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M. Greco and A. F. Grillo: RADIATIVE ASYMMETRY IN $e^+e^- \to \mu^+\mu^-$ NEAR A NARROW RESONANCE WITH POLARIZED BEAMS.

In this note we present the results of the evaluation of forward-backward asymmetry in $e^+e^- \to \mu^+\mu^-$ near a narrow vector resonance, in the case of polarized beams. The infrared factors are considered to all orders in α, as explicity given in the recent analysis[1] of the radiative corrections for colliding beam resonances. The purely electromagnetic asymmetry in presence of polarization of the incident particles has been evaluated by Khriplovich[2] and Brown et al.[3] in the leading order of soft-photon emission, while the presence of a narrow resonance has been taken into account by Altarelli et al.[4] and Kraemmer and Lautrup[5], for non polarized beams to the same order in α. The full soft photon contribution to the asymmetry can be easily read off from I, in absence of polarization of the incident beams.

The main results of the above calculations is that strong cancellations occur, near the resonance, between the emission of real and virtual soft photons, such to make the angular asymmetry considerably small and even smaller of the pure QED value. Our evaluation shows that the inclusion of the possible radiative polarization of the electrons and positrons in the storage ring[6] reduces even more the charge asymmetry in the angular distribution.

The results obtained in I, for the case of nonpolarized beams, are the starting point of our analysis. The differential cross section to all orders in the soft photon emission is given by[1] :

$$\frac{d\sigma}{d\Omega} = C^{RES}_{INFRA}\left(\frac{d\sigma}{d\Omega}\right)_{RES}(1 + C^{RES}_{F}) + + C^{INT}_{INFRA}\left(\frac{d\sigma}{d\Omega}\right)_{INT}(1 + C^{INT}_{F}) + + C^{QED}_{INFRA}\left(\frac{d\sigma}{d\Omega}\right)_{QED}(1 + C^{QED}_{F}) , \tag{1}$$

with

$$\left(\frac{d\sigma}{d\Omega}\right)_{RES} = \frac{\alpha^2}{4s}(1 + z^2)\left(\frac{3\Gamma e}{2\alpha}\right)^2 \frac{1}{y^2 + (\frac{\Gamma}{2})^2} , \tag{2a}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{INT} = \frac{\alpha^2}{4s}(1 + z^2)\left(\frac{3\Gamma e}{\alpha}\right) \frac{1}{y^2 + (\frac{\Gamma}{2})^2} , \tag{2b}$$

$$\left(\frac{d\sigma}{d\Omega}\right)_{QED} = \frac{\alpha^2}{4s}(1 + z^2) , \tag{2c}$$

$$C^{RES}_{INFRA} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu}\left\{\frac{y^2 + (\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right\}^{\frac{\beta_e}{2}} \times \left\{1 + \beta_e \frac{2y}{\Gamma}\left[\frac{\pi}{2} + tg^{-1}\left(\frac{2y}{\Gamma}\right)\right]\right\} , \tag{3a}$$

$$C^{INT}_{INFRA} = \left(\frac{\Delta\omega}{E}\right)^{\beta_\mu + \beta_{int}}\left\{\frac{y^2 + (\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right\}^{\frac{\beta_e}{2}} \times \left\{1 - \beta_e \frac{\Gamma}{2y}\left[\frac{\pi}{2} + tg^{-1}\left(\frac{2y}{\Gamma}\right)\right]\right\} , \tag{3b}$$

$$C^{QED}_{INFRA} = \left(\frac{\Delta\omega}{E}\right)^{\beta_e + \beta_\mu + 2\beta_{int}} , \tag{3c}$$

$$C_F^{RES} \equiv \delta_{RES} + Y(\theta) = \frac{3}{4}(\beta_e + \beta_\mu) + + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{1}{2}\right) + \frac{4\alpha^2 \Gamma}{9\Gamma_e} + Y(\theta) , \tag{4a}$$

$$C_F^{INT} \equiv \delta_{INT} + \frac{1}{2}X(\theta) = \frac{11}{12}(\beta_e + \beta_\mu) + + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{13}{18}\right) + \frac{1}{2}X(\theta) , \tag{4b}$$

$$C_F^{QED} \equiv \delta_{QED} + X(\theta) = \frac{13}{12}(\beta_e + \beta_\mu) + + \frac{2\alpha}{\pi}\left(\frac{\pi^2}{3} - \frac{17}{18}\right) + X(\theta) , \tag{4c}$$

where, with obvious notations, $s = 4E^2$ is the total energy squared of the incoming electrons, $M, \Gamma, \Gamma_e \simeq \Gamma_\mu$ are the mass, the total and leptonic widths of the 1^{--} resonance, $y = 2E - M$, $z = \cos\theta$, $\Delta\omega$ the energy resolution of the experiment, $\beta_{e,\mu} = \frac{4\alpha}{\pi}(\ln\frac{2E}{m_{e,\mu}} - \frac{1}{2})$ and $\beta_{int} = \frac{4\alpha}{\pi}\ln(\mathrm{tg}\,\frac{\theta}{2})$. The functions $X(\theta)$ and $Y(\theta)$ are defined as:

$$X(\theta) = -\frac{4\alpha}{\pi}\left\{\frac{1}{1+z^2}\left[z\left(\ln^2\sin\frac{\theta}{2} + \ln^2\cos\frac{\theta}{2}\right) + + \sin^2\frac{\theta}{2}\ln\cos\frac{\theta}{2} - \cos^2\frac{\theta}{2}\ln\sin\frac{\theta}{2}\right] + \ln^2\cos\frac{\theta}{2} - - \ln^2\sin\frac{\theta}{2} + \frac{1}{2}\left[Li_2(\sin^2\frac{\theta}{2}) - Li_2(\cos^2\frac{\theta}{2})\right]\right\} , \tag{5a}$$

$$Y(\theta) = \frac{4\alpha^2}{3}\frac{\Gamma}{\Gamma_e}\left\{\frac{1}{1+z^2}\left[z - 2z\left(\ln\sin\frac{\theta}{2} + \ln\cos\frac{\theta}{2}\right)\right] + + 2\ln\mathrm{tg}\,\frac{\theta}{2}\right\}, \tag{5b}$$

and contribute only to the antisymmetric cross-section, being odd in the exchange $\theta \to \pi - \theta$.

In the case of polarized beams, it is easy to see that the infrared corrections (eqs. (3)) remain unchanged. This simply follows from the observation that the factorization properties of the matrix elements in the classical current factors do not depend on the polarization of the initial electrons.

The only changes in (1) are therefore those arising from the lowest order matrix elements and their interference with next order virtual photon exchange diagrams, which affect $(d\sigma/d\Omega)_i$ and C_F^i respectively (i = RES, INT, QED). In particular, because of the cancellations between real and virtual photon exchanges, discussed in detail in refs. (1, 4, 5), it is enough to take into account the pure QED box diagrams and the corresponding interference with the Born terms.

Assume, in the c. m. frame, the positive direction as given by the incident electron, and a polarization $p_\pm$ of the $e^\pm$ along the x-axis, the direction of the magnetic field. The μ^- will emerge at angles (θ, ϕ) in this system. Then a straightforward calculation shows that eqs. (2) are simply modified by the substitution

$$1+\cos^2\theta \to f(\theta,\phi) = 1+\cos^2\theta - |p_+p_-|\sin^2\theta\cos 2\phi \,.$$

Finally, a standard calculation of the interference between the Born terms and the QED box diagrams leads to the following modification of the functions $X(\theta)$ and $Y(\theta)$ and therefore of the finite factors C_F^i (eqs. (4)):

$$X_{pol}(\theta,\phi) = \frac{\alpha}{\pi}\left\{ -\cos\theta\left(\frac{\ln^2\sin\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} + \frac{\ln^2\cos\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right) + \right.$$

$$+\left(\frac{\ln\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} - \frac{\ln\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\right) + 4\left[\ln^2\sin\frac{\theta}{2} - \ln^2\cos\frac{\theta}{2} + \right. \tag{6a}$$

$$\left. + \frac{1}{2}\mathrm{Li}_2\left(\cos^2\frac{\theta}{2}\right) - \frac{1}{2}\mathrm{Li}_2\left(\sin^2\frac{\theta}{2}\right)\right] + \frac{2\cos\theta}{1+\cos^2\theta - |p_+p_-|\sin^2\theta\cos 2\phi} \,.$$

$$\left. \cdot \left[-\cos\theta\left(\frac{\ln^2\sin\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} - \frac{\ln^2\cos\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right) + \left(\frac{\ln\sin\frac{\theta}{2}}{\cos^2\frac{\theta}{2}} + \frac{\ln\cos\frac{\theta}{2}}{\sin^2\frac{\theta}{2}}\right)\right]\right\},$$

and

$$Y_{pol}(\theta,\phi) = -\frac{2\alpha^2}{3}\frac{\Gamma}{\Gamma_e}\left\{\left[2\ln tg\frac{\theta}{2} - \frac{1}{2}\cos\theta\left(\frac{\ln\sin\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} + \frac{\ln\cos\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right) - \frac{\cos\theta}{\sin^2\theta}\right] + \frac{2\cos\theta}{1+\cos^2\theta - |p_+p_-|\sin^2\theta\cos 2\phi} \times \right. \tag{6b}$$

$$\left. \times\left[\frac{-\cos\theta}{2}\left(\frac{\ln\sin\frac{\theta}{2}}{\cos^4\frac{\theta}{2}} - \frac{\ln\cos\frac{\theta}{2}}{\sin^4\frac{\theta}{2}}\right) + \frac{1}{\sin^2\theta}\right]\right\}.$$

It is trivial to verify that eqs. (6) reduce to (5) in absence of polarization of the incident beams, or when $\phi = \pm 45^o$.

With all that in mind, the charge asymmetry, defined by

$$A(\theta,\phi) = \frac{\frac{d\sigma}{d\Omega}(\theta,\phi) - \frac{d\sigma}{d\Omega}(\pi-\theta,\phi)}{\frac{d\sigma}{d\Omega}(\theta,\phi) + \frac{d\sigma}{d\Omega}(\pi-\theta,\phi)}, \tag{7}$$

can be easily obtained through eqs. (1-4) with the suitable modification factors. However, due to the smallners of β_{int} in the kinematical regions of experimental interest, we can safely expand in $\beta_{int}\ln(\Delta\omega/E)$ and obtain the simpler expressions :

$$\frac{1}{2}\left|\frac{d\sigma}{d\Omega}(\theta,\phi) - \frac{d\sigma}{d\Omega}(\pi-\theta,\phi)\right| = \frac{\alpha^2}{4s}\left(\frac{\Delta\omega}{E}\right)^{\beta_\mu} f(\theta,\phi) \times$$

$$\times\left\{\left[\frac{y^2+(\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right]^{\frac{\beta_e}{2}}\left(\frac{3\Gamma_e}{2\alpha}\right)^2\frac{1+\beta_e\frac{2y}{\Gamma}(\frac{\pi}{2}+tg^{-1}(\frac{2y}{\Gamma}))}{y^2+(\frac{\Gamma}{2})^2} \times\right.$$

$$\times\, Y_{pol}(\theta,\phi) + \left[\frac{y^2+(\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right]^{\frac{\beta_e}{2}}\left(\frac{3\Gamma_e}{2\alpha}\right) \times \tag{8}$$

$$\times\frac{y-\beta_e\frac{\Gamma}{2}(\frac{\pi}{2}+tg^{-1}(\frac{2y}{\Gamma}))}{y^2+(\frac{\Gamma}{2})^2}\left[(1+\delta_{int})2\beta_{int}\ln\left(\frac{\Delta\omega}{E}\right) + X_{pol}(\theta,\phi)\right] +$$

$$+ \left(\frac{\Delta\omega}{E}\right)^{\beta_e} \left[(1+\delta_{QED})\, 2\beta_{int} \ln\left(\frac{\Delta\omega}{E}\right) + X_{pol}(\theta,\phi) \right] \Bigg\} ,$$

$$\frac{1}{2}\left[\frac{d\sigma}{d\Omega}(\theta,\phi) + \frac{d\sigma}{d\Omega}(\pi-\theta,\phi)\right] = \frac{\alpha^2}{4s}\left(\frac{\Delta\omega}{E}\right)^{\beta_\mu} f(\theta,\phi) \ x$$

$$x \left\{ \left[\frac{y^2+(\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right]^{\frac{\beta_e}{2}} \left(\frac{3\Gamma_e}{2\alpha}\right)^2 \frac{1+\beta_e \frac{2y}{\Gamma}(\frac{\pi}{2}+tg^{-1}(\frac{2y}{\Gamma}))}{y^2+(\frac{\Gamma}{2})^2} \ x \right. \tag{9}$$

$$x\ (1+\delta_{RES}) + \left[\frac{y^2+(\frac{\Gamma}{2})^2}{(\frac{M}{2})^2}\right]^{\frac{\beta_e}{2}} \left(\frac{3\Gamma_e}{\alpha}\right) \frac{y-\beta_e\frac{\Gamma}{2}(\frac{\pi}{2}+tg^{-1}(\frac{2y}{\Gamma}))}{y^2+(\frac{\Gamma}{2})^2} \ x$$

$$\left. x\ (1+\delta_{INT}) + \left(\frac{\Delta\omega}{E}\right)^{\beta_e} (1+\delta_{QED}) \right\} ,$$

where we have assumed $(\beta_{int} \ln\frac{\Delta\omega}{E})^2 \ll 1$ $(\beta_{int} \ln\frac{\Delta\omega}{E} \simeq 0.02$ for $\theta = 45^{\circ}$ and $\Delta\omega/E = 0.1)$.

Our results are plotted in Fig. 1, in the region of $\psi/J(3100)$, for $\Delta\omega/E = 0.2$, in the case of non-polarized beams (full lines) and in the best conditions to enhance the polarization effects, when $|p_+| = |p_-| = 0.924$ and $\phi = 0$ (dotted lines).

In the first case, which is nothing but the results of I, a few lines are in order to compare with those of refs. (4) and (5). In ref. (4) the antisymmetric cross section (our eq. (8)) has been evaluated in the lowest order of the soft photon emission, and the factor

$$\left\{\left[y^2+(\Gamma/2)^2\right]/(M/2)^2\right\}^{\beta_e/2} ,$$

both for the resonant and interference terms, has been taken as unity. The symmetric cross section however is identical to our eq. (9). The inclusion of the full soft photon effects makes dips before the resonance almost completely filled up and shifts on the right the position of the first zero, as shown in Fig. 1. Similarly, the above factor is missing in ref. (5), both in the symmetric and antisymmetric cross sections, giving rise to a sharp structure in the asymmetry before the resonance, which is also smoothened when all orders are included in exponentiated form.

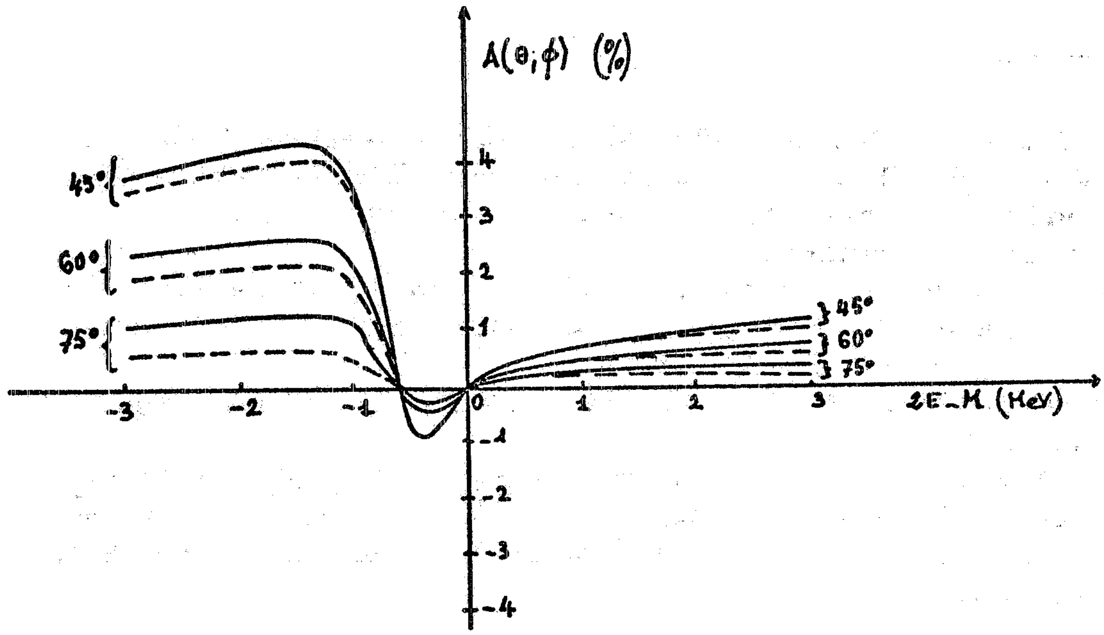

FIG. 1 - Asymmetry $A(\theta,\phi) = [d\sigma/d\Omega]_A / [d\sigma/d\Omega]_S$ as a function of the distance from the resonance, for various values of θ. The full and dotted lines refer to the unpolarized and polarized cases respectively (see text). The parameters are chosen as follows : M = 3100 MeV, Γ = 69 keV, Γ_e = 4.8 keV and $\Delta\omega/E$ = 0.2.

The effect of polarization of the incident beams always reduce the asymmetry, as explicitly shown in Fig. 1, the reduction factors ranging from about 10% for $\theta = 45^o$ to 50% and more at large angles. Of course such effect depends on $|p_\pm|$ and ϕ and decreases smoothly for $0 < \phi \leq 45^o$.

In all the above considerati ons we have neglected the effect of the folding in the beam energy resolution function. When this is included, the resulting asymmetry becomes always positive and smaller, of the order of a few parts for thousand, in the same energy range considered above.

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