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P. Nunberg, E. Pace and D. Prosperi: AN APPLICATION OF A NEW HARMONIC-OSCILLATOR BASIS TO THE CALCULATION OF TRINUCLEON GROUND-STATE OBSERVABLES.

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P. Nunberg[x], E. Pace[o] and D. Prosperi: AN APPLICATION OF A NEW HARMONIC-OSCILLATOR BASIS TO THE CALCULATION OF TRINUCLEON GROUND-STATE OBSERVABLES.

ABSTRACT. -

A new harmonic-oscillator basis for trinucleon ground-state calculations is introduced, featuring different oscillator radii for the two intrinsic variables. This basis allows charge-dependent interactions to be handled and seems to yield a better convergence with respect to previously used oscillator bases. A test calculation with the Reid soft-core interaction is presented. The resulting (extrapolated) triton binding energy is 7.3 ± 0.2 MeV, and the first minimum in the ^{3}He charge farm factor occurs at $q^2 = 13.1$ fm^{-2}. A discussion of trinucleon binding energies derived from different realistic interactions is also included, with special regard to the role of the low-energy N-N parameters and of the deuteron D-state probability.

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1. - INTRODUCTION. -

In the framework of recent attempts to solve the trinucleon ground-state problem[1,2], a very thorough investigation has been performed by Strayer and Sauer[3]. These authors diagonalize the intrinsic hamiltonian built starting from Reid soft-core interaction[4] in an harmonic oscillator (h. o.) basis featuring definite orbital particle permutation symmetry. A total of 4654 states containing up to 50 oscillator quanta was employed. In spite of the impressive computational effort, some uncertainties are left, mainly due to the poor convergence of the charge form-factor at high momentum transfers and of the mixed-symmetry S-state percentage ($P_{S'}$). The situation would become even worse if the charge-dependence, which is present in realistic interactions fitting both n-p and p-p data, had to be taken into account, as suggested among others by the present authors[1] and by Afnan and Read[5]. Isospin mixing induced by such interactions leads in fact to an inevitable increase in the number of required basis states.

In an attempt to overcome this difficulty we introduce a new h. o. basis characterized by the presence of a further non linear parameter in addition to the oscillator radius used in ref. (3). More exactly the method here employed is based on the diagonalization of the intrinsic nuclear hamiltonian in an L-S basis in two intrinsic variables. By choosing two different h. o. parameters for the two different coordinates, we are able to reproduce a reasonable value both for $P_{S'}$ and for the ratio between the r. m. s. radii of ^3H and ^3He already in the zero order state. This method can in principle improve the convergence of the diagonalization procedure as a function of the maximum number of oscillator quanta characterizing the basis states (Q_{max}). The method is discussed in more detail in Sect. 2.

To test our method we apply it to Reid soft-core (RSC) interaction, which although charge independent, is a realistic interaction for which many trinucleon calculations, performed by different methods, seem to be converging towards a common result.

Our results for the triton binding energy (B_T) and the charge form

factors of ^{3}H and ^{3}He ($F_{ch}(q^2)$) are presented and discussed in Sect. 3. Including up to 820 basis states ($Q_{max} = 28$) we obtain $B_T = 6.7$ MeV. Extrapolating our results up to $Q_{max} = \infty$, we obtain $B_T = (7.3 \pm 0.2)$ MeV, in agreement with other available calculations. The convergence of our procedure is also discussed and found more satisfactory than that obtained by Strayer and Sauer.

Our results are finally compared with those obtained by other realistic potentials in Sect. 4. It is stressed that, at least for the class of realistic potentials which are local for any LSJT wave and for one-boson exchange (OBE) potentials, the off-shell effects seem to be less important that the present uncertainty in the knowledge of the D-state deuteron percentage $P_D(^2H)$.

2. - THE CALCULATION METHOD. -

2.1. - General definitions. -

We choose the following internal coordinates:

$$(1) \qquad \underline{a} = \frac{1}{\sqrt{2}} (\underline{r}_1 - \underline{r}_2) \quad , \quad \underline{b} = \frac{1}{\sqrt{2}} (2\,\underline{r}_3 - \underline{r}_1 - \underline{r}_2) \, .$$

We use these coordinates rather than the usual Jacobi ones ($\underline{a}_{Jac.} = \underline{a}$; $\underline{b}_{Jac.} = \frac{1}{\sqrt{3}}\,\underline{b}$) in order to simplify some of the following formulas. Here and in the following the indices 1 and 2 label the two like particles.

Neglecting three-body forces the hamiltonian of the internal motion takes the form

$$(2) \qquad H = \sum_{i\ 1}^{3} (P_i^2/2m) - P^2/2M + \sum_{i<j\ 1}^{3} V_{ij} = \frac{1}{2m} (P_a^2 + 3\,P_b^2) + \sum_{i<j\ 1}^{3} V_{ij} ,$$

where $\underline{P} = \sum_{i\ 1}^{3} \underline{P}_i$ and $M = 3m$ are, respectively, the total momentum and the total mass.

Let us now introduce two sets of oscillator wave functions

($|n_a l_a m_a\rangle_{\varrho_a}$ and $|n_b l_b m_b\rangle_{\varrho_b}$), where $\varrho_{a(b)} = \sqrt{\hbar/m\,\omega_{a(b)}}$ are usual h. o. parameters. In the following we introduce the additional parameter $\varepsilon = \varrho_b/\varrho_a$. In the case $\varepsilon = 1$ we obtain the basis employed in our previous works[6], while $\varepsilon = \sqrt{3}$ gives the h. o. orbital components employed by Jackson and coworkers[7] and by Strayer and Sauer[3]. Criteria allowing to select the best-values for ϱ_a, ϱ_b (or ε) will be given in Sect. 3.

We choose an L-S coupling scheme and a basis of the kind

$$|\Phi_K\rangle = |(n_a l_a, n_b l_b)L, (S_{12}, S_3)S; JM\rangle_{\varrho_a \varrho_b} . \tag{3}$$

Our basis states are not coupled to a definite isospin. The parity ($\pi = +1$) of the studied states fixes the parity of $(l_a + l_b)$ so that $(-1)^{l_a + l_b} = \pi$. Antisymmetry under the exchange of the space-spin coordinates of the two identical particles (1 and 2) enforces the further limitation $(-1)^{l_a + S_{12}} = +1$.

2.2. - Matrix elements of the hamiltonian. -

The matrix elements of the kinetic energy can be immediately calculated. We obtain

$$\begin{aligned}
\langle \Phi_K | P_a^2/2m | \Phi_{K'} \rangle &= \delta^{[n_a n_a']}_{K,K'} \langle n_a l_a | P_a^2/2m | n_a' l_a \rangle_{\varrho_a} , \\
\langle \Phi_K | P_b^2/2m | \Phi_{K'} \rangle &= \delta^{[n_b n_b']}_{K,K'} \langle n_b l_b | P_b^2/2m | n_b' l_b \rangle_{\varrho_b} = \\
&= \delta^{[n_b n_b']}_{K,K'} \; \varepsilon^{-2} \langle n_b l_b | P_b^2/2m | n_b' l_b \rangle_{\varrho_a} ,
\end{aligned} \tag{4}$$

where $\delta^{[nn']}_{K,K'}$ denotes a product of Krönecker symbols involving all pairs (l_a, l_a'), (l_b, l_b')... of quantum numbers labelling states Φ_K and $\Phi_{K'}$, except those explicitly indicated (n, n').

The matrix elements $\langle \Phi_K | V_{12} | \Phi_{K'} \rangle$ can also be easily calculated, being $\underline{r}_{12} \propto \underline{a}$. After standard manipulations and a suitable change of the coupling scheme ($\underline{l}_a + \underline{l}_b = \underline{L}$, $\underline{S}_{12} + \underline{S}_3 = \underline{S}$, $\underline{L} + \underline{S} = \underline{J} \longrightarrow \underline{L} + \underline{S}_{12} = \underline{I}$, $\underline{I} + \underline{S}_3 = \underline{J}$) we obtain:

$$\langle \phi_K | V_{12} | \phi_{K'} \rangle = \delta_{S_{12}, S'_{12}} \delta_{n_b, n'_b} \delta_{l_b, l'_b} (\hat{L}\hat{L}'\hat{S}\hat{S}')$$

$$\sum_I (2I+1) \begin{Bmatrix} L & S_{12} & I \\ 1/2 & J & S \end{Bmatrix} \begin{Bmatrix} L' & S_{12} & I \\ 1/2 & J & S' \end{Bmatrix} \sum_j (2j+1) \tag{5}$$

$$\begin{Bmatrix} l_b & l_a & L \\ S_{12} & I & j \end{Bmatrix} \begin{Bmatrix} l_b & l'_a & L' \\ S_{12} & I & j \end{Bmatrix} \langle n_a (l_a S_{12}) j | V_{12} | n'_a (l'_a S_{12}) j \rangle_{\varrho_a}$$

where 6-j symbols have been introduced and $\hat{j} = (2j + i)^{1/2}$.

We have now to evaluate separately the matrix elements of V_{13}. In principle this is not necessary if $\varepsilon = \sqrt{3}$ is chosen, since in this case a completely antisymmetric basis can be used, yielding $\langle V_{13} \rangle = \langle V_{12} \rangle$ [3,7]. For $\varepsilon \neq \sqrt{3}$ this is however not possible, since the construction of a completely antisymmetric basis from our h. o. states would require an infinite expansion (this point can be easily proved using the transformation brackets defined by two of us [8]). Since V_{13} is a function of both $\underline{a}$ and $\underline{b}$, to compute $\langle \phi_K | V_{13} | \phi_{K'} \rangle$ we shall use the so called "Generalized Moshinsky transformation (G. M. T.)", originally introduced by Smirnov[9] to describe the motion of two particles of different masses in a common oscillator potential

$$|(n_a l_a, n_b l_b); LM_L\rangle_{\varrho_a \varrho_b} = \sum_{n_r n_R} \sum_{l_r l_R} \langle (n_r l_r, n_R l_R); L | (n_a l_a, n_b l_b); L \rangle_\varepsilon \cdot$$

$$\cdot |(n_r l_r, n_R l_R); LM_L\rangle_{\varrho_r \varrho_R} , \tag{6}$$

where

$$\begin{aligned} \underline{r} &= \underline{a} - \underline{b} = \sqrt{2}\,(\underline{r}_1 - \underline{r}_3) , \\ \underline{R} &= (\varrho_a^2 + \varrho_b^2)^{-1} (\varrho_b^2 \underline{a} + \varrho_a^2 \underline{b}) = \\ &= (2)^{-1/2} (1 + \varepsilon^2)^{-1} (2\underline{r}_3 - [1 - \varepsilon^2]\,\underline{r}_1 - [1 + \varepsilon^2]\,\underline{r}_2) \end{aligned} \tag{7}$$

and

$$\varrho_r = (\varrho_a^2 + \varrho_b^2)^{1/2} = \varrho_a (1 + \varepsilon^2)^{1/2} , \tag{8}$$

$$\varrho_R = (\varrho_a^2 + \varrho_b^2)^{-1/2} \varrho_a \varrho_b = \varrho_a \varepsilon(1+\varepsilon^2)^{-1/2} . \tag{8}$$

The coefficients $\langle (n_r l_r, n_R l_R); L \,|\, (n_a l_a, n_b l_b); L \rangle_\varepsilon$ are called "Generalized Moshinsky brackets" and reduce to standard Moshinsky brackets when $\varepsilon = 1$. By the use of the G. M. T. the interparticle distance $r_{13} = (1/\sqrt{2})r$ appears explicitly in the terms of our basis allowing a simple solution of our problem.

We finally obtain the expression

$$\begin{aligned}
\langle \phi_K | V_{13} | \phi_{K'} \rangle = (-1)^{L+L'+S+S'+S_{12}+S'_{12}+1} \; & (\hat{L}\hat{L}'\hat{S}\hat{S}'\hat{S}_{12}\hat{S}'_{12}) \\
\sum_{n_r l_r} \sum_{n'_r l'_r} \sum_{n_R l_R} & \langle (n_a l_a, n_b l_b); L \,|\, (n_r l_r, n_R l_R); L \rangle_\varepsilon \, \langle (n'_r l'_r, n_R l_R); L' | \\
& | (n'_a l'_a, n'_b l'_b); L' \rangle_\varepsilon \sum_{S_{13}} (2 S_{13} + 1) \begin{Bmatrix} 1/2 & 1/2 & S_{12} \\ 1/2 & S & S_{13} \end{Bmatrix} \begin{Bmatrix} 1/2 & 1/2 & S'_{12} \\ 1/2 & S' & S_{13} \end{Bmatrix} \\
\sum_{I,j} (2j+1)(2I+1) & \begin{Bmatrix} l_R & 1/2 & I \\ l_r & S_{13} & j \\ L & S & J \end{Bmatrix} \begin{Bmatrix} l_R & 1/2 & I \\ l'_r & S_{13} & j \\ L' & S' & J \end{Bmatrix} \\
& \langle (n_r l_r) S_{13}; j \,|\, V_{13} \,|\, (n'_r l'_r) S_{13}; j \rangle ,
\end{aligned} \tag{9}$$

where standard 9-j symbols have been introduced.

2.3. - Charge form factor. -

In the non relativistic Born approximation the charge form factor for elastic electron scattering by ^3H is given by

$$F_{ch}(q^2) = 2 F_1(q^2) G_n(q^2) + F_3(q^2) G_p(q^2) \tag{10}$$

where $\underline{q}$ is the momentum transfer and G_n and G_p are the neutron and proton electric form factors, for which we have used the parametrization given by Janssens et al.[10]. The functions F_i ($i = 1, 3$) are form factors of point particles, given by the relation

$$F_i(q^2) = \frac{1}{4\pi} \langle \phi_T | \int d\Omega_q \, e^{i \underline{q} \cdot \underline{r}'_i} | \phi_T \rangle , \tag{11}$$

where $|\phi_T\rangle = \Sigma_K C_K |\phi_K\rangle$ is the triton ground state wave function and

$$\begin{aligned} \underline{r}'_1 &= \underline{r}_1 - \underline{R}_{C.M.} = \frac{1}{\sqrt{2}} \left(\underline{a} - \frac{1}{3} \underline{b}\right) , \\ \underline{r}'_3 &= \underline{r}_3 - \underline{R}_{C.M.} = \frac{\sqrt{2}}{3} \underline{b} \end{aligned} \tag{12}$$

Neglecting Coulomb and other isospin-breaking effects the form factor of ^{3}He is obviously given by

$$F_{ch}(q^2) = \frac{1}{2} \left[2 F_1(q^2) G_p(q^2) + F_3(q^2) G_n(q^2) \right] . \tag{13}$$

If we write the radial h. o. wave functions in the form:

$$R_{nl}(r, \varrho) = \varrho^{-\frac{3}{2}} e^{-\frac{1}{2}(\frac{r}{\varrho})^2} \sum_{\alpha_o}^{n} D^{\alpha}_{nl} \left(\frac{r}{\varrho}\right)^{l+2\alpha} , \tag{14}$$

where

$$D^{\alpha}_{nl} = \sqrt{\frac{n!}{\sqrt{\pi}} \frac{(2n+2l+1)!!}{2^{n+l-2}}} \cdot \frac{(-1)^{\alpha} 2^{l+\alpha}}{\alpha! (n-\alpha)! (2l+2+1)!!} , \tag{15}$$

we obtain the following expression for the matrix elements of the point form factor for the unlike particle:

$$\begin{aligned} &\frac{1}{4\pi} \langle \phi_K | \int d\Omega_q \, e^{i \underline{q} \cdot \underline{r}'_3} | \phi_{K'} \rangle = \\ &= \delta^{[n_b, n'_b]}_{K,K'} \frac{1}{2} e^{-(\varepsilon^2 \varrho_a^2 q^2/18)} \sum_{o\gamma}^{n_b} \sum_{o\delta}^{n'_b} D^{\gamma}_{n_b l_b} D^{\delta}_{n'_b l'_b} \Gamma(\beta + \tfrac{3}{2}) F(-\beta | \tfrac{3}{2} | \varepsilon^2 \varrho_a^2 q^2/18) \end{aligned} \tag{16}$$

where

$$\beta = \gamma + \delta + \frac{l_b + l'_b}{2} ; \tag{17}$$

F and Γ are the usual confluent hypergeometric and Γ functions.

For the identical particles we have:

$$\frac{1}{4\pi}\langle \phi_K \left| \int d\Omega_q \, e^{i\underline{q}\cdot\underline{r}'_1} \right| \phi_{K'} \rangle =$$

$$= \delta_{S_{12},S'_{12}} \, \delta_{S,S'} \, \delta_{L,L'} \, \frac{\pi}{16} \, e^{-\frac{\varrho_a^2 q^2}{8}(1+\frac{\varepsilon^2}{9})} (-1)^L \cdot$$

$$\cdot \hat{l}_a \hat{l}'_a \hat{l}_b \hat{l}'_b \sum_\lambda (2\lambda+1) \begin{Bmatrix} \lambda & l_b & l'_b \\ L & l'_a & l_a \end{Bmatrix} \begin{pmatrix} l_a & l'_a & \lambda \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} l_b & l'_b & \lambda \\ 0 & 0 & 0 \end{pmatrix} \cdot$$

(18)

$$\cdot \left(\frac{\varepsilon \varrho_a^2 q^2}{24}\right)^\lambda \frac{1}{\left[\Gamma(\lambda+\frac{3}{2})\right]^2} \sum_0^{n_a} \gamma' \sum_0^{n'_a} \delta' \, D^{\gamma'}_{n_a l_a} \, D^{\delta'}_{n'_a l'_a} \, \Gamma(\alpha+\frac{\lambda}{2}+\frac{3}{2})$$

$$F\left(\frac{\lambda}{2}-\alpha \left| \lambda+\frac{3}{2} \right| \frac{\varrho_a^2 q^2}{8}\right) \sum_0^{n_b} \gamma \sum_0^{n'_b} \delta \, D^{\gamma}_{n_b l_b} \, D^{\delta}_{n'_b l'_b}$$

$$\cdot \Gamma(\beta+\frac{\lambda}{2}+\frac{3}{2}) \, F\left(\frac{\lambda}{2}-\beta \left| \lambda+\frac{3}{2} \right| \frac{\varepsilon^2 \varrho_a^2 q^2}{72}\right) \quad ,$$

where

(19)
$$\alpha = \gamma' + \delta' + \frac{l_a + l'_a}{2}$$

and β is given by eq. (17). We note that all the hypergeometric functions appearing in the above formulae reduce to polynomials.

3. - NUMERICAL RESULTS. -

3.1. - Choice of non linear parameters. -

Let us start with a short discussion of the possibilities arising from the presence of ε as a free parameter. These can be clarified by analysing the properties of the lowest oscillator state $|(0S)_a, (0S)_b\rangle$ which is always present with a strong amplitude in our wave functions. In particular it is convenient to analyse the dependence on ε of the ratio η between the radii of the

identical particles and of the different one (i. e., η = proton radius/neutron radius in ^{3}He) and of the mixed symmetry S-state percentage $P_{S'}$. For the $|(0S)_a, (0S)_b>$ state we have:

$$\eta = \frac{\langle r_1^2 \rangle^{1/2}}{\langle r_3^2 \rangle^{1/2}} = \left(\frac{9 + \varepsilon^2}{4\,\varepsilon^2}\right)^{1/2} , \tag{20}$$

$$P_{S'} = \frac{2}{3}\left\{1 - \left[\frac{16\,\varepsilon^2}{(1+\varepsilon^2)(9+\varepsilon^2)}\right]^{3/2}\right\} .$$

We also introduce the mass-radius r_{mass} by the definition:

$$r_{mass} = \left(\frac{2\langle r_1^2 \rangle + \langle r_3^2 \rangle}{3}\right)^{1/2} = \varrho_a \left(\frac{3 + \varepsilon^2}{6}\right)^{1/2} . \tag{21}$$

When $\varepsilon = 1$, we have $\eta = 1,58$, $P_{S'} = 19\%$ and $r_{mass} = \varrho_a \cdot 0,815$; when $\varepsilon = \sqrt{3}$, we have $\eta = 1$, $P_{S'} = 0$ and $r_{mass} = \varrho_a$. It is clear that our old basis ($\varepsilon = 1$)[6] gives too high zero-order values both for η and $P_{S'}$ (we recall that the experimental value for η is 1.13, while the values of $P_{S'}$, resulting from most trinucleon calculations lie in the range 1-2%), giving rise to a rather slow convergence. On the other hand, the basis employed in refs.(3) and (7) ($\varepsilon = \sqrt{3}$) starts from more reasonable zero-order values for η and $P_{S'}$ and features a complete antisymmetrization of the basis states which results in an appreciable shrinking of the size of the matrices involved in the calculation. The convergence of $P_{S'}$ is however rather slow[3]. This last observation leads to the suspicion that a choice of ε giving rise to a larger zero-order value of $P_{S'}$ might yield a better convergence of B_T as a function of the maximum number Q_{max} of oscillator quanta included in the basis.

To check this point we performed a series of small trial calculations with different values of ε. It should be noted that the possibility of leaving ε arbitrary has been achieved at no cost in terms of code complexity or running time; the difference between our old code and the new one consisting only in the introduction of generalized Moshinsky brackets, which are

as convenient to compute as the standard ones used in our previous works.

Table I shows values of B_T obtained including up to twelve oscillator quanta ($Q = 2n_a + l_a + 2n_b + l_b \leq Q_{max} = 12$). The mass-radius was fixed to the value $r_{mass} = 0.85$ fm used in ref. (3) and ε^2 was varied in the range 1.5 - 3.0; there appears to be a distinct advantage in choosing $\varepsilon^2 \neq 3$. This calculation seems to indicate that the optimum value of ε^2 lies somewhere in the range $1.5 \leq \varepsilon^2 \leq 2.25$. Similar results were obtained by varying r_{mass} around 0.85 fm. Our final choice is $\varepsilon^2 = 2.25$, corresponding to $\eta = 1.12$, in agreement with the value determined by the experimental charge

TABLE I

$-B_T$ (MeV) versus ε^2 for small values of the maximum number of oscillator quanta Q_{max}, $r_{mass} = 0.85$ fm.

ε^2 \ Q_{max}	0	2	4	6	8	10	12
1.5	1022.8	240.1	36.0	25.3	8.6	4.9	-0.1
2.0	943.8	219.9	36.3	22.0	9.0	4.1	-0.2
2.25	916.9	213.9	37.2	21.4	9.4	4.2	-0.1
2.5	897.7	210.4	38.2	21.2	9.9	4.5	0.2
3.0	872.8	207.5	40.3	21.6	10.9	5.1	0.7

radii of ^3He and ^3H (corrected for the finite proton dimensions). This choice also corresponds to a value $P_S = 1.53\%$ lying in the range of the most commonly accepted estimates. In this way only the ratio ϱ_b/ϱ_a is fixed. On the other hand also the absolute values of ϱ_a and ϱ_b (or, equivalently, of r_{mass}) strongly affect the convergence of B_T as a function of Q_{max}. The dependence of the convergence rate on r_{mass} has been preliminarly studied for $2.0 \leq \varepsilon^2 \leq 2.5$ and $Q_{max} \leq 12$. The results obtained for $\varepsilon^2 = 2.25$ are shown in Table II. The optimum value for r_{mass} is seen to lie somewhere around 0.75 fm; as however hamiltonian matrix elements increase with decreasing r_{mass}, leading to larger numerical errors both in the construction of the matrix and in the diagonalization procedure, we chose

r_{mass} = 0.85 fm, which also allows a meaningful comparison with the result of ref. (3).

TABLE II

$-B_T$(MeV) versus r_{mass} (fm) for small values of the maximum number of oscillator quanta Q_{max}; $\varepsilon^2 = 2.25$.

r_{mass} \ Q	0	2	4	6	8	10	12
0.55	3152.9	695.9	80.6	40.2	21.8	12.4	7.1
0.65	1983.0	446.8	52.2	26.3	10.8	4.6	0.6
0.75	1318.4	304.0	42.2	22.5	8.6	3.1	-1.0
0.85	916.9	213.9	37.2	21.4	9.4	4.2	-0.1
0.95	661.7	156.0	33.5	20.6	10.7	5.9	1.8
1.05	493.9	118.1	30.2	19.5	11.4	7.3	3.7
1.15	377.3	91.8	27.2	18.1	11.6	8.1	5.0

3.2. - Triton binding energy and charge radii. -

We included in our computation all partial waves which are present in the RSC interaction[4], namely those having $j \leqslant 2$.

The calculation were performed with a basis including states up to a maximum number of oscillator quanta $Q_{max} = 28$. The total number of states retained in this case was 820; this number must be compared with the exact number of states with $Q \leqslant 28$, namely 2360. To achieve this reduction we enlarged our basis step by step, starting from $Q_{max} = 16$ and increasing Q_{max} by two units each time. At each step we performed a diagonalization and subsequently scratched from our basis all states having an amplitude less than 10^{-3}. The effect of discarded states on B_T was checked with some further diagonalizations and found to be of the order of 10 KeV at each step. Moreover, an analysis of the discarded components allowed us to conclude that some categories of states could be systematically neglected

starting from sufficiently high values of Q_{max}; more precisely, P states were retained only up to $Q_{max} = 12$ and states with $L = 0$, $S_{12} = 1$ up to $Q_{max} = 22$. We note that P states are usually assumed to have a small effect on B_T and are completely ignored by Strayer and Sauer[3] as well as by many other authors. As far as $L = 0$, $S_{12} = 1$ states are concerned, their smallness can be understood in terms of the lack of any space-symmetric component.

Figs. 1a) and 1b) show the dependence of B_T and of the r. m. s. charge radii $\langle r_{ch}^2 \rangle^{1/2}$ of ^3H and ^3He upon Q_{max}. The values here reported for B_T were calculated by the expression

$$B_T = -\langle \phi_T \mid H \mid \phi_T \rangle \tag{22}$$

which is remarkably more precise than the eigenvalues directly given by the diagonalization program. A short discussion of this point can be found in App. 1. In Fig. 2 the P- and D- state percentages are plotted versus Q_{max}. The values obtained for the same quantities with our largest basis ($Q_{max} = 28$) are also reported in Table III. In the same table we also show extrapolated values obtained by the fit formula

$$B_T(Q_{max}) = B_T(\infty) - \alpha\, Q_{max}^{-\beta} \tag{23}$$

for the energies and an exponential formula for the charge radii (corrected for the finite proton dimension). Theoretical arguments leading to justify extrapolation formulas of kind (23) for sufficiently high values of Q_{max} are given in ref. (11). This method is also commonly used in calculations made by variational methods[3, 7, 12, 13]. The best parameters α, β and $B_T(\infty)$ were determined for different sets of $B_T(Q_{max})$ values with $Q_1 \leqslant Q_{max} \leqslant Q_2$. Our final value $B_T = 7.3$ MeV was obtained for $Q_1 = 16$, $Q_2 = 28$, while the error quoted for B_T in Table III takes into account the dependence of $B_T(\infty)$ on Q_1 and Q_2.

A detailed comparison of the results obtained in recent works for the RSC potential is displayed in Table IV. It can be seen that the bin-

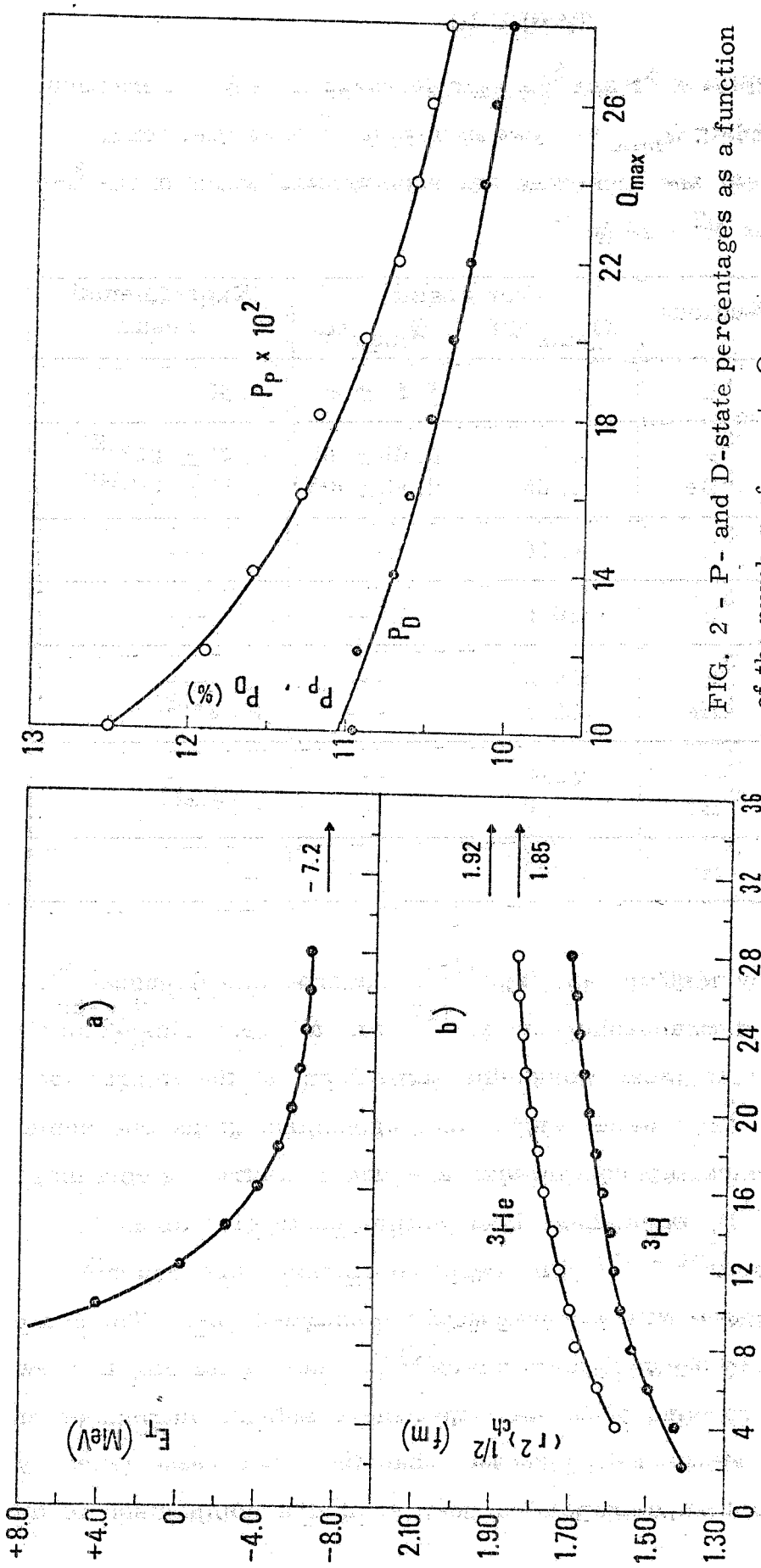

FIG. 1 - Dependence of $E_T = -B_T$ and of the r.m.s. radii $[\langle r^2 \rangle_{ch}]^{1/2}$ of ^{3}H and ^{3}He upon the number of quanta Q_{max}. The values adopted for the non linear parameters were $\varepsilon^2 = 2.25$ and $r_{mass} = 0.85$ fm.

FIG. 2 - P- and D-state percentages as a function of the number of quanta Q_{max}.

TABLE III

Ground state observables of ^{3}H and ^{3}He calculated for the RSC interaction. Values under the heading $Q_{max} = \infty$ are extrapolated ones (see text). R is the ratio between the computed and experimental value of the ^{3}He charge form factor at $q^2 = 20$ fm^{-2}.

Observable	Nucleus	Our result $Q_{max}=28$	Our result $Q_{max}=\infty$	Experimental result
B_T (MeV)	^{3}H	6.7	7.3 ±.2	8.48
$[\langle r^2 \rangle_{ch}]^{1/2}$ (fm)	^{3}H	1.71	1.85±.05	1.70±.05[19]
	^{3}He	1.85	1.92±.05	1.87±.05[18]
P_P(%)	^{3}H	0.10	---	---
P_D (%)	^{3}H	10.0	---	---
q^2_{min} (fm^{-2})	^{3}H	15.3	---	---
	^{3}He	13.1	---	11.6[18]
q^2_{max} (fm^{-2})	^{3}H	22.0	---	---
	^{3}He	18.0	---	~17[18]
R	^{3}He	0.48	---	1.0

ding energies given by Malfliet and Tjon[14], Laverne and Gignoux[15], Afnan and Read[5] and Brandenburg et al.[17] are all very close to each other ($B_T \sim 7.0$ MeV). Since in these works the same form of the interaction is used ($^1S_0 + (^3S_1 - {}^3D_1)$ waves only) this agreement gives one some confidence that numerical approximations are under control. Remaining waves (odd and $^1D_2$, $^3D_2$ ones) have been estimated to give about 0.2 ÷ ÷ 0.3 MeV extra binding[12 ÷ 17]. The result by Strayer and Sauer[3] seems therefore to agree with the previously mentioned ones. The some what different result by Hennell and Delves[12] is not disturbing in view both of the quoted error and of the fact that these authors introduced an interaction in partial waves with j greater than two. The value given by Demin et al.[16] is not an extrapolated one, so that a comparison is dif

TABLE IV

Results obtained by various authors by the R.S.C. potential. (V) stands for variational technique, (F) for solution of Faddeev equations, (Ho) for harmonic oscillator basis, (HH) for hyperspherical harmonic basis, (UPA) for unitary pole approximation, (TPT) for T matrix perturbation theory.

Reference	B_T (MeV)	r^2_{ch} (^{3}H) (fm)	r^2_{ch} (^{3}He) (fm)	$P_{S'}$	P_P (%)	P_D	Method
Malfliet[14]	6.9	1.80	2.05	1.6	---	8.5	F (a)
Brandenburg[17]	6.98	----	2.25	1.7	---	8.1	F (a)
Laverne[15]	7.0	1.65	1.90	1.8	---	9.0	F (a)
Afnan[5]	7.02 7.15	----	----	---	---	---	TPT UPA (a)
Demin[16]	6.64	1.77	1.90	0.4	0.06	8.9	HH (b)
Hennel[12]	7.75 ± .5	1.78	----	1	---	9.5	V (c,e)
Strayer[3]	7.3 ± .2	1.85	2.07	1.4	---	8.8	Ho (c)
Present work	7.3 ± .2	1.85	1.92	---	.10	10.0	Ho (d)
Experiment[18, 19]	8.48	1.70 ± .05	1.87 ± .05	---	---	(6÷11)	---

a) $^1S_0 + (^3S_1 - ^3D_1)$ waves only; b) All waves with $j \lesssim 2$ included; c) As case b), extrapolated value of B_T; d) As case b), extrapolated values of B_T and charge radii; e) contribution from waves with $j > 2$ included.

ficult; the very low value of $P_{S'}$ might also indicate an inadequacy of the basis.

From the above discussion we can conclude that some sort of consensus is beginning to emerge around a value of 7.3 MeV for the triton binding energy with the full Reid potential. Our result, if taken at face value, seems to corroborate this conclusion; it should be noted however that our calculation differs from those in the previously mentioned group in that P--states are included in our basis. A trial diagonalization performed at $Q_{max} = 28$ excluding these states showed that they contribute 0.31 MeV to B_T, in qualitative agreement with the estimate by Demin et al.[16] (0.22 MeV). Since however Fig. 2 shows that the P-state percentage is still noticeably decreasing when we reach our largest basis, this result can only be interpreted as an upper limit to the contribution of P-states. It seems therefore that the inclusion of P-states in the computation by Strayer and Sauer[3] would lead to a somewhat stronger binding with respect to our work; it should be noted however that the discrepancy is within the limits of estimated numerical errors even if the contribution of P-states is assumed to be 0.3 MeV.

A direct assessment of the improvement in the convergence rate obtained in the transition from the basis employed in ref. (3) to our one is not possible since, in addition to the uncertainty on the contribution of P-states, our results stop at $Q_{max} = 28$ while those in ref. (3) start from $Q_{max}=36$. Extrapolation of our data to $Q_{max} = 36$ (which is of course much more reliable than that to $Q_{max} = \infty$) yields B_T $(Q_{max} = 36) = 7.0$ MeV, while the corresponding value by Strayer and Sauer is 6.5 MeV. We thus appear to have gained at least 0.2 MeV at $Q_{max} = 36$.

As far as the r.m.s. charge radii are concerned, we observe some spread among the computed values, which are also somewhat larger than the experimental ones.

3.3. - Charge form factors. -

Figs. 3a) and 3b) show our results for the charge form factors of ^{3}He and ^{3}H respectively, together with experimental curves fitting the data

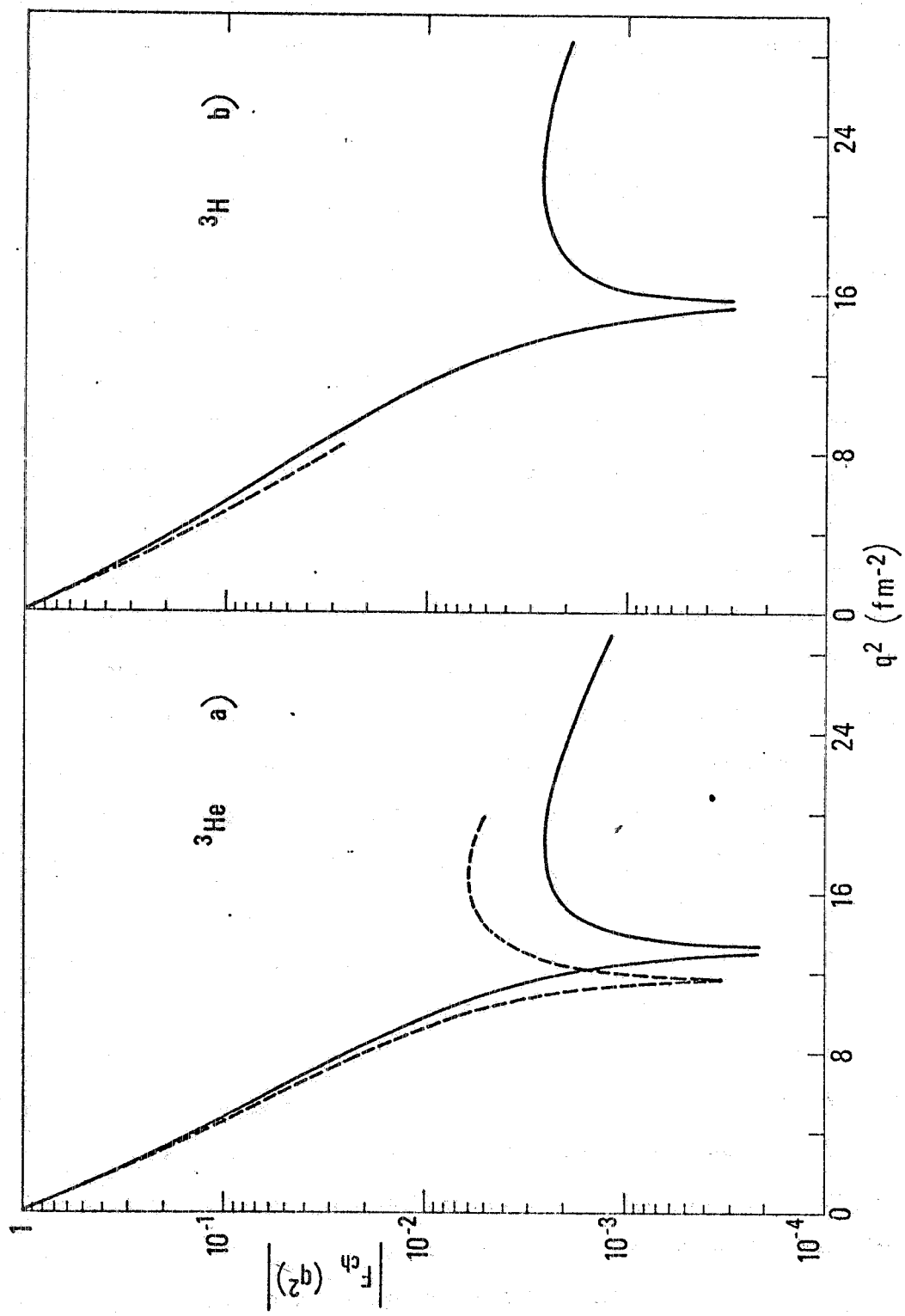

FIG. 3 - Charge form factors of ^{3}He and ^{3}H. Full curves : present computation. Dashed curves : experiment (18, 19).

of refs. (18, 19). The results obtained for the position of the diffraction minimum and of the height of the secondary maximum are also reported in Table III. The height of the secondary maximum is expressed by the ratio of the theoretical and experimental values of the ^{3}He charge form factor at $q^2 = 20$ fm^{-2}:

$$R = \left\{ \frac{\left[F_{ch}(q^2)\right]_{th.}}{\left[F_{ch}(q^2)\right]_{exp}} \right\}_{q^2 = 20 \text{ fm}^{-2}} \tag{24}$$

A comparison with the results obtained by other authors for the RSC potential is shown in Table V. Only the most recent results obtained with the complete form or with the $^1S_0$ and ($^3S_1$ - $^3D_1$) waves of the interaction were included: as shown in refs. (21, 3), neglecting odd waves has a small effect on the charge form factor (the position of the diffraction minimum is raised by about 0.3 ÷ 0.7 fm^{-2}) and so neglecting $^1D_2$, $^3D_2$ waves[21].

It is clear that the old trend of Faddeev calculations yielding diffraction minima at higher values of q^2 than variational ones is now fading out and there is a fair agreement among computed values of this parameter. Larger discrepancies persist for the height of the secondary maximum. In this connection it is interesting to note that we obtain a much better convergence for this quantity than Strayer and Sauer[3]. The slope of the relevant curve ($F_{ch}(q^2 = 20 \text{ fm}^{-2})$ versus Q_{max}) computed by us at $Q_{max} = 28$ is of the opposite sign but at least twice as small as that computed by the above authors at $Q_{max} = 50$. This seems to suggest that the poor convergence found in ref. (3) is mostly due to the lack of convergence of $P_{S'}$ rather than to the neglect of high Fourier components due to basis truncation. There is thus a very indirect indication that our basis allows a better treatment of the S' component; a positive check would however require a direct computation of $P_{S'}$, which implies a large computational effort. To analyse the dependence of $F_{ch}(q^2)$ on the details of the wave function, we carried out further calculations by changing P_P and P_D. No visible effect was obtained in the ranges $0\% \leq P_P \leq 0.1\%$ and $8.8\% \leq P_D \leq 10.0\%$.

Concluding we have the usual result of a too high value of q^2 for the

TABLE V

Diffraction minimum and height of the secondary maximum in $F_{ch}(q^2)$ for the RSC potential.

Reference	^{3}He		^{3}H		Method
	Diffr. minimum q^2 (fm^{-2})	Heigth of secondary maximum (x 10^3)	Diffr. minimum q^2 (fm^{-2})	Heigth of secondary maximum (x 10^3)	
Laverne[15]	14.0	2	---	---	F (a)
Brandenburg[17]	13.9	1.7	---	---	F (a)
Hennell[12]	12.8 $\pm$ 0.3	2	13.8 $\pm$ 0.5	2.3	V (c)
Demin[21]	13.8	1.9	15.0	2.5	HH (b)
Strayer-Sauer[3]	12.6	2	13.4	2	HO (b)
Present work	13.1	2.5	15.3	2.6	HO (b)
Experiment[18]	11.6	6	---	---	---

a) $^1S_0 + (^3S_1 - {}^3D_1)$ waves only; b) All waves with $j \leq 2$ included; c) Contribution from waves with $j > 2$ included;

diffraction minimum and a secondary maximum which is too low by a factor ≈3. No simple explication of this disagreement is known at present. Preliminary studies of the effect of schematic three-body forces[22] and relativistic corrections[23] both yield a displacement of the minimum in the wrong direction. An explanation of these discrepancies must await for a more detailed evaluation of relativistic corrections and exchange current effects[20]. The effects arising from nucleon polarization must also be systematically investigated.

4. - TRINUCLEON CALCULATIONS AND THE N-N INTERACTION. -

A list of triton binding energies calculated employing different realistic potentials is shown in Table VI. We selected results obtained with the whole interaction whenever possible. The interactions have been divided in three sets:

1) interactions local for any LSJT wave and charge-independent;
2) O. B. E. velocity dependent interactions;
3) charge-dependent interactions.

To the last group belong two potentials linearly dependent on J^2 and local for any LST wave (Rutherford lab. interactions)[33] studied by the present authors[34].

The results reported in the table show an overall spread of about 2 MeV. In fact we have a minimum value of 6. 73 MeV for the Yale potential and a maximum value of 8. 83 MeV for the O. B. E. potential due to Ueda and Green (UG2).

If we neglect computational uncertainties this difference can be mainly ascribed to three effects (1) :

1) Difference between the $P_D(^2H)$ values of the various N-N interactions;
2) Difference between low energy parameters (a_s, a_t; r_s, r_t) and phases given by the interactions;
3) Off-energy shell effects (other than those included in $P_D(H^2)$) due to the different parametrization of the interactions.

TABLE VI

Triton binding energy B_T obtained by different realistic interactions. $P_D(^2H)$ is the deuteron D-state percentage.

Interaction		B_T (MeV)			$P_D(^2H)(\%)$
GPT	(24)	(a)	8.61	(16)	3.8
SSC-B	(25)	(b, c)	7.60	(15)	4.25
SSC-A	(25)	(b, c)	7.58	(15)	4.43
SSC-C	(25)	(b, c)	7.34	(15)	5.45
EH	(26)	(a)	7.45	(16)	6.21
RSC'	(4)	(b)	7.32	(5)	6.22
RSC	(4)	(a)	7.3 ±.2	(p. w.)	6.47
RHC	(4)	(b)	6.96	(5)	6.50
Yale	(27)	(b)	6.73	(5)	6.95
HJ	(28)	(b)	6.96	(5)	7.02
SRG	(29)	(b)	8.31	(5)	4.58
UG2	(30)	(b)	8.83	(5)	4.6
BG	(31)	(b)	7.81	(5)	5.06
BS	(32)	(b)	7.94	(5)	5.47
RUTH-A	(33)	(a)	8.1 ±.2	(34)	4.7
RUTH-B	(33)	(a)	7.8 ±.3	(34)	5.4

a) all partial waves of the interaction included; b) $^1S_0+(^3S_1-^3D_1)$ interaction only; c) contribution of other waves of the interaction calculated perturbatively.

Let us now examine the first point. The values of B_T reported in Table VI are plotted versus $P_D(^2H)$ in Fig. 4. There is an evident correlation between these two quantities, the binding energy decreasing with increasing $P_D(^2H)$. On the other hand there is a wide spreading of the single points with respect to the average behaviour. Incidentally, the variation of $P_D(^2H)$ reflects in a corresponding variation of $P_D(^3H)$, in fact we have $P_D(^3H)/P_D(^2H) \sim 1.4$.

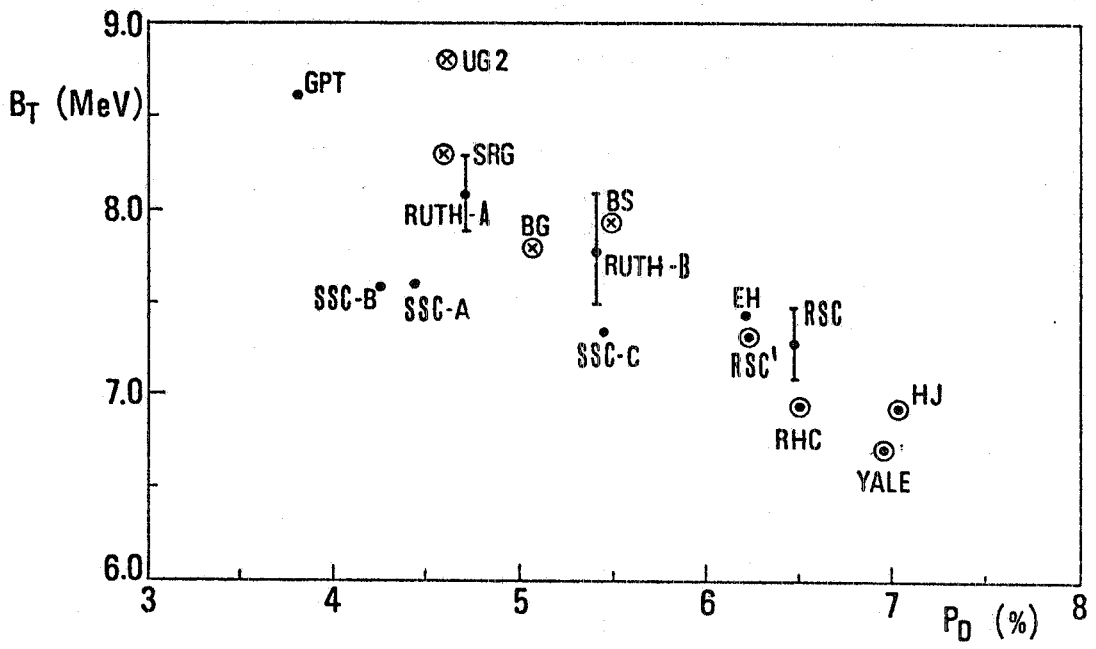

FIG. 4 - B_T versus $P_D(^2H)$ for realistic potentials. (SeeTable VI).

As far as point 2) is concerned, there are sizeable differences in the values of the low-energy parameters given by different interactions. As an istance, OBEP's are fitted to n-p data while other potentials usually fit p-p data. We can attempt to correct the B_T values shown in Fig. 4 for the differences in the $\mathcal{C}_i$ (a_s, a_t; r_s, r_t) parameters of the corresponding potentials by utilizing the derivatives $\partial B_T / \partial \mathcal{C}_i$ estimated by Van Wageningen et al.[35]. The reference values here assumed for the $\mathcal{C}_i$ parameters are:

$$
\begin{aligned}
a_s &= -17.1 \text{ fm} \quad , \quad a_t = 5.40 \text{ fm}, \\
r_s &= 2.83 \text{ fm} \quad , \quad r_t = 1.73 \text{ fm}.
\end{aligned}
\tag{25}
$$

The singlet parameters are those commonly accepted for p-p scattering, corrected for Coulomb effects[36]. To correct the B_T values obtained by the charge-dependent interactions RUTH-A and RUTH-B, we schematically assumed the binding energy to be given for 1/3 by the n-n pair and for 2/3 by the two n-p pairs. The corrected energies are finally shown in Fig. 5; the spread is remarkably reduced and we can observe a clear linear behaviour with the slope $\partial B_T / \partial P_D(^2H) \approx -0.27$ MeV. A similar behaviour was previously ob-

served for schematic non local separable interactions[37]. Applying the same procedure as above, we can also estimate that for charge-dependent interactions fitted to experimental parameters [a_s(n-p) = -23.7 fm, r_s(n-p)=2.73fm[36]] the line of Fig. 5 is shifted upwards by about 0.3 MeV[5]. It is very interesting to observe that the point which shows the largest departure from the average behaviour corresponds to an interaction which does not exhibit a OPE tail (EH interaction). It is also noteworthy that the velocity dependent OBEP's follow the same pattern as other interactions.

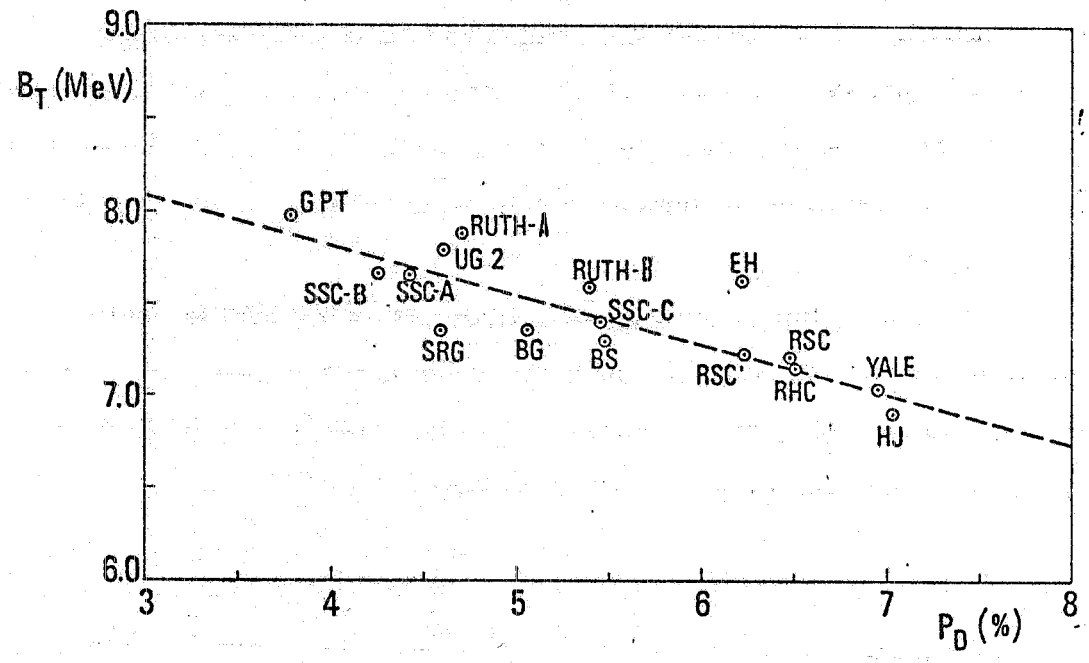

FIG. 5 - B_T versus $P_D(^2H)$ for realistic potentials. Corrected values (see text).

If we keep in mind all the uncertainties in the points of Fig. 5 (neglect of some partial waves in the interaction, computational errors, corrections from differences in low-energy parameters amounting up to 1 MeV), we feel justified in saying that the only parameter which appreciably affects B_T is the deuteron D-state percentage, at least as far as realistic interactions which are local for any wave or velocity-dependent OBEP's are concerned. For this kind of interactions the off-energy shell effects mentioned under point 3) above do not therefore seem to play a significant role. It appears

however that results obtained by the method of phase-equivalent potentials (which feature strong non-localities) are somewhat different. If the only constraints which are imposed are phase-equivalence and constant deuteron binding energy, very large off-shell effects (several MeV) can be found[38]. Even when further constraints are added, such as the wishful one of a complete knowledge of the deuteron wave function[39] and that of charge--symmetry or OPE tail in the $^1S_0$ channel[40], variations in excess of 1 MeV are still found. It should be borne in mind, however, that the situation of these investigations is rapidly evolving, so that the above result is not final. A better understanding of the theoretical implications of the non localities which are introduced in phase-equivalent interactions as well as an accurate investigation of the effect of realistic constraints in the $^3S_1 - {}^3D_1$ channel (e. g. fixing $P_D(^2H)$ and the deuteron quadrupole moment) would be of considerable help in this respect.

To conclude, the main aim of trinucleon calculations, namely that of gaining information on the two-and three-body nuclear interaction seems still far from being achieved. To advance in this direction further experimental two-body data are also needed (especially $P_D(^2H)$).

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APPENDIX. -

Since we are dealing with the diagonalization of rather large matrices, it is important that numerical errors introduced by this procedure are kept under control. To this end we use some rigorous a posteriori bounds on the errors affecting the computed value of the lowest eigenvalue and the corresponding computed eigenvector. The relevant formulas which are due to Wilkinson[41] can also also be found in a work by Ortega[42], together with an explanation of the Givens-Householder diagonalization method, which is employed in our work.

Let H be a N x N matrix. We define:

$\underline{S}$ ≡ true normalized ground-state eigenvector of H,

$\underline{S}'$ ≡ computed value of $\underline{S}$,

$E_i (i = 1, 2, \ldots, N)$ = true eigenvalues of H,

$E_i' (i = 1, 2, \ldots, N)$ = computed eigenvalues of H.

The eigenvalues are assumed to be ordered in ascending order. We also introduce the definition

$$D \equiv \left\| (H - E_1')\underline{S}' \right\|$$

with the usual definition of vector norm.

The following upper bound obtains for the error on the ground-state eigenvalue

(A1) $$\Delta E \equiv \left| E_1 - E_1' \right| \leq D.$$

If we define $\alpha \equiv E_2 - E_1'$, the error on the ground-state eigenvector is bound by the following relation

(A2) $$\left\| \underline{S}_1' - \underline{S}_1 \right\| \leq \frac{D}{\alpha}\left(1 + \frac{1}{4}\frac{D^2}{\alpha^2}\right)^{1/2}.$$

To apply this formula we approximate α by $E_2' - E_1'$. This is probably a good approximation since $E_2' - E_1'$ turns out to be large in our calculation (of the order of 10 MeV at $Q_{max} = 28$). When expression (A1) is applied to the results

of our computation, inordinately large error bounds result; for instance already at $Q_{max} = 12$ (252 states) a bound of 80 KeV is found for Reid potential, increasing to 498 KeV at $Q_{max} = 28$ (820 states). It should be noted that, not unexpectedly, we found error bounds to decrease for interactions yielding smaller matrix elements.

A better estimate of the eigenvalue can be obtained by computing the expectation value of H with the computed eigenvector:

(A3) $$\mu \equiv (\underline{S}', H\underline{S}').$$

In fact the following bound can be shown to hold:

(A4) $$|\mu - E_1| \leq \frac{D^2}{\alpha} \left(1 - \frac{D^2}{\alpha^2}\right)^{-1}$$

Since α is large compared to D this bound is considerably more stringent than that expressed by eq. (A1): at $Q_{max} = 28$ we get $|\mu - E_1| \leq 13$ KeV. The availability of this better estimate for E_1 incidentally allowed us to find that eq. (A1) gives a much too large bound for the error on E_1'; at $Q_{max}=28$, $|\mu - E_1'|$ is only 231 KeV and thus a factor of about 2 less than D.

Numerical errors introduced by the diagonalization procedure turn out to be much less influent in the determination of the extrapolated binding energy than the selection of the set of points included in the fitting procedure. As far as the ground-state eigenvector is concerned, eq. (A2) yields, at $Q_{max} = 28$:

$$\| \underline{S}_1 - \underline{S}_1' \| \leq 0.026.$$

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