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P. Nunberg, E. Pace and D. Prosperi: CALCULATION OF THE TRINUCLEON GROUND-STATE PROPERTIES BY CHARGE-DEPENDENT INTERACTIONS. -

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SUMMARY.-

Trinucleon ground-state properties are computed with two realistic charge-dependent interactions (Rutherford laboratory potentials) using a new harmonic oscillator basis. Extrapolated binding energies of 8.1 and 7.8 MeV are obtained. The ^{3}He charge form factors display minima at $q^2 = 13.1$ and 12.6 fm^{-2}.

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Aim of the present work is to contribute to the study of the trinucleon ground-state properties by using realistic charge-dependent interactions. The need for calculations employing interactions different as far as the singlet n-p and p-p low energy parameters are concerned has been stressed in refs. (1), (2).

We employ the soft-core Rutherford Laboratory interaction RUTH-A[3]. This can be written in the usual form

$$v = v_C + v_{SL}(\vec{S}\cdot\vec{L}) + v_T S_{12} \,. \tag{1}$$

All terms v_i ($i \equiv C$, SL, T) are linearly dependent on $\vec{J}^2$ and their radial behaviour is given as a superposition of Yukawa-potentials; their form is

$$v_i(x) = \frac{1}{x} \sum_{n}^{N} \left[a_n^{(i)'} + J(J+1)\, a_n^{(i)''} \right] e^{-nx} \tag{2}$$

where $x = \mu r$ ($\mu = 0.707\ \mathrm{fm}^{-1}$). The coefficients a_n depend on spin and parity.

A version (RUTH-B) of the interaction is also used, which differs from RUTH-A in the triplet-even $J = 1$ channel only[4].

Both these interactions yield an excellent overall reproduction of the phase-shifts, the value of χ^2/(degree of freedom) ranging from 0.94 for singlet even ones to 1.6 for triplet ones. Unfortunately however the fit to some low energy parameters is not very accurate: the effective range $r_{os}(pp)$ and r_{ot} are respectively 3.06 and 2.02 fm, to be compared with the experimental ones $r_{os}^{exp}(pp) = 2.79 \pm 0.01$ fm and $r_{ot}^{exp} = 1.73 \pm 0.01$ fm[5]. Let us finally note that, for Ruth-A and Ruth-B, the deuteron D-state percentage is, respectively, $P_D(^2H) = 4.7\%$ and $P_D(^2H) = 5.4\%$.

In our calculation we used a method discussed in detail in ref. (6) in connection with a similar calculation using Reid soft-core interaction. The method is based on the diagonalization of the intrin

sic nuclear hamiltonian in an L-S basis in the two intrinsic variables $\underline{a} = (\underline{r}_1 - \underline{r}_2)/\sqrt{2}$ and $\underline{b} = (2\underline{r}_3 - \underline{r}_1 - \underline{r}_2)/\sqrt{2}$, where the indices 1 and 2 label the like nucleons. Harmonic oscillator wave functions have been employed. Since the basis states do not have a definite isospin, charge-dependent interactions do not pose any problem. To optimize the convergence rate of the triton binding energy (B_T), we used two different harmonic oscillator parameters, ρ_a and ρ_b, for the two coordinates. We can thus reproduce a reasonable value both for the mixed symmetry S-state percentage ($P_{S'}$) and for the ratio between the r.m.s. radii of ^3H and ^3He already in the zero order $(os)^2$ state.

All results reported in this note were obtained by using the parameters $\varepsilon^2 \equiv (\rho_b/\rho_a)^2 = 2.25$ (we recall that, with our choice of coordinates, a standard oscillator basis is characterized by $\varepsilon^2 = 3$) and $r_{mass} = \rho_a\left[(3+\varepsilon^2)/6\right]^{1/2} = 0.85$ fm as in ref. (6). The calculations were performed including states up to a maximum number of oscillator quanta $Q_{max} = 28$. The total number of states retained in this case was 820; all states with amplitudes less than 10^{-3} were discarded by a procedure explained in detail in ref. (6). The Coulomb interaction was neglected.

Fig. 1 shows the dependence of B_T and of the r.m.s. charge radii $\langle r_{ch}^2 \rangle^{1/2}$ of ^3H and ^3He upon Q_{max} for both interactions. The values obtained for the same quantities with our largest basis ($Q_{max} = 28$) are also shown in Table I, together with the P- and D-state percentages in the triton ground state wave function. In the same table we also display extrapolated values obtained by using a fit formula of the kind

$$B_T(Q_{max}) = B_T(\infty) - \alpha Q_{max}^{-\beta} \qquad (3)$$

for the energies[7] and an exponential formula for the charge radii.

It would be interesting to know the effect of the poor fitting of low-energy parameters. We can attempt to estimate this effect using the derivatives of the binding energy with respect to these parameters given by Van Wageningen et al.[8]. It turns out that the contributions of δr_{os} and δr_{ot} are about +400 and -300 keV respectively, so that the overall correction to be applied to B_T is about +100 keV.

Let us compare the values of B_T computed by us with those obtained by realistic charge-independent interactions featuring similar values of $P_D(^2H)$, namely the SSC-A and SSC-C interactions[9] (B_T = 7.58 MeV, P_D = 4.43% and B_T = 7.34 MeV, P_D = 5.45% respectively[10]). It thus appear that the charge-dependent RUTH interactions give an extra binding of about 0.5 MeV with respect to the SSC ones.

Should we attempt to correct the binding energies obtained with RUTH and SSC interactions by the above outlined procedure in such a way as to correspond to common (charge-independent) low energy parameters, we would discover that the difference would decrease to about 0.2 MeV only. It thus appears reasonable to assume that a sizeable fraction of the extra binding obtained by the RUTH interactions derives from charge-dependence.

Fig. 2 shows our results for the charge form factors of ^3H and ^3He for RUTH-A interaction: all calculations have been done in the Born non relativistic approximation (see ref. (6)). The values obtained for the position of the diffraction minimum and the height of the secondary maximum are listed in Table I together with those obtained by RUTH-B interaction. For both interactions we have the usual result of a too high value of q^2 for the diffraction minimum and a secondary maximum which is too low by a factor ~ 3.

In conclusion the results obtained with RUTH interactions do not show any major difference with respect to those derived by other

TABLE I

Trinucleon ground-state properties given by RUTH interactions. The parameters here used for the calculation are $\varepsilon^2 = 2.25$, $r_{mass} = 0.85$ fm. For B_T and $|\langle r^2_{ch} \rangle|^{1/2}$ the first number is the value calculated for $Q_{max} = 28$, the second is an extrapolated one.

Nucleus	Quantity	Inter-action	Calculated value	Extrapol. value	Experim. value
^{3}H	B_T(MeV)	A	7.13	8.1 ± 0.2	8.48
		B	6.65	7.8 ± 0.3	
	$\langle r^2_{ch} \rangle^{1/2}$(fm)	A	1.75	1.79	1.70 ± 0.05 [12]
		B	1.76	1.79	
	P_P (%)	A	0.06		
		B	0.07		
	P_D (%)	A	7.6		6 - 11
		B	8.2		
	q^2_{min}(fm^{-2})	A	14.9		
		B	14.1		
	$\|F_{ch}\|_{max} \times 10^3$	A	2.1		
		B	2.4		
^{3}He	$\langle r^2_{ch} \rangle^{1/2}$ (fm)	A	1.88	1.91	1.87 ± 0.05 [11]
		B	1.88	1.90	
	q^2_{min} (fm^{-2})	A	13.1		11.6 [11]
		B	12.6		
	$\|F_{ch}\|_{max} \times 10^3$	A	1.9		~ 6
		B	2.2		

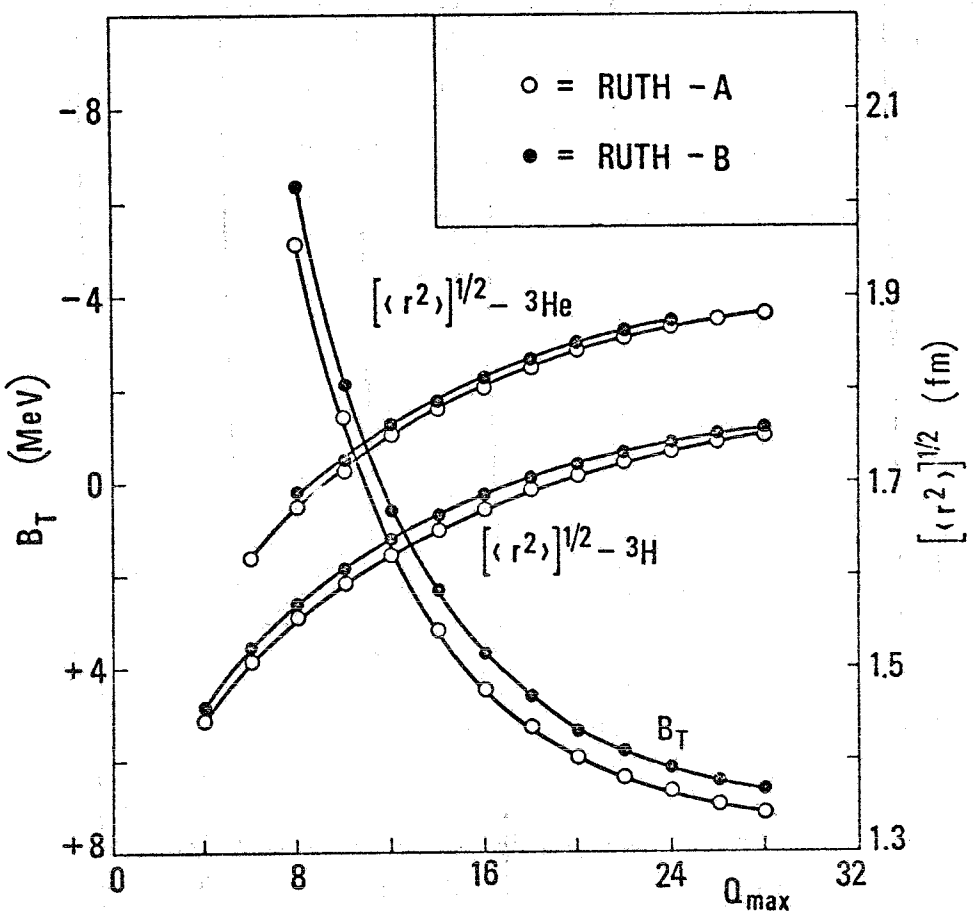

FIG. 1 - Dependence of B_T and the r.m.s. charge radii $\left[\langle r^2_{ch} \rangle\right]^{1/2}$ for ^{3}H and ^{3}He on Q_{max}.

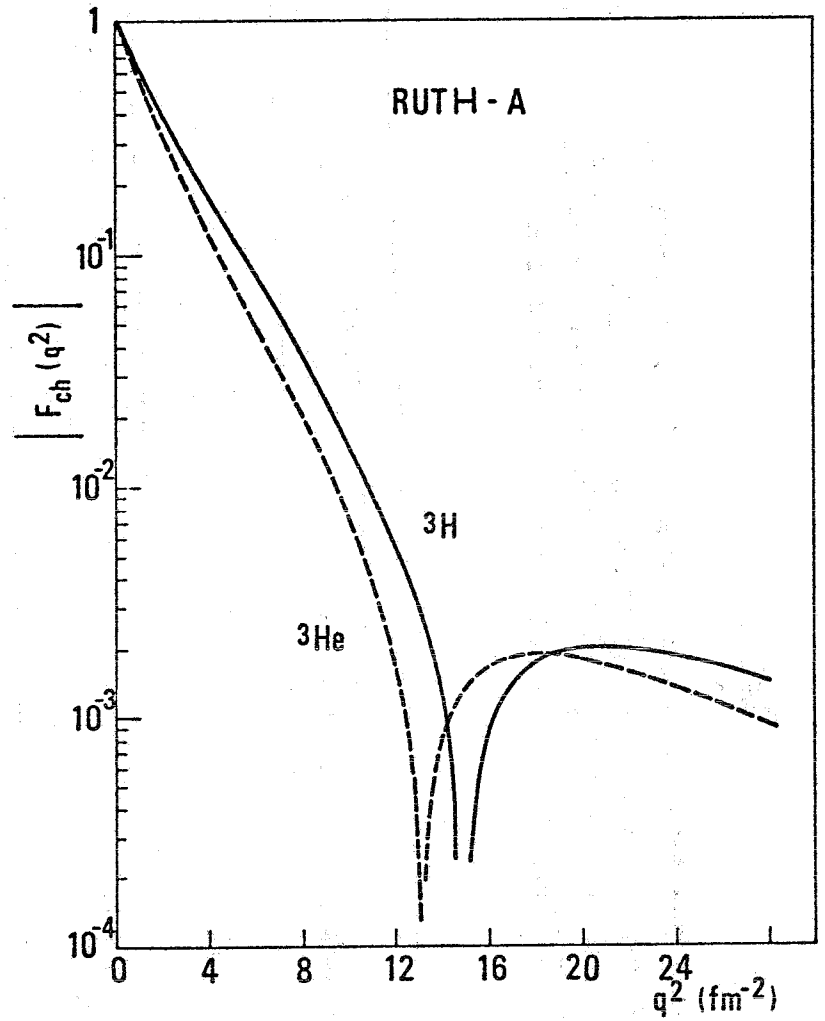

FIG. 2 - Charge form factors of ^{3}H and ^{3}He calculated by the RUTH-A interaction.

realistic interactions featuring comparable values of $P_D(^2H)$, except for those originating from different values of the low-energy parameters[6]. In order to have a more quantitative estimate of the effect of the charge dependence on B_T, it would be useful to have charge dependent interactions reproducing all experimental low--energy parameters.

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