To be submitted to
Astronomy and Astrophysics

COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-75/48(P)
6 Novembre 1975

L. Donazzolo, E. Etim, A.F. Grillo, M. Grilli and F. Occhionero: BARYON NUMBER INSTABILITY AND THE MATTER CONTENT OF A CHARGE-SYMMETRIC UNIVERSE.

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-75/48(P)
6 Novembre 1975

L. Donazzolo[(x)], E. Etim, A. F. Grillo, M. Grilli and F. Occhionero[(x)]:
BARYON NUMBER INSTABILITY AND THE MATTER CONTENT OF A CHARGE-SYMMETRIC UNIVERSE. -

ABSTRACT. -

It is proposed that as a charge-symmetric universe cools through the hadron era it undergoes a phase transition which breaks the underlying charge conjugation symmetry giving rise thereby to separate regions of matter and antimatter. The matter content of the universe at the critical temperature is calculated using the statistical bootstrap-model. The observed value of the entropy per baryon is deduced therefrom and shown to be related to the ratio of electromagnetic to strong interaction couplings. The relation of our results to recent work on the same subject is discussed.

KEY WORDS:

- Charge-symmetric universe -
- Spontaneous symmetry breakdown -
- Hadron phase transition -
- Baryon-antibaryon separation -
- Entropy per baryon.

(x) - Laboratorio di Astrofisica Spaziale del CNR, Frascati.

Until recently the kinematic aspects of the universe (global homogeneity and isotropy of space), supplemented with the principles of general relativity, formed the foundations of cosmology (Bondi, 1961). With the discovery of the microwave background radiation (Penzias et al., 1965) much attention is being paid to the dynamical aspects, in particular to a less sweeping representation of the conditions prevailing in the early universe (Harrison, 1968). From this point of view the ratio

$$\eta = N_B/N_\gamma \tag{1}$$

of the total number of baryons and antibaryons to photons is an extremely important parameter. Since the total number of particles of all kinds in a co-moving volume is a constant and proportional to the total entropy, and since presently the photon contribution to the entropy dominates overwhelmingly, η is essentially the reciprocal of the entropy per baryon. Because it is proportional to the matter content of the universe, theories of galaxy formation must be concerned with it. In a charge-asymmetric cosmology the initial baryon number contrast $\Delta B/B$ can be shown to be of the order of η (Harrison, 1968). Other interesting thermodynamic parameters are related to it e.g. the ratio of the specific heats of matter and radiation (Gamow, 1956) and the ratio of pressure to energy density (Alpher et al., 1967; Zeldovich, 1967). Its observed value is

$$\eta_{ob} \gtrsim 10^{-9}. \tag{2}$$

Theoretically (Chiu, 1966; Steigman, 1973; Zeldovich, 1965) it is grossly under-estimated

$$\eta_{th} \lesssim 10^{-18} \tag{3}$$

by at least nine orders of magnitude. This large discrepancy is not yet understood.

In this paper we present a simple model of a charge-symmetric universe in which η can be calculated completely thermodynamically at a uniquely determined temperature characteristic of the hadron era. The importance of this approach is that by stipulating a mechanism which prevents complete annihilation of baryons and antibaryons, and which need be only a few percent efficient, it is possible to accommodate the observed order of magnitude of η. A natural mechanism is suggested whose efficiency is determined by the ratio of electromagnetic to strong interaction couplings. For purposes of comparison we recall briefly the main arguments of the existing calculation of η (Steigman, 1973). By assumption the universe is charge-asymmetric, with an excess of nucleons over antinucleons. The number of nucleons N_B (each of mass m) and photons N_γ in equilibrium at a temperature T in a box of volume V are given respectively by

$$N_B = 4V\left(\frac{mT}{2\pi}\right)^{3/2} \exp(-m/T)\,, \qquad (4)$$

$$N_\gamma = \frac{2VT^3}{\pi^2}\,\zeta(3)\,, \qquad (5)$$

where $\zeta(3) \simeq 1.2$ is the zeta function.

The next step is to compare the expression for η gotten from these equations

$$\eta = 4\pi^2\left(\frac{m}{2\pi T}\right)^{3/2} \exp(-m/T)\,, \qquad (6)$$

with that obtained from dynamics. The argument involves the age of the universe as given by the (radiation-dominated) mass-energy density

$$\varrho_e = \frac{3}{32\pi G}\,\frac{1}{t^2} \simeq \rho_\gamma \simeq 8.4\times 10^{-36}\,T^4 \;\;(\mathrm{gm\,cm^{-3}}) \qquad (7)$$

and the time t_a when nucleons and antinucleons fail to meet each other and annihilate, as given by the annihilation mean free path

$$t_a = \frac{1}{(\frac{N_B}{V})\, v\, \sigma_a} , \tag{8}$$

where v is the relative velocity of the nucleon-antinucleon pair and σ_a their total annihilation cross section. Eqs. (5), (7) and (8) give another determination of η

$$\eta = \left(\frac{32\pi G}{3}\right)^{1/2} \left(\frac{1.2\,\pi^2}{v\,\sigma_a}\right) \frac{10^{-18}}{T} , \tag{9}$$

which compared with (6) fixes the decoupling temperature and hence η.

The above arguments are fairly straightforward and seemingly hard to fault. Perhaps for this reason the prediction in eq. (2) has been described as a catastrophe. Actually from the point of view of hadronic interactions these arguments are inadequate. Hadronic interactions enter through the product $v\,\sigma_a$ whose value has been estimated from scattering data (Steigman, 1973). There is no connection between it and the thermodynamic coexistence of the hadrons themselves. This is a serious limitation, for we must expect a strong dynamical correlation between the interactions of hadrons and their thermodynamic coexistence during the hadron era and after. The statistical bootstrap model (Hagedorn, 1965; 1970) provides a connection between thermodynamics and strong interactions of hadrons. The main equation of this model (Frautschi, 1971)

$$\rho(m) = \sum_{N=2}^{\infty} \frac{1}{N!} \left(\frac{V_o}{(2\pi)^3}\right)^{N-1} \prod_{i=1}^{N} \int d^3\vec{p}_i \; dm_i\, \rho(m_i) \;\times$$

$$\times \;\; \delta^3\Big(\sum_i p_i\Big)\, \delta\Big(\sum_i E_i - m\Big) , \tag{10}$$

consistently generates the hadron mass spectrum $\rho(m)$ from itself as input. Its solution for large mass is (Hagedorn, 1965; Frautschi, 1971)

$$\rho(m) \xrightarrow[m \to \infty]{} a\, m^{-b} \exp(m/T_o) \, , \tag{11}$$

with a, b constants and $T_o = 160$ MeV, the maximum equilibrium temperature of hadronic matter.

We propose to apply this model to investigate the stability of the early universe, with respect to baryon number composition, as it cools through the hadron era. We assume an initially uniform state with zero net baryon number. Consider then a cell of volume V large enough to contain average conditions of the universe. For times $t < 10^{-4}$ sec, V contains a large number of different species of hadrons described by the mass spectrum $\rho(m)$. Their free energy is (Bernstein et al., 1969; Omnès, 1972)

$$\begin{aligned} F(V,T,N,n) = {} & F(\pi, \tfrac{N+n}{2}) + F(\pi, \tfrac{N-n}{2}) - NT + \frac{T}{4V}(B_2+B_2')N^2 + \\ & + \frac{T}{4V}(B_2-B_2')n^2 + \frac{T}{24V^2}(B_3+3B_3')N^3 + \\ & + \frac{T}{8V^2}(B_3-B_3')Nn^2 + \cdots \end{aligned} \tag{12}$$

where

$$F(\pi, N) = -2NT \int_M^\infty dE \left(\frac{E}{2\pi T}\right)^{1/2} K_2\left(\frac{E}{T}\right) \rho_1(E) , \tag{13}$$

is the free energy of a background gas of pions interacting with N baryons and antibaryons immersed in it and $K_2(z)$ is the modified Bessel function of order two

$$B_2(T) = \frac{4\pi}{m^3 T^2} \int_{2m}^\infty dE\, E^2 K_2\left(\frac{E}{T}\right) \rho_2(E) , \tag{14}$$

$$B_2'(T) = \frac{4\pi}{m^3T^2} \int_0^\infty dE\, E^2\, K_2(\frac{E}{T})\, \rho_o(E) \quad , \tag{15}$$

are second virial coefficients from baryon-baryon (antibaryon-antibaryon) and baryon-antibaryon scattering respectively. If N is the number of baryons and $\overline{N}$ that of antibaryons, then the third virial coefficients B_3 and B_3' are the coefficients of the three-body terms $\frac{1}{2!}(N^2\overline{N} + \overline{N}^2 N)$ and $\frac{1}{3!}(N^3 + \overline{N}^3)$ respectively. They are related to the second virial coefficients and the so-called third cluster functions b_3 and b_3' by (Pais et al., 1959)

$$\begin{aligned} \frac{B_3}{6} &= -2(2B_2^2 - b_3) \ , \\ \frac{B_3'}{2} &= -2(2B_2'^2 - b_3') \ . \end{aligned} \tag{16}$$

In the application below we shall make the simplifying assumption

$$b_3 = b_3' \ . \tag{17}$$

The spectral functions $\rho_o(E)$, $\rho_1(E)$ and $\rho_2(E)$ corresponds to the channels with baryon number zero, one and two respectively. They behave for large argument as the total mass density (Frautschi, 1971)

$$\rho(m) = \sum_B \rho_B(m) \ . \tag{18}$$

The approximation involved in identifying $(S^{-1} \frac{\overleftrightarrow{\partial}}{\partial E} S)_B$, where S is the S-matrix (Bernstein et al., 1969), with the B-channel mass spectrum $\rho_B(E)$ amounts to assuming that asymptotically the S-matrix counting of resonances coincides with thermodynamic counting. This assumption provides the correlation between thermodynamics and strong interactions mentioned previously. The variable n in eq. (12) is an order parameter in the thermodynamic use of the term. It is the net baryon number of the system and is, by assumption, initially zero. Notice that in eq. (12) there are no non-interacting terms. This is required by the statistical bootstrap model because all the hadrons present are fully interacting. However if one considers a sum over all of them it is allowed for instance (Hagedorn, 1965; Frautschi, 1971) to replace $F(\pi, \frac{N \pm n}{2})$ by the non-interacting free energies $F_o(V, T, \frac{N \pm n}{2})$ of baryons (+) and antibaryons (-). The connection between them, in first approximation, is expressed through the chemical potential by

$$\mu_o = \frac{\partial F_o(N)}{\partial N} = - 2T \int_m^\infty dE \left(\frac{E}{2\pi T} \right)^{1/2} K_2\left(\frac{E}{T}\right) \rho_1(E), \qquad (19)$$

where, neglecting statistics, $F_o(V, T, N)$ is given by

$$F_o(V, T, N) = NT\left(\ln \frac{N}{VC(T)} - 1\right), \qquad (20)$$

with

$$C(T) = \frac{T}{2\pi^2} \int_m^\infty dE\, E^2 K_2\left(\frac{E}{T}\right) \rho_1(E) \; . \qquad (21)$$

The condition of thermodynamic equilibrium is obtained by minimising the free energy with respect to N; this gives

$$(B_2' + B_2)\left(\frac{N}{2V}\right) = 1 - \frac{1}{2}(B_3 + 3B_3')\left(\frac{N}{2V}\right)^2 + \cdots\cdots , \qquad (22)$$

where dots stand for terms we shall neglect. Our main idea is that there exists a regime of temperatures where this (mass stable) equilibrium (i.e. $\frac{\partial^2 F}{\partial N^2} > 0$) becomes unstable with respect to perturbations in the baryon number about $n = 0$.

The phase transition which accompanies this instability is due to the spontaneous breakdown of charge conjugation symmetry (Omnès, 1972). The condition defining the critical temperature T_c is

$$(B_2' - B_2)(\frac{N}{2V}) = 1 + (B_3 - B_3')(\frac{N}{2V})^2 + \cdots ; \quad T = T_c \;, \tag{23}$$

and is obtained by equating to zero the second derivative of the free energy with respect to n. For $T > T_c$ stable equilibrium prevails with $n = 0$ in all parts of V while for $T < T_c$ the free energy is minimised by non-zero values of n. Because of overall baryon number conservation in V the equilibrium values of n occur in pairs, say $\pm n_0(T)$, for each subvolume in which $n \neq 0$. The volume V is consequently partitioned into regions of matter and antimatter. These matter inhomogeneities are the galactic germs in this model.

We shall solve eqs. (22) and (23) for the critical temperature in two kinematical limits. An important consequence of the bootstrap postulate is that the energy density is dominated by non-relativistic hadrons. For small collision energies in fact B_3 and B_3' can, to a good approximation, be set equal to zero, so that eqs. (22) and (23) yield

$$B_2'(T)(\frac{N}{2V}) = 1 ; \qquad T = T_c \;, \tag{24}$$

$$B_2(T_c) = 0 \;. \tag{25}$$

The last equation is compatible with

$$B_2(T) = 0 \;, \tag{25'}$$

which is argued for, in this regime, by the exoticity of the channel with baryon number two, i.e. $\rho_2(E) \simeq 0$. Eqs. (24) and (25') were first obtained by Omnès (1972). They can also be understood by means of a simple argument of dimensional analysis, which relates the total cross sections to the second virial coefficients. For a totally absorptive scattering in a force field of range R the total cross section is

$\sigma = 2\pi R^2$, while from eq. (24) $B_2' \propto V \propto R^3$. Since for small energies the total baryon-antibaryon cross section ($\propto B_2'^{2/3}$) is much greater than the total baryon-baryon cross section ($\propto B_2^{2/3}$), eqs. (24) and (25') are consistent with the low energy behaviour of the cross sections.

If the dominant hadrons in equilibrium are relativistic we can again invoke the relationship between the second virial coefficients and total cross sections together with the Pomeranchuk theorem on total cross sections to conclude that

$$B_2(T) = B_2'(T) \ . \tag{26}$$

From eqs. (16), (17), (22), (23) and (26) we then have

$$B_2'(T)\left(\frac{N}{2V}\right) = \frac{5}{4} \ ; \qquad T = T_c \ , \tag{27}$$

together with a constraint on the third cluster function $b_3(T)$

$$b_3\left(\frac{N}{2V}\right)^2 = 3 \ . \tag{28}$$

Eqs. (24) and (27) are not substantially different. By making use of eq. (20) to express $(N/2V)$ in terms of the chemical potential $\mu_o(T)$ it is now easy to calculate the entropy per baryon at the critical temperature from either eq. (24) or (27). To do this it is necessary to fix the value of the power exponent b in the hadron mass spectrum. Various arguments have been given for considering values of $b \geqslant 5/2$ in applications (Hagedorn, 1965; 1970; Frautschi, 1971). Eq. (19) is consistent with eqs. (20), (21), (22) and (23) for b in this range. In all the calculations below we shall use $b = 5/2$. With this input one obtains from eqs. (15) and (21)

$$B_2'(T)\,C(T) = \frac{a_1 a_2}{m^3}\left(\frac{2T_o}{T_o+T}\right)\left|\ln\left(\frac{T_o-T}{2T_o}\right)\right|^2 \ , \tag{29}$$

where a_1, a_2 are defined similarly to a in eq. (11) and m is the mass of the dominant baryons present at the temperature T. From the thermodynamic relation

$$d\mu_0 = -\frac{S}{N}dT + \frac{V}{N}dp \ , \tag{30}$$

where S is the entropy and p the pressure, one gets from eqs. (20), (24), (27) and (30)

$$\frac{S}{N} = T\frac{\partial}{\partial T}(B_2' C(T)) - \frac{\mu_0}{T} \ . \tag{31}$$

For b= 5/2 and $T \rightarrow T_0$ the first term in (31) tends to infinity faster than the second which does so only as $\ln\ln(T_0-T)$. For T_c near T_0 we shall therefore set $\mu_0(T_c) = 0$ for simplicity. Making use of the fact that for given a_1 and a_2 eq. (29) relates the critical temperature to a critical mass, m_c, namely that of the dominant baryons, one finds at $T = T_c$

$$\left(\frac{N}{S}\right)_c = \left(\frac{T_0-T_c}{2T_c}\right)\left|\ln\left(\frac{T_0-T_c}{2T_0}\right)\right| \simeq \left(\frac{\lambda m_c^3}{a_1 a_2}\right)^{1/2} \times \\ \times \exp\left(-\left(\frac{\lambda m_c^3}{a_1 a_2}\right)^{1/2}\right) , \tag{32}$$

where $\lambda = 1$ or $5/4$ according as one uses eq. (24) or (27). Eq. (32) is amusingly similar to (6); the role of the fixed nucleon mass is now assumed by the constant $(a_1a_2)^{1/3}$. In fact for $b = 5/2$ the mass dimension of the a's is 3/2. The role of the temperature variable in eq. (6) is taken up by m_c in eq. (32). This relationship between mass and temperature is much deeper than we have been able to boring out in this simple application. In a wider context it is a consequence of the connection between the singularity structure of the analytic S-matrix (e.g. mass poles) and that of the thermodynamic partition function (e.g. temperature singularities at phase transitions).

Table I gives $\eta_c = (N/S)_c$ with the corresponding critical temperatures for typical values of m_c up to ten nucleon masses. For lack of a better estimate we have put $a_1 = a_2 = 2.6 \times 10^4$ $(MeV)^{3/2}$ (Hagedorn,1965).

TABLE I

The table shows values of η_c and the temperature difference T_o-T_c from eqs.(24) and (27) as they vary with $m_c = km_p$ (k = 1, 2, ..., 10) where m_p is the mass of the proton.

	$\lambda = 1$		$\lambda = 5/4$	
m_c/m_p	η_c	T_o-T_c(MeV)	η_c	T_o-T_c(MeV)
1	0.37	108.5	0.36	92.8
2	0.14	14	0.11	9.6
3	1.81×10^{-2}	1.01	1.03×10^{-2}	0.51
4	1.26×10^{-3}	4.54×10^{-2}	4.95×10^{-4}	1.6×10^{-2}
5	5.45×10^{-5}	1.52×10^{-3}	1.33×10^{-5}	3.07×10^{-4}
6	1.39×10^{-6}	2.73×10^{-5}	2.27×10^{-7}	4.0×10^{-6}
7	2.52×10^{-8}	3.94×10^{-7}	2.4×10^{-9}	4.83×10^{-8}
8	3.38×10^{-10}	4.32×10^{-9}	1.86×10^{-11}	2.13×10^{-10}
9	2.81×10^{-12}	3.0×10^{-11}	9.1×10^{-14}	8.67×10^{-13}
10	2.2×10^{-14}	2.01×10^{-13}	3.67×10^{-16}	3.0×10^{-15}

From a theoretical point of view Table I is interesting by itself in as much as η_c has been calculated thermodynamically in a self-consistent theory using eqs.(20), (24), (27) and (30) in conjuction with the strong interaction input eq.(29). This fact suggest the most direct connection between η_c and the observed value η_{ob} based on the possibility of this latter being calculated in the same theoretical framework with the electromagnetic coupling substituted for the strong interaction coupling. To begin with note that in order to conserve the created regions of matter and antimatter, which must eventually evolve into galaxies and antigalaxies, it is necessary to stipulate a separation mechanism which prevents their complete annihilation. On the basis of the efficiency of this mechanism it is clear that the entries in Table I discriminate between possible models of evolution for $T < T_c$. For instance since η_c is necessarily greater than η_{ob} (some annihilation being inevitable) it follows that models with $m_c \gg m_p$, where m_p is the proton mass, are not acceptable. This low mass restriction is important in that it encourages extrapolation of laboratory data to our cosmological problem. This extrapolation is not natural in the model (Alexanian et al., 1975) where matter-antimatter separation takes place between superbaryons of mass $10^5\ m_p$.

On the other hand since the entropy is essentially proportional to the total number of particles of all kinds, models with low values of m_c (e.g. $m_c = m_p$, $2m_p$, $3m_p$) envisage a relatively large contribution from baryons and antibaryons to the total entropy in the hadron era. This situation obtains in classical cosmology where nucleons are the only hadrons contemplated. However for such models the separation mechanism must be highly inefficient, the efficiency ($\sim 10^{-6}$% for $m_c = m_p$) being itself of the order of η_{ob}. Thus as far as understanding the smallness of η_{ob} is concerned, knowledge of η_c in these models is not helpful since the problem of finding the small efficiency is equivalent to that of η_{ob}. One is therefore left with models in which η_c is itself a small quantity. In the hadron era this corresponds to neutral background (pion gas) dominance of the total entropy, exactly as in the radiation era. The interaction of baryons and antibaryons with the background is represented thermodynamically by the chemical potential $\mu_o(T)$ (cf. eq.(19)). At equilibrium this coupling permits just a definite amount, $n_B(\pi) = N/V$, of baryons and antibaryons per unit volume, given by eq. (23), to coexist with pions. Consequently annihilation of baryons and antibaryons for $T < T_c$ cannot be a blind search- and- destroy. Only that much annihilation occurs which allows the photon gas in the radiation era and after to coexist thermodynamically with a definite amount $n_B(\gamma)$ of baryons and antibaryons per unit volume, given by an equation identical to (23). Making use of the constancy of entropy per unit comoving volume and applying eq. (30) one finds immediately

$$\eta_{ob} = \eta_c \frac{n_B(\gamma)}{n_B^c} = \eta_c \frac{(\partial \mu_o/\partial p)_c}{(\partial \mu_o/\partial p)_\gamma} , \qquad (33)$$

where n_B^c is the value of $n_B(\pi)$ at $T = T_c$. An extremely appealing estimate for $n_B(\gamma)/n_B^c$ suggested by these considerations is that it is equal to the ratio between electromagnetic and strong interaction coupling constants i.e.

$$\frac{n_B(\gamma)}{n_B^c} \sim 10^{-2} . \qquad (34)$$

From eqs. (33) and (34) a critical mass is easily found from Table I to reproduce the right order of magnitude of η_{ob}.

We conclude with a few remarks. Applied to a created matter inhomogeneity our results are the same as those of Huang and Weinberg (1970). The limit temperature fixed by them from the value of η_{ob} coincides with our critical temperature. We have shown already that the model of Omnès (1971) is a special solution of our own. It is possible to obtain the results of Alexanian and Mejia-Lira (Alexanian et al., 1975) from our equations. In fact saturating eq. (21) with a particle pole at $E = m_o$ and making use of eqs. (19), (20) and (24) and setting $B_2^{\frac{1}{2}}(T) = V_c$ (V_c some fundamental volume) one recovers their principal equation

$$\frac{V_c T_c}{2\pi^2} m_o^2 K_2 (m_o/T_c) = 1 . \qquad (35)$$

The limit of this equation for $m_o \to 0$ is

$$\frac{V_c T_c^3}{\pi^2} = 1 . \qquad (36)$$

By equating together two apparently unrelated large numbers Alexanian (1975) has used eq. (36) to calculate the fine structure constant $\alpha = e^2/4\pi$. This ad hoc procedure can be physically motivated in our approach using arguments analogous to those leading to eq. (34). The main inputs are : (i) the constancy of the total entropy; (ii) neutral background dominance of the entropy; and (iii) in any era the total entropy measures the ratio of the coupling mediated by the quanta of the background medium to the gravitational coupling. Since, with our choice of units, entropy is dimensionless, appropriate combinations of these coupling constants can always be found such that the entropy can be written as

$$S = \frac{g^2/4\pi}{G\, m_1 m_2} , \qquad (37)$$

where $g^2/4\pi$ is the strength of the interaction mediated by the quanta of the background medium and $G\, m_1\, m_2$ the gravitational coupling between the masses m_1 and m_2.

Assuming the pion gas in the hadron era behaves radiation-like one can use eq. (36) and the first part of eq. (7) to write, at time $t \sim \sim 1/m_\pi$ and temperature T_π

$$S = S_\pi = \frac{V_\pi \rho_e(\pi)}{T_\pi} = \left(\frac{m_\pi}{T_\pi}\right)^4 \frac{3\pi}{32\, G\, m_\pi^2} . \qquad (38)$$

If we let $T_\pi \sim m_\pi$ then S_π is of the form (37) with $g_\pi^2/4\pi \simeq 1$. By the same token the entropy in the radiation era can be written as

$$S = S_\gamma = \frac{V_\gamma \rho_e(\gamma)}{T_\gamma} = \frac{\pi^2}{T_\gamma^4} \rho_e(\gamma) = \frac{g_\gamma^2/4\pi}{G m_e m_p} , \qquad (39)$$

where m_e is the mass of the electron and m_p that of the proton. Equating (38) and (39) and putting in the known values of the masses one finds

$$g_\gamma^2/4\pi \simeq e^2/4\pi . \qquad (40)$$

The choice $m_1 = m_e$ and $m_2 = m_p$ therefore correctly normalizes eq. (37). This is the observation of Alexanian (1975).

REFERENCES. -

- Alexanian M., Phys. Rev. D4, 722 (1975).
- Alexanian M. and Mejia-Lira F., D4, 716 (1975).
- Alpher R., Gamow G. and Herman R., Proc. Nat. Acad. Sci. 58, 2179 (1967).
- Bernstein H. J., Dashen R. and Ma S., Phys. Rev. 187, 345 (1969).
- Bondi H., Cosmology (Cambridge Univ. Press, London, 1961).
- Chiu H. Y., Phys. Rev. Letters 17, 712 (1966).
- Frautschi S., Phys. Rev. D3, 2821 (1971).
- Gamow G., Physics of the expanding universe in vistas in Astronomy (Ed. A. Beer) (Pergamon Press, London, 1956), Vol. 2, pag. 1726.
- Hagedorm R., Nuovo Cimento Suppl. 3, 147 (1965).
- Hagedorn R., Astron. and Astrophys. 5, 184 (1970).
- Harrison E. R., Physics Today 5, 31 (1968).
- Huang K. and Weinberg S., Phys. Rev. Letters 25, 895 (1970).
- Omnès F., Phys. Report 3C, 1 (1972).
- Pais A. and Uhlenbeck G., Phys. Rev. 116, 2501 (1959).
- Penzias A. A. and Wilson R. W., Astrophys. J. 142, 419 (1965).
- Steigman G., The case against antimatter in the universe, in Cargese Lectures in Physics (Ed. E. Schatzman) (Gordon and Breach, New York, 1973), pag. 505.
- Zeldovich Ya. B., Adv. Astron. Astrophys. 3, 241 (1965).
- Zeldovich Ya. B., Soviet Phys. -Uspekhi 9, 602 (1967).