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F. Palumbo and Yu. A. Simonov: FRACTIONAL EXPONENTIAL
ASYMPTOTIC FALL OFF OF THE FORM FACTOR OF MANY-
-BODY SYSTEMS WITH A CLASS OF INTERACTIONS.

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ABSTRACT:

It is shown that for a class of interactions the form factor of many-body systems has a fractional exponential asymptotic fall off. Potentials more singular than r^{-2} at the origin belong to this class. Another interaction belonging to this class is the Skyrme interaction.

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In a recent paper⁽¹⁾ the asymptotic behaviour of the form factor of non relativistic systems has been derived for local and non-local, two - and three-body potentials taking into account the Pauli principle, which has been found to have a very remarkable effect, making the form factor to fall off faster by a power of the order of $A^{8/3}$ for heavy nuclei (A being the particle number). In the case of local potentials the derivation has been confined to potentials less singular than r^{-2} at the origin, because the technique employed did not allow to account in a simple manner for the vanishing of the total wave function when any interparticle distance vanishes in the presence of potentials more singular than r^{-2} . In the present paper by using a different technique, we shall establish the asymptotic behaviour of the form factor of non-relativistic many-body systems with local two-body potentials more singular than r^{-2} at the origin

$$(1) \quad v(r) \sim \frac{\hbar^2}{mr_0} \left(\frac{r_0}{r} \right)^{2(1+\gamma)}, \quad 0 < \gamma < 1/2,$$

where m is the mass of the particles and the condition on γ is necessary for the method we use to be applicable. Our results will actually be true for a larger class of interactions including the Skyrme interaction⁽²⁾.

Drell et al.⁽³⁾ showed that in the two-body case the asymptotic form factor with potentials (1) has a fractional exponential fall-off

$$(2) \quad F_2(r_0 q) \sim (r_0 q)^{-2 - \frac{\gamma}{2(1+\gamma)}} f(r_0 q),$$

where

$$(3) \quad f(x) = \exp \left\{ -\frac{\gamma+1}{\gamma} \sin \left(\frac{\pi}{2} \frac{1}{1+\gamma} \right) (2x^\gamma)^{\frac{1}{1+\gamma}} \right\} \cdot \cos \left\{ \frac{\pi}{4} \frac{\gamma}{1+\gamma} + \frac{\gamma+1}{\gamma} \sin \left(\frac{\pi}{2} \frac{2+\gamma}{1+\gamma} \right) (2x^\gamma)^{\frac{1}{1+\gamma}} \right\}.$$

They conjectured that a similar behaviour should also be true for the three-body form factor. In this note we shall generalize the result of eq. (2) to many-body systems, by using the hyperspherical expansion⁽⁴⁾ for the total wave function

$$(4) \quad \psi = \rho^{-\frac{3A-4}{2}} \sum_{K\nu} \chi_{K\nu}(\rho) U_{K\nu}(\Omega).$$

In this equation A is the number of particles, ρ is the hyperradius of the system and Ω a set of $3A-4$ angular coordinates plus spin-isospin variables. The index K is the degree of the harmonic polynomial $U_{K\nu}(\Omega)$. It runs from a minimum value which is zero for boson constituents and is given in ref. (5) for fermion constituents, to infinity. The wave functions $\chi_{K\nu}$ satisfy the infinite system of coupled equations

$$(5) \quad - \left[\frac{d^2}{d\rho^2} - \frac{\mathcal{L}(\mathcal{L}+1)}{\rho^2} + \frac{2mE}{\hbar^2} \right] \chi_{K\nu}(\rho) + \frac{2m}{\hbar^2} \sum_{K'\nu'} V_{K\nu}^{K'\nu'}(\rho) \chi_{K'\nu'}(\rho) = 0$$

where E is the intrinsic energy of the system and

$$(6) \quad \mathcal{L} = K + \frac{3A-6}{2}.$$

The functions $V_{K\nu}^{K'\nu'}(\rho)$ behave at the origin like the two-body potentials⁽⁶⁾, so that we have

$$(7) \quad V_{K\nu}^{K'\nu'}(\rho) \sim V_{0K\nu}^{K'\nu'} \left(\frac{r_0}{\rho} \right)^{2(1+\gamma)}, \quad \rho \rightarrow 0.$$

In order that the hyperspherical expansion be applicable to the potentials⁽¹⁾, γ must be smaller than $1/2$. In what follows, however, we shall only make use of the property that the functions $V_{K\nu}^{K'\nu'}(\rho)$ should diverge faster than $\rho^{-2(1+\gamma)}$ at the origin. Other particle interactions different from (1) can give rise to such a behaviour with values of γ greater than $1/2$. The Skyrme interaction provides an example⁽⁶⁾ for which $\gamma = 2$.

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So our results hold true for all the particle interactions giving rise to $V_{K\nu}^{K'\nu'}$ functions satisfying eq. (7).

Eqs. (5) are satisfied for small ϱ by⁽²⁾

$$(8) \quad \chi_{K\nu}(\rho) \sim P_{K\nu} \rho^{\frac{\gamma+1}{2}} e^{-\frac{\beta}{\gamma} \left(\frac{r_0}{\rho}\right)^\gamma},$$

provided the constants $P_{K\nu}$ and β satisfy the eigenvalue equation

$$(9) \quad \sum_{K'\nu'} V_{oK\nu}^{K'\nu'} P_{K'\nu'} = \frac{\hbar^2}{2mr_0^2} \gamma^2 \beta^2 P_{K\nu}.$$

It should be noted that the $\chi_{K\nu}$'s do not depend on K for small ϱ , so that

$$(10) \quad \psi \sim \rho^{-\frac{3A-4}{2}} \rho^{-\frac{1+\gamma}{2}} e^{-\frac{\beta}{\gamma} \left(\frac{r_0}{\rho}\right)^\gamma} \phi(\Omega),$$

$$(11) \quad \phi(\Omega) = \sum_{K\nu} P_{K\nu} U_{K\nu}(\Omega).$$

The form factor is

$$F(q) = \int d\underline{r}_1 \dots \int d\underline{r}_A \delta(\underline{r}_1 + \dots + \underline{r}_A) |\psi|^2 e^{iq \left(\underline{r}_A - \frac{1}{A} \sum_{i=1}^A \underline{r}_i \right)}.$$

Using the relations⁽⁷⁾

$$U_{K\nu}^*(\Omega) U_{K'\nu'}(\Omega) = \sum_{\bar{K}\bar{\nu}} C_{K\nu K'\nu'}^{\bar{K}\bar{\nu}} U_{\bar{K}\bar{\nu}}(\Omega)$$

$$\int d\Omega U_{\bar{K}\bar{\nu}}(\Omega) e^{iq \left(\underline{r}_A - \frac{1}{A} \sum_{i=1}^A \underline{r}_i \right)} = J_{\bar{K} + \frac{3A-5}{2}}(q_e \rho) (q_e \rho)^{-\frac{3A-5}{2}}$$

$$U_{\bar{K}\bar{\nu}}(\Omega(q));$$

$$q_e = \sqrt{\frac{A-1}{A}} q,$$

where J_ν is the Bessel function of order ν , we obtain

$$(12) \quad F(q) = \sum_{K\nu} \sum_{K'\nu'} \frac{\sum P_{K\nu} P_{K'\nu'}}{\bar{K}^\nu} C_{K\nu K'\nu'}^{\bar{K}^\nu} U_{\bar{K}^\nu}(\Omega(q)) I_{\bar{K}}(q)$$

$$(13) \quad I_{\bar{K}}(q) = \int_0^\infty d\rho \rho^{1+\gamma} e^{-\frac{2\beta}{\gamma} \left(\frac{r_0}{\rho}\right)^\gamma} J_{\bar{K} + \frac{3A-5}{2}}(q_e \rho) (q_e \rho)^{-\frac{3A-5}{2}}$$

Now $I_{\bar{K}}$ turns out to be independent of \bar{K} for large q , allowing to extract the q dependence in eq. (12). We shall do this explicitly for A even. In this case we use the relation⁽⁸⁾

$$J_{m+\nu}(z) z^{-(m+\nu)} = \left(\frac{1}{z} \frac{d}{dz}\right)^m \left[z^{-\nu} J_\nu(z) \right],$$

and integrate by parts a sufficient number of times to obtain

$$(14) \quad I_{\bar{K}} \sim q^{-(3A-5)+\bar{K}} \int_0^\infty d\rho \sin(q_e \rho) e^{-2 \frac{\beta}{\gamma} \left(\frac{r_0}{\rho}\right)^\gamma} \cdot$$

$$\cdot \left[\frac{3A-6}{2} - \gamma + (\gamma+1)\left(\bar{K} + \frac{3A-6}{2}\right) \right],$$

where only the term with highest negative exponent has been retained in the integrand. This kind of integral has been evaluated by the steepest descent method in ref. (3)

$$I_{\bar{K}} = (\rho_0 q)^{-\frac{\gamma}{1+\gamma} \frac{3A-6}{2} - 2 - \frac{\gamma}{2(1+\gamma)}} \exp \left\{ -\frac{\gamma+1}{\gamma} \sin\left(\frac{\pi}{2} \frac{1}{1+\gamma}\right) \cdot \right.$$

$$\cdot \left[2(q\rho_0)^\gamma \right]^{\frac{1}{1+\gamma}} \left. \sin \left\{ \frac{\pi}{4} \frac{\gamma}{1+\gamma} + \frac{\gamma+1}{\gamma} \sin\left(\frac{\pi}{2} \frac{2+\gamma}{1+\gamma}\right) \right\} \right.$$

$$\cdot \left[2(q\rho_0)^\gamma \right]^{\frac{1}{1+\gamma}} + \frac{\pi}{2} - \frac{3A-6}{2} \frac{\gamma+2}{\gamma+1} \frac{\pi}{2} - \frac{\pi}{2} \bar{K} \left. \right\},$$

where we have put

$$\rho_0 = r_0 \beta^{\frac{1}{\gamma}} \sqrt{\frac{A-1}{A}}.$$

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Taking into account that \bar{K} is even because of parity conservation we finally obtain

$$\begin{aligned}
 F_A = (\rho_0 q)^{-\frac{\gamma}{\gamma+1} \frac{3A-6}{2} - 2 - \frac{\gamma}{2(1+\gamma)}} \exp \left\{ -\frac{\gamma+1}{\gamma} \sin \left(\frac{\pi}{2} \frac{1}{1+\gamma} \right) \right. \\
 \cdot \left. \left[2(q\rho_0)^\gamma \right]^{\frac{1}{1+\gamma}} \right\} \cos \left\{ \frac{\pi}{4} \frac{\gamma}{1+\gamma} + \frac{\gamma+1}{\gamma} \sin \left(\frac{\pi}{2} \frac{2+\gamma}{1+\gamma} \right) \right. \\
 \cdot \left. \left[2(q\rho_0)^\gamma \right]^{\frac{1}{1+\gamma}} - \frac{3A-6}{2} \frac{\gamma+2}{\gamma+1} \frac{\pi}{2} \right\} \Lambda(\Omega(q))
 \end{aligned}$$

with

$$\Lambda(\Omega(q)) = \sum_{K\nu} \sum_{K'\nu'} \sum_{\bar{K}\bar{\nu}} P_{K\nu} P_{K'\nu'} C_{K\nu K'\nu'}^{\bar{K}\bar{\nu}} (-1)^{\bar{K}} U_{\bar{K}\bar{\nu}}(\Omega(q)).$$

It should be noted that $\Lambda(\Omega(q))$ does not depend on the modulus of q . For $A=2$ the results of ref. (2) is reproduced.

We see that the asymptotics of the form factor does not depend on the constituents being fermions or bosons. This is due to the fact that the potential is so repulsive at the origin that the wave function always vanishes at zero interparticle distance irrespective of the nature of the particles.

REFERENCES -

- (1) - I.M. Narodetsky, F. Palumbo and Yu. A. Simonov, Phys. Lett. 58 B, 125 (1975).
- (2) - T.H.R. Skyrme, Phyl. Mag. 1, 1043 (1956).
- (3) - S.D. Drell, A.C. Finn and M.H. Goldhaber, Phys. Rev. 157, 1402 (1967).
- (4) - Yu. A. Simonov in The Nuclear Many Body Problem (F. Calogero and C. Ciofi degli Atti Editors), Roma 1972.
- (5) - A.M. Badalyan, F. Calogero and Yu. A. Simonov, Nuovo Cimento 68 A, 572 (1970).
- (6) - J. Navarro, to be published. It should be noted, however, that the Skyrme interaction has to be used only in the K_{\min} approximation.
- (7) - M. Fabre de la Ripelle, Comptes Rendus de la Session d'Etude de l'Institut de Physique Nucleaire de Lyon. La Toussuire IPNO/TH 71-13 (1971).
- (8) - I. S. Gradshteyn and I. M. Ryzhik, Tables of Integrals series and products (Academic Press, 1965), p. 968.