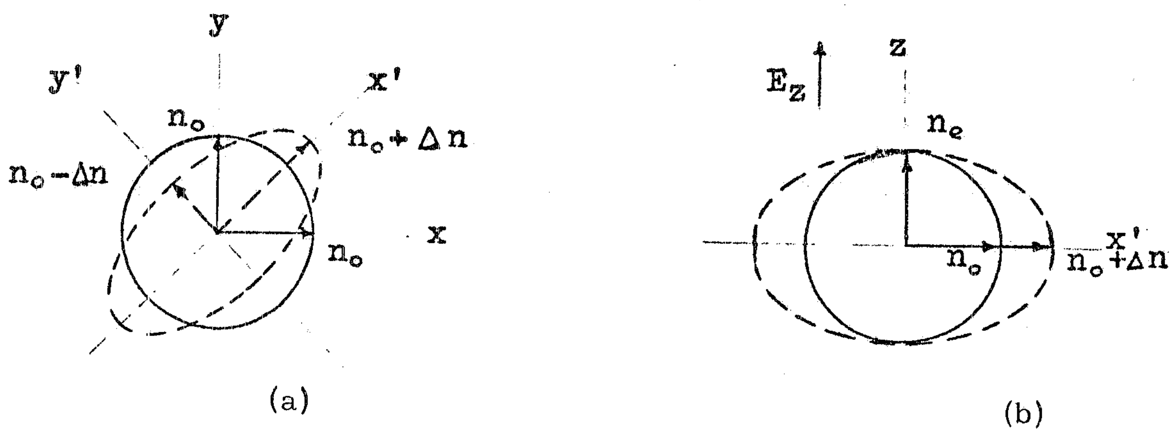

FIG. 1 - Indicatrix before (solide line) and after (dashed line) a field is applied. Indicatrix of KDP when a field is applied along the z-axis.

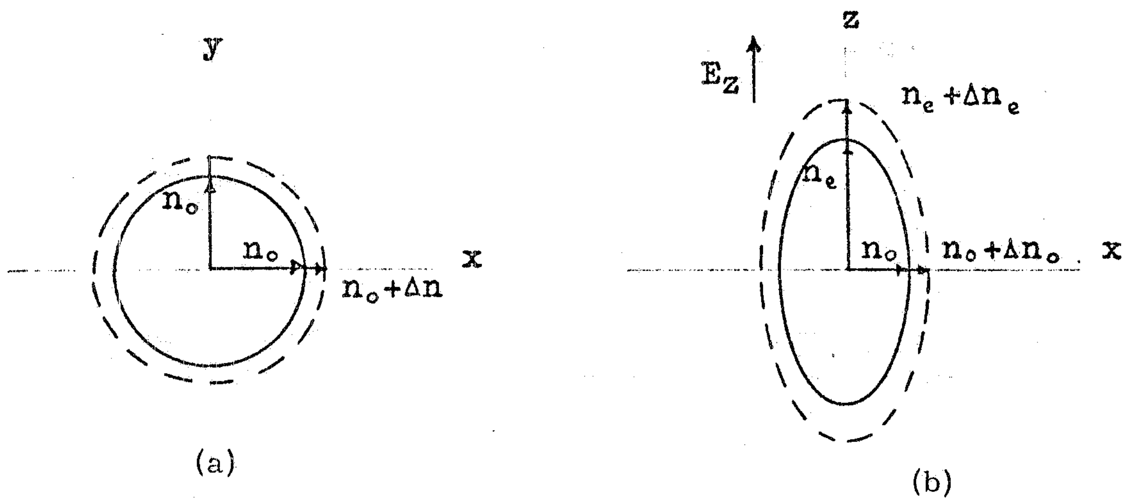

FIG. 2 - Indicatrix of $LiTaO_3$ when a field is applied along the z-axis.

(3)
$$\frac{x^2}{(n_o+\Delta n_o)^2} + \frac{y^2}{(n_o+\Delta n_o)^2} + \frac{z^2}{(n_e+\Delta n_e)^2} = 1$$

where:

$$\Delta n_o = -\frac{n_o^3}{2} r_{13} E_z \qquad r_{13} E_z \ll \frac{1}{n_o^2}$$

and

$$\Delta n_e = -\frac{n_e^3}{2} r_{33} E_z \qquad r_{33} E_z \ll \frac{1}{n_e^2}$$

3.- ELECTROOPTIC RETARDATION.-

3.1.-Longitudinal electrooptic effect.-

The electrooptic effect is called longitudinal when the electric field is applied along the optical path of the travelling wave inside the crystal. In particular when this direction is the z-axis in a KDP, (see Fig. 1a), the x' and y' electric components of wave polarized along the y-axis, propagate as

$$A_{x'} = A e^{i[\omega t - \frac{\omega}{c_o} n_{x'} z]} = A e^{i\{\omega t - \frac{\omega}{c_o}[n_o + \frac{n_o^3}{2} r_{63} E_z] z\}}$$

$$A_{y'} = A e^{i[\omega t - \frac{\omega}{c_o} n_{y'} z]} = A e^{i\{\omega t - \frac{\omega}{c_o}[n_o - \frac{n_o^3}{2} r_{63} E_z] z\}}$$

At an arbitrary output plane z=1, the phase difference between the two components is called the "retardation" and is given by:

(4)
$$\Gamma = \frac{\omega n_o^3 r_{63} V}{c_o} \qquad V = E_z 1$$

Furthermore in terms of the "half wave voltage"

$$V_\pi = \frac{\lambda_o}{2 n_o^3 r_{63}} \qquad \lambda_o = \frac{2\pi c_o}{\omega} \text{ (free space wavelengths)}$$

which yields a retardation $\Gamma = \pi$, the expression (4) can be written as:

$$\Gamma = \pi \frac{V}{V_\pi}$$

3.2.- Transverse electrooptic effect.-

In the transverse electrooptic effect, the applied field is normal to the light path.

If the wave propagates in the same KDP crystal with $\bar{E}$ applied along the z -direction and its polarization lies in the x'-z plane at $\pi/4$ from the z-axis, the retardation one gets at z = l is (see Fig. 1b):

$$\Gamma = \frac{\omega l}{c_o} \left[(n_o - n_e) + \frac{n_o^3}{2} r_{63} E_z \right] \qquad V = E_z d$$

where d is the crystal size along the applied field direction.

When $\bar{E}$ is high enough for the static birifrangence term $(n_o - n_e)$ to be neglected, in first approximation one has:

(5) $$V_\pi = \frac{\lambda_o}{n_o^3 r_{63}} \frac{d}{l} \qquad \Gamma = \pi V \frac{n_o^3 r_{63}}{\lambda_o} \left(\frac{l}{d}\right)$$

Instead for the $LiTaO_3$ crystal, when the electric field is along the z-direction, it turns out from eq. (3) and Figs. 2a, 2b that only the transverse effect is present. Furthermore by neglecting the static birifrangence term, the half wave voltage comes out to be:

$$V_\pi = \frac{\lambda_o}{n_e^3 r_{33} - n_o^3 r_{13}} \frac{d}{l}$$

4.- ELECTROOPTIC AMPLITUDE MODULATION.-

Fig. 3 shows a typical electrooptic amplitude modulator. It consists of an electrooptic crystal placed between two crossed polarizers which, in turn, are at an angle of $\pi/4$ with respect to the electrically induced birifrangence axes x' and y'. This arrangement can be achieved using a KDP crystal. Also included in the optical path is a naturally birifrangence crystal that introduces a fixed retardation Γ_b, so the total retardation Γ_t is the sum of the retardation due to this crystal and the electrically induced one.

The incident electric field $\bar{A}$ is parallel to x at the input face of the crystal and hence, having equal components along x' and y', we can take them as:

$$A_{x'}(z=0) = A_{y'}(z=0) = B \quad \text{and} \quad I_i = |A_{x'}|^2 + |A_{y'}|^2 = 2B^2$$

upon emerging from the output face z = 1, the x' and y' components have acquired, according to section 3) a relative retardation of Γ_t radiants

$$A_{x'}(z=1) = Be^{i\varphi} \qquad A_{y'}(z=1) = Be^{i\varphi}\, e^{-i\Gamma_t}$$

The common phase-angle φ is related to the optical path 1 inside the crystal.

The total field emerging from the output polarizer is the sum of the y-components of $A_{x'}(1)$ and $A_{y'}(1)$

$$A_y = \frac{B}{\sqrt{2}}\,(e^{-i\Gamma_t} - 1)\, e^{i\varphi}$$

that corresponds to an output intensity

$$I_o = A_y \cdot A_y^{*} = \frac{B^2}{2}\,(e^{-i\Gamma_t} - 1)(e^{i\Gamma_t} + 1) = 2B^2 \,\mathrm{sen}^2 \frac{\Gamma_t}{2}$$

and its ratio with respect to the incident one is:

$$\frac{I_o}{I_i} = \mathrm{sen}^2 \frac{\Gamma t}{2} = \mathrm{sen}^2 \left(\frac{\Gamma b}{2} + \frac{\pi}{2}\, \frac{V}{V\pi}\right)$$

At zero-bias, a small sinusoidal modulation voltage

$$V = V_m \,\mathrm{sen}\, \omega_m t$$

would cause only a small nearly sinusoidal modulation of the transmitted intensity as shown in Fig. 4a.

On the other hand the presence of a fixed retardation will act as an amplifier of such a modulation.

As a matter of fact when the inserted bias is a quarter wave plate one has:

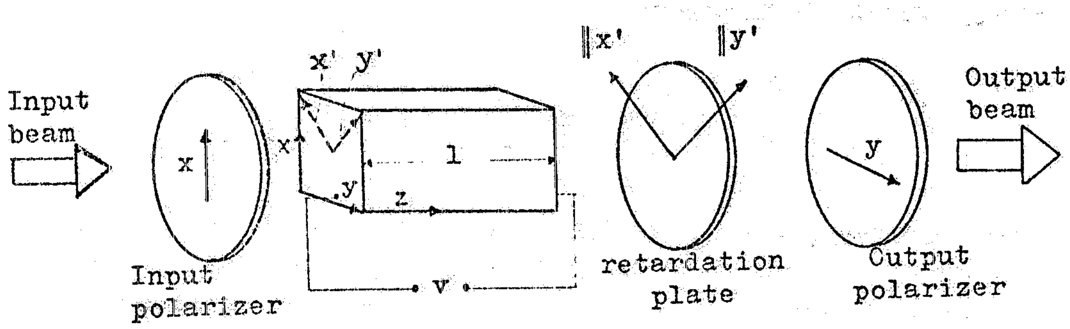

FIG. 3 - A typical electrooptic modulator. The total retardation is the sum of fixed retardation bias introduced by the "quarter-wave" plate and that attributable to the electrooptic crystal.

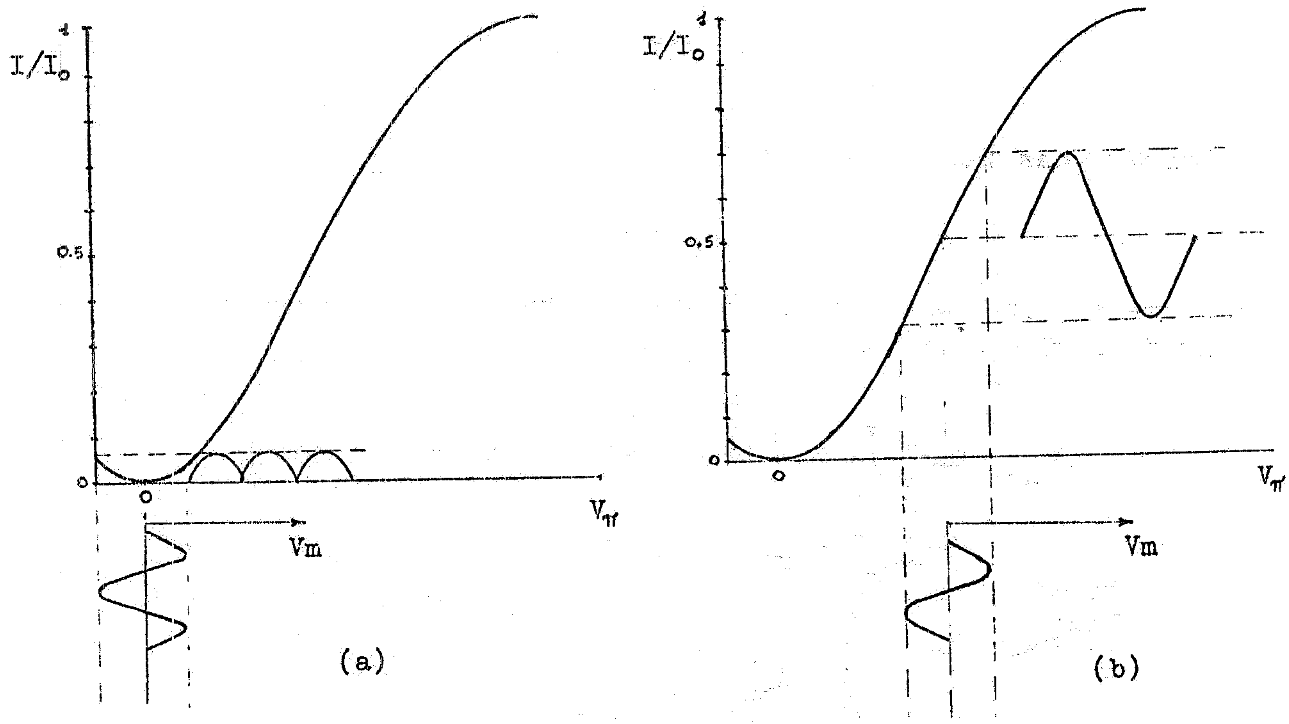

FIG. 4 - Relative intensity vs crystal voltage for (a) zero bias and (b) quarter-wave bias.

$$\frac{I_o}{I_i} = \operatorname{sen}^2\left(\frac{\pi}{4} + \frac{\pi}{2V_\pi} V_m \operatorname{sen}\omega_m t\right) = \frac{1}{2}\left[1+\operatorname{sen}\left(\pi \frac{V_m}{V_\pi} \operatorname{sen}\omega_m t\right)\right]$$

that for $\pi(V_m/V\pi) \ll 1$ becomes

$$\frac{I_o}{I_i} \simeq \frac{1}{2}\left(1+\pi \frac{V_m}{V_\pi} \operatorname{sen}\omega_m t\right)$$

This expression shows that with the inserted bias, the starting point of the transmitted modulation has been pushedup to 50% of the incident intensity value and represents a direct linear replica of the tuning voltage $V_m \operatorname{sen}\omega_m t$ (see Fig. 4b).

5.- ELECTROOPTIC PHASE MODULATION.-

In Fig. 5 a situation is shown where the incident beam is polarized along the x' axis. The polarization is unaffected by the applied field, whereas the phase, after the optical path l, changes by the amount:

$$\Delta\phi = -\frac{\omega n_o^3 r_{63}}{2c_o} E_z l$$

when the bias field is sinusoidal

$$E_z = E_m \operatorname{sen}\omega_m t$$

and we neglect the initial phase factor, the wave-electric field emerging from the output face (z = l) is

$$A_{out} = B\cos\left[\omega t - \frac{\omega}{c_o}\left(n_o - \frac{n_o^3}{2} r_{63} E_m \operatorname{sen}\omega_m t\right) l\right]$$

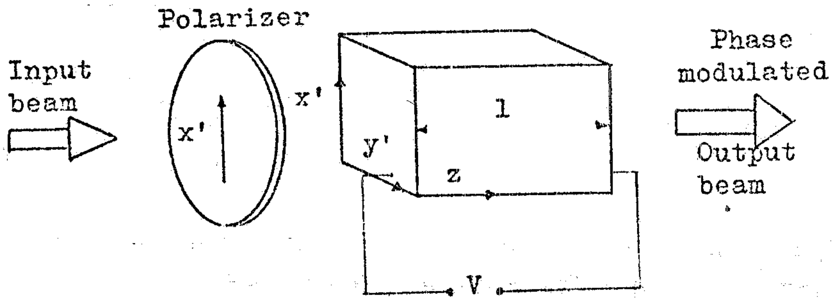

FIG. 5 - An electrooptic phase modulator. The orientation and applied directions are appropiate to KDP.

Dropping off the unessential phase-constant as well, the last expression can be rewritten as follows:

$$A_{out} = B\cos\left[\omega t + \delta \,\mathrm{sen}\, \omega_m t\right]$$

where

$$\delta = \frac{\pi n_o^3 r_{63} E_m l}{\lambda_o}$$

is referred to as the phase modulation index.

The optical field is thus phase-modulated with a modulation index δ .

6.- MODULATORS DESIGN.-

According to section 3, for the transverse electrooptic effect, the half wave voltage is proportional to the geometrical factor d/l . Thus the optimum geometry is obtained when the aperture of the crystal is just large enough for the beam to pass through the sample.

In general the beam evolution inside the medium where it propagates, is usually described by the following complex parameter[2, 3]

(6)
$$\frac{1}{q} = \frac{1}{R} - i \frac{\lambda}{\pi w^2}$$

where R is the curvature radius of the beam front wave and w is the beam radius measured to point where the field is 1/e its value on the beam axis. In particular at the minimum spot-size the radius is named waist (w_o). Following ABCD law[2, 3], the q-value in a generic point of the beam line, can be expressed through its value at the waist q_o, and the transport matrix elements A, B, C, D:

(7)
$$q = \frac{Aq_o + B}{Cq_o + D}$$

For a modulator with refraction index n, the transport matrix is

(8)
$$\begin{Vmatrix} 1 & z/n \\ 0 & 1 \end{Vmatrix}$$

and from equations (6), (7), (8), one has:

(9) $$q = q_o + \frac{z}{n} \qquad w^2(z) = w_o^2 \left[1 + \left(\frac{\lambda z}{\pi n w_o^2}\right)^2\right]$$

On the other hand, it has been shown that to minimize the diffraction losses, for fixed waist the maximum crystal length l is given by the following relation[4]

(10) $$l = 2\pi \frac{w_o^2 n}{\lambda}$$

By focousing the beam into the cylinder centre of the modulator, the beam outline appears as the sketch shown in Fig. 6. Consequently, from eqs. (9) and (10), the beam dimensi on at the two and-faces is $2\sqrt{2}\,w_o$ and therefore:

$$d = 2\sqrt{2}\, s\, w_o \qquad \frac{d}{l} = \frac{\sqrt{2}\,\lambda s}{\pi n w_o}$$

where s is a safety factor in general greater than 1.

Identical considerations, applied to the case shown in Fig. 7 yield

$$d = 2\sqrt{5}\, s\, w_o \qquad \frac{d}{l} = \frac{\sqrt{5}\,\lambda s}{\pi n w_o}$$

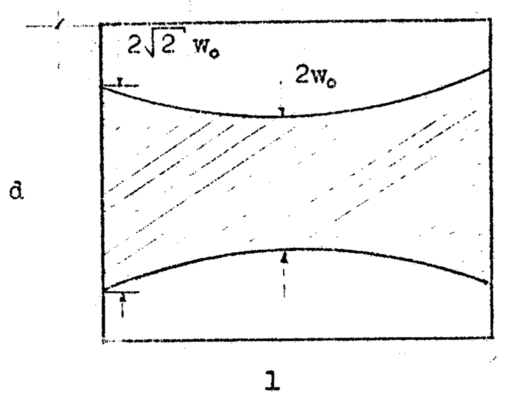

FIG. 6 - Propagation of a gaussian beam within the modulator.

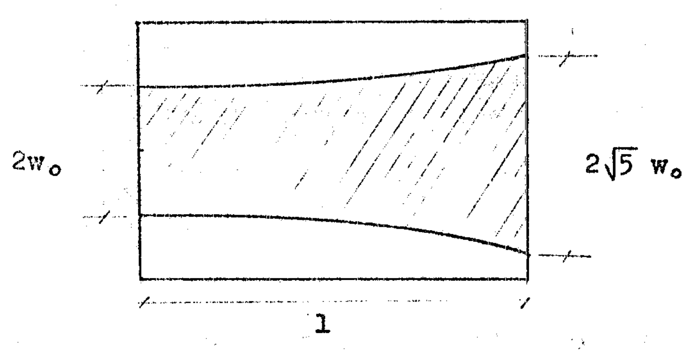

FIG. 7 - Beam profile.

7.- EFFECT OF VELOCITY MATCHING.-

In order to have a cumulative interaction between the modulating wave and the light wave, the component of the modulating wave group velocity in the direction of the light propagation must be equal to the group velocity of the light beam.

The group velocity of light is approximatively the same as the phase velocity if the dispersion is small, and for broad-band modulation, the group velocity of the modulating wave must be equal to its phase velocity over the desired band-width. Thus the condition for matching the group velocities will be met if the phase velocities are matched. It has been proòved[5] that for imperfect velocity matching, the optical phase retardation is reduced by a factor:

$$\frac{\operatorname{sen} u}{u}$$

where

$$u = \frac{\omega_m l}{2}\left[\frac{1}{V_o} - \frac{1}{V_m}\right] = \frac{\omega_m l}{2c}(n - \sqrt{\varepsilon}) \tag{11}$$

in which V_o and V_m are the parallel components of the phase velocities of the light and modulating wave, respectively, and ω_m is the angular frequency of the modulating wave.

For instance, the dielectric constant of KDP is about 2.25 for light waves, and about 20 over a fairly range of microwave frequencies. Therefore the u-value expressed by eq. (11) could in general be substantially different from zero.

8.- DIFFERENT SCHEMES.-

8.1.- Lumped modulators.-

The lumped modulator condition is obtained when l is much smaller than the modulating wave-length λ

$$l \ll \frac{\pi c}{\omega_m \sqrt{\varepsilon}}$$

that is, if the field in the modulator is nearly uniform during the transit time for the light to pass through the crystal. As soon as this transit time becomes a significant fraction of the period of the modula ting voltage, the depth of modulation decreases and, the reduction factor becomes $1/\sqrt{2}$ at the frequency where $u = 1.4$.

By inserting this value in eq. (11) and within the approximation that for lumped modulators $1/V_m \simeq 0$, the band-width from the finite transit time effect comes out to be:

$$\Delta\omega \simeq \frac{2.8c}{nl} \tag{12}$$

A typical driving circuit for a lumped modulator is shown in Fig. 8.

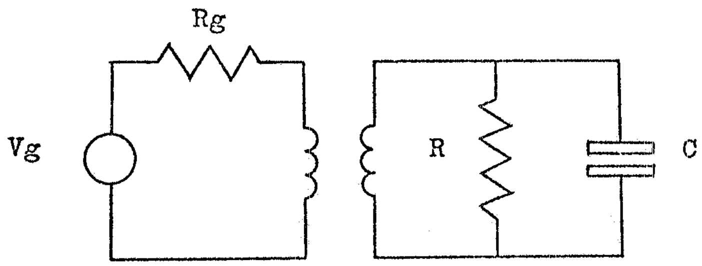

FIG. 8 - Equivalent driving circuit for a lumped modulator.

The lossy component of the crystal can be generally ignored, and the modulator is terminated with a load resistence R that is matched to the general impedence Rg through an ideal transformer with a band width assumed to be as broad as necessary. The band width of such a circuit is given by the usual expression

$$\Delta\omega = \frac{2}{RC} \tag{13}$$

where C includes the capacitance of the crystal and the parasitic capacitance of the crystal mount.

Depending on which effect is dominant, the band width for a lumped modulators is given either by eq. (12) or eq. (13).

The percent modulation can be defined as[(6)]

$$P_M = 2\, J_1\left(\pi \frac{V_m}{V_\pi}\right)$$

where J_1 is the Bessel function of the first order and V_m is the peak voltage across the crystal. A 100 percent modulation is obtained when $V_m = 0.383\ V_\pi$ (7). The reactive power stored into the crystal in order to obtain 100 percent modulation can be expressed as

$$W = \frac{\Delta\omega}{2}\, C \left(\frac{V_m}{\sqrt{2}}\right)^2$$

and, in terms of the modulation index δ, for the KDP transverse electrooptic effect one has:

$$W = \frac{\Delta\omega}{4}\, C \left(\frac{\lambda\,\delta}{\pi\, n_o^3\, r_{63}}\right)^2 \left(\frac{d}{l}\right)^2$$

Furthermore, since $C = \varepsilon_o \varepsilon\, l$, the dependence of W on the geometrical characteristics of the modulator is

(14) $$W = \frac{\Delta\omega}{4}\, \varepsilon_o\, \varepsilon \left(\frac{\lambda\,\delta}{\pi\, n_o^3\, r_{63}}\right)^2 \frac{d^2}{l}$$

The main features of same lumped modulators are listed in Table I. For all the modulators listed, the measured values of $W/\Delta f$ agree within a factor two with the calculated values using eq. (14). Other lumped modulator have been reported in (8)÷(13).

8.2.-Travelling wave modulators.-

The band width limitation resulting from the mismatch of velocities of light and modulating wave can be extended by using a substantially dispersionless structure in which both waves can propagate with equal velocities.

This is achieved by propagating part of the energy of the modulating wave outside of the crystal. Since this part does not contribute to the interaction of light and modulating waves, the velocity synchronization is achieved at the expense of the increased modulation power.

The ideal structure, neglecting dispersion and losses, is a two-plates guide partially filled with the medium and the remainder filled with a matching dielectric material(14).

It has been prooved(19) that for the configuration shown in Fig. 9 the phase velocity can be expressed in terms of an effective dielectric constant ε_e, which under the condition:

TABLE I

Comparison of lumped modulators at $\lambda = 0.63\ \mu$

Material	$LiTaO_3$ (b)	KDP	$LiTaO_3$	$LiNbO_3$
$f_o(GH_z)$	base band	base band	4.2	1.5
$\Delta f(GH_z)$ (a) (limited by capacitance)	1.3	0.22	0.04	1.5
$\Delta f(GH_z)$ (limited by transit time)	3	0.53	—	—
d (mm)	0.25	0.75	0.23	0.5
l (mm)	10	57	4.2	5
s	2.9	4.3	6.7	11.6
$W/\Delta f$ (c) (mW/MH_z)	1.1 (6)	60 (d)	24	37

(a) - Assume the load resistance R = 50 ohms
(b) - Based on round-trip mode
(c) - For 100 percent intensity modulation
(d) - A capacite equalization network effective for Δ f = 100 MH_z is used.

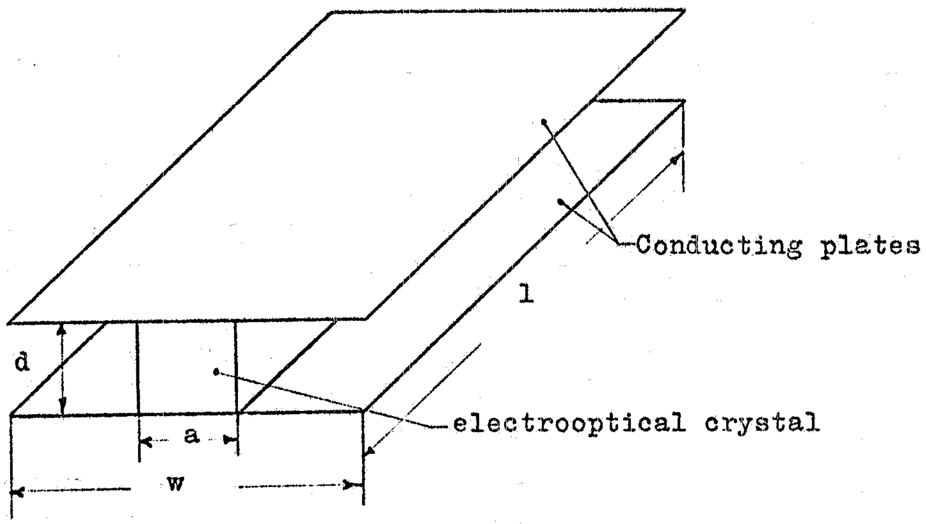

FIG. 9 - Transmission line for travelling wave modulator.

$$\frac{\omega_m \sqrt{\varepsilon}\, a}{c} \ll 1$$

can be approximated by the following expression:

$$\varepsilon_e \approx \varepsilon \frac{d}{w}$$

where w and d are respectively the width and the distance between the plates.

With such an arrangement, the bandwidth due to the finite transit time effect has been enlarged up to the value

$$\Delta\omega = \frac{2.8\, c}{(\sqrt{\varepsilon_e} - n)\, l}$$

and the stored power to obtain 100 percent modulation is[14]

$$W = \frac{1}{2}(0,383\, V_{\pi})^2 \frac{\sqrt{\varepsilon_e}\, w}{377\, d}$$

Travelling-wave modulators using parallel-plate guides have been constructed[15÷17] with l ∼100 cm using many pieces of ADP or KDP. Bicknall et al. (18) have constructed a travelling wave modulator with l = 16 cm using a chain of KDP crystal loading a coaxial transmission line. A travelling wave modulator using a rod of $LiTaO_3$ of dimensions (0,25 x 0,25 x 10) mm^3 mounted in microstrip has been described by White and Chin[19]. A travelling-wave modulator at 6 GH_z and 10 percent bandwidth has also been constructed using KDP in the ring-plane travelling-wave circuit[20].

Several types of modulator, based on the electrooptic effect, are now available for infrared and visible range[21], [22].

8.3. - Zig-Zag modulator. -

The zig-zag modulator is shown in Fig. 10. The light is assumed to propagate in the y-z-plane, between mirrors located in xy-plane and to have its electric vector polarized either perpendicular to or parallel to the yz-plane. The modulating wave propagates in y-direction, along the axis of the modulator. We may distinguish two cases. In the first (see Fig. 11a) the electric vector of the modulating field lies in the optical plane of incidence (this we call the 0^o modulator). In the second (see Fig. 11b) the modulating electric field is perpendicular to the optical plane of incidence (90^o modulator).

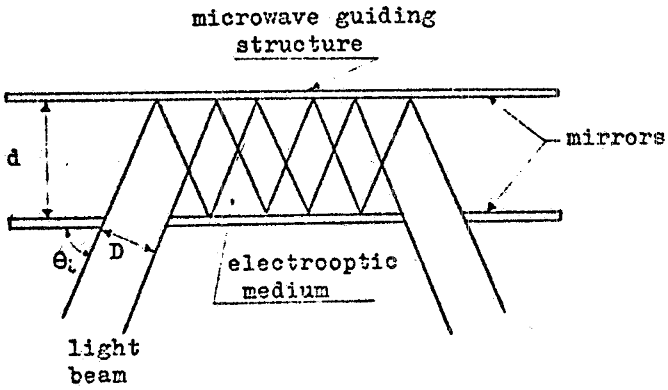

FIG. 10 - Basic zigzag modulator configuration.

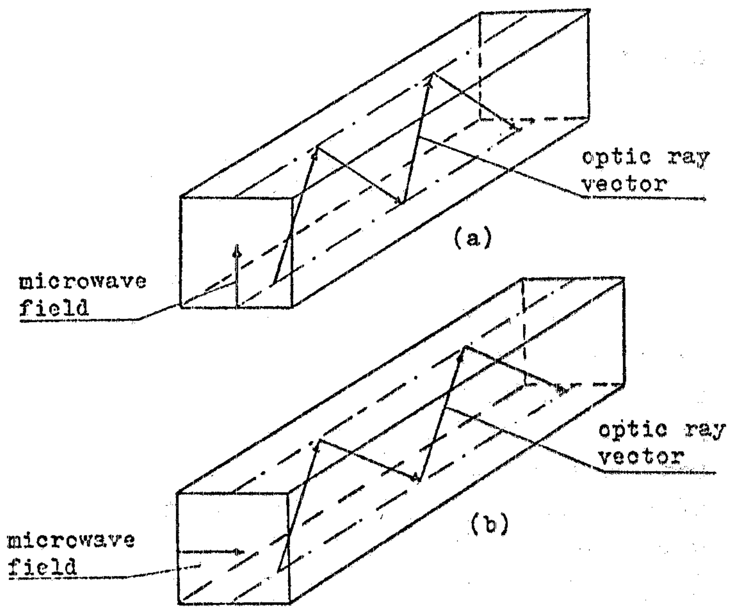

FIG. 11 - Relative orientation of modulating electric field and optical plane of incidence in two zig-zag modulator configuration: (a) 0^o modulator; (b) 90^o modulator.

In order to have a perfect velocities matching, the angle of incidence of the light must be:

$$\theta_i = \text{arcosen} \frac{n}{\sqrt{\varepsilon}} \tag{15}$$

The total optical path L is (see Fig. 12)

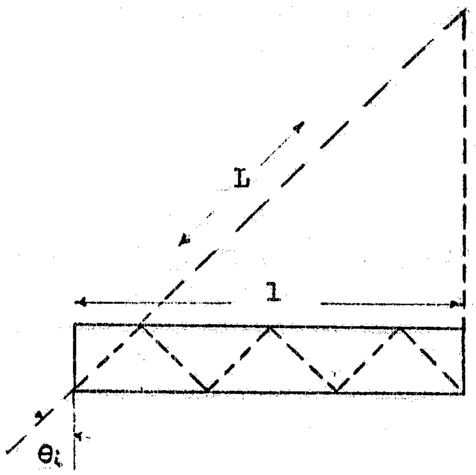

FIG. 12 - Effective light path length.

$$L = \frac{l}{\operatorname{sen}\theta_i} = l\,\frac{\sqrt{\varepsilon}}{n}$$ l = modulator length

and the retardation given by expression (5), (for a KDP crystal becomes (see Fig. 11a, 11b)

$$\Gamma_z = \frac{\pi}{\lambda}\, n_o^3\, r_{63}\, V_m \frac{L}{d} = \frac{\pi\, n_o^2\, r_{63} \sqrt{\varepsilon}\, V_m}{\lambda} \left(\frac{l}{d}\right)$$

and consequently:

$$V_{\pi_z} = \pi \frac{V_m}{\Gamma_z} = \frac{\lambda}{n_o^2\, r_{63} \sqrt{\varepsilon}} \; \frac{d}{l} = V_\pi \;\operatorname{sen}\theta_i$$

The power for 100 percent intensity modulation is:

$$W = \frac{1}{2}\, \frac{\sqrt{\varepsilon}}{377}\, \frac{b}{d} \left(0.383\, V_{\pi_z}\right)^2$$

$$W = \frac{1}{2}\, \frac{\sqrt{\varepsilon}}{377}\, \frac{b}{d} \left(0.383\, V_\pi \operatorname{sen}\theta_i\right)^2$$

It is easy to see that for the same geometrical feature the power to obtain 100 percent intensity modulation in the zig-zag modulator is reduced by a factor $\operatorname{sen}^2\theta_i$ respect to the travelling wave modulator.

In order to bounce a parallel light beam of diameter D more than once inside the crystal, it is necessary that

(16) $$D \leq 2\, d\, \mathrm{sen}\theta_i$$

from purely geometrical consideration.

Since the dispersion of this structure is known only approximately, the bandwidth of zig-zag modulators will be somewhat arbitrarily defined as(7):

(17) $$d \approx 0.2\ \lambda_m$$

where $\lambda_m = 2\pi c_o/\omega_m\sqrt{\varepsilon}$ is the modulating wavelength in the medium. Upon substitution of (15) and (16) into (17), the bandwidth becomes

$$\Delta\omega = \frac{0.84\,\pi\, c_o n}{\varepsilon D}$$

Auth(23) reported costruction of such a modulator using KDP. Modulation was observed from 7.8 GH_z to 12.4 GH_z but the depth of modulation versus the modulating power has not been reported. The scheme of noncollinear velocity matching has also been used to modulate a 0.63 μ laser beam with a HCN laser beam at 964 GH_z using $LiNbO_3$(24).

REFERENCES. -

(1) - J.F. Nye, Properties of crystals (Oxford University Press, London 1960).
(2) - H. Kogelnik and T. Li, Proc. IEEE 54, 1312 (1966).
(3) - A. Tranquilli, LNF-75/10 (1975).
(4) - G.D. Boy and J. Gordon, Bell. Sys. Tech. J. 40, 489 (1961).
(5) - W. Rigrod and I.P. Kaminow, Proc. IEEE 51, 137 (1963).
(6) - R.T. Denton et al., J. Appl. Phys. 38, 1611 (1967).
(7) - F.S. Chen, Proc. IEEE 58, 1440 (1970).
(8) - I.P. Kaminow, Phys. Rev. Letters 6, 528 (1961).
(9) - R.H. Blumenthal, Proc. IRE 50, 450 (1962).
(10) - K.M. Johnson, Proc. IRE 51, 1368 (1963).
(11) - H. Brond et al., Electronics 2, 317 (1966).
(12) - R. Dare et al., IEEE J. Q.E. E- 555 (1967).

(13) - R.P. Riesz et al., Appl. Opt. 8, 1393 (1969).
(14) - I. P. Kaminow and J. Liu, IRE 51, 132 (1963).
(15) - C.J. Peters, Proc. IRE 51, 132 (1963).
(16) - C.J. Peters, Proc. IRE 53, 455 (1965).
(17) - J.P. Kaminow et al., Proc. IEEE 54, 1374 (1966).
(18) - W.E. Bickenell et al., Proc. IEEE 55, 225 (1967).
(19) - G. White and G. Chin, Proc. IEEE Conf. San Francisco (1970).
(20) - J.L. Putz, IEEE Trans. Electr. Devices 15, 695 (1968).
(21) - L.N. Magdich et al., Sov. J. Quant. Elect. 2, 571 (1973).
(22) - U.N. Mozhaiskii et al., Sov. J. Quant. Elect. 3, 337 (1974).
(23) - D.C. Auth, IEEE J.Q.E. QE5, 622 (1969).
(24) - I.P. Kaminow et al., Appl. Phys. Letters 16, 416 (1970).