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G. Parisi: TOWARD A FIELD THEORY OF CONFINEMENT.

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ABSTRACT. -

We propose the use of compact technicques to obtain exact and approximate results in field theories in which a non local long range interaction is present.

It is fashionable to assume that quarks are the fundamental costituents of the hadronic matter(1, 2). A very long range force among quarks has been postulated(3). It is apparent that in a four dimensional world such an interaction cannot arise from the exchange of a zero mass quanta: models have been constructed in which a similar interaction is produced by non linear phenomena(4, 5). Unfortunately these models can be treated only qualitatively and it is very difficult to go beyond semiclassical approximations.

A different approach would be to start from a phenomenological Lagrangian in which a non local long range interaction among quarks has been introduced and to use this Lagrangian to get quantitative predictions on the properties of hadrons. We have in mind a Lagrangian of the type:

$$\mathcal{L} = \mathcal{L}_{FREE} + \int dx\, dy\ \rho(x)\ \rho(y)\ V(x-y) \qquad (1)$$

where ρ is a density operator, bilinear in the quark field and $V(x)$ is a phenomenological potential. If $V(x)$ increases at infinity, one expects naturally that quarks are confined also for small values of the coupling costant. Our aim is to compute the spectrum of the bound states, the S-matrix, to verify the Regge behaviour and the duality properties, to check the parton model results... It is evident that the success of this program would be an important step toward the construction of a dynamical theory of hadrons ; perturbation theory is useless and more powerful techniques must be used. In this note we argue that a suitable approximation is : firstly to neglect all the quarks loops (NQLA) and only at the end to make a perturbation expansion in the number of the quarks loops.

The approximate validity of the Zweig selection rule(2) suggestes that diagrams containing quarks loops give a negligible contribution to the hadronic mass spectrum ; moreover in a beatiful and unfortunately not enough well known series of paper on Euclidean quantum field theory Symanzik(6) has shown that the procedure we have outlined is convergent also for not small values of the coupling costant, if infrared and ultraviolet divergences are absent. The real advantage of this approach is the existence of compact expressions for the sum of all Feynmann diagram in the NQLA.

Let us illustrate these techniques in the simplest possible example. We consider a Lagrangian as in eq. (1) : two scalar fields are present q and $\bar{q}$; they will be denoted respectively quarks and anti-quarks. The density ρ and the potential V(x) are :

$$\rho = q^2 - \bar{q}^2 , \qquad V(x) = g(x^2)^{\alpha} \qquad g > 0 \qquad (2)$$

The interaction is attractive among quarks and anti-quarks, but it is repulsive among particles of the same kind.

In the NQLA approximation we have proved the following results :

a) In the Euclidean region when x goes to infinity the quark propagator satisfies the following inequalities :

$$\begin{aligned} &0 \le G(x) \le \exp - A V(x) && A > 0 && \tfrac{1}{2} < \alpha < 1 \\ &0 \le G(x) \le \exp - B x^2 && B > 0 && \alpha \gtrsim 1 \end{aligned} \qquad (3)$$

The Fourier transform of the propagator is holomorfic in the complex p^2 plane, a property that whas suggested in ref. (7) and proved to be true in a soluble model in ref. (8).

b) Although no singularities are present in the quark propagator the quark anti-quark scattering amplitude has a singularity in the "zero triality" channel. This results follows from the inequality :

$$\langle 0 | T \left[q(\lambda x + y) \bar{q}(\lambda x - y) q(z) \bar{q}(t) \right] | 0 \rangle \qquad (4)$$

$$f(x, y, z, t) \exp(-\lambda C | x |) \qquad C > 0 \qquad \lambda \to \infty$$

c) The short distance behaviour of the Green functions is the same as in free field theory.

d) If $\alpha = 1$ the model is soluble (in NQLA) and the mass spectrum can be exactly computed.

We want only the sketch the technique used to prove these results. We start from the following rappresentation for the propagator in the NQLA (similar rappresentations are valid for higher order Green functions[6, 9, 10]):

$$G(x) = \int_0^\infty \exp(-m^2 t)\, P^t_{ox}(d\omega) \exp(-U[\omega]) \equiv$$

$$\equiv \int_0^\infty \exp(-m^2 t)\, \mathcal{D}^t_{ox}(d\omega) \exp\left(-\frac{1}{2}\int_0^t \dot{\omega}_\mu(\tau)\, \dot{\omega}^\mu(\tau)\, d\tau - U[\omega]\right) \qquad (5)$$

$$\dot{\omega}_\mu = \frac{d}{d\tau}\, \omega_\mu \,, \qquad U[\omega] = g \int_0^t d\sigma \int_0^t d\tau\, V\left[\omega(\sigma) - \omega(\tau)\right]$$

Let us explain our notations which are the same as ref. (6) $P^t_{xy}(d\omega)$ is the conditional Wiener measure on the space of continuous paths in R^4 from x to y in time t; it is normalized in such way that

$$\int P^t_{xy}(d\omega) = \exp\left[-(x-y)^2/2t\right] / (2\pi t)^2 \equiv K(x-y, t) \qquad (6)$$

$\mathcal{D}^t_{ox}(\omega)$ is a shift invariant measure on trajectory space which is formally defined by

$$P^t_{xy}(d\omega) = \exp -\frac{1}{2}\int_0^t \dot{\omega}_\mu(\tau)\, \dot{\omega}^\mu(\tau)\, d\tau\, \mathcal{D}^t_{ox}(d\omega) \qquad (7)$$

Having written the sum of all Feynmann diagrams in a compact form we can prove points a)-d) using standard mathematical techniques. The upper bound in a) is proved using the saddle point integration technique: the convexity properties of the exponent of the integrand imply that the output of the saddle point integration is actually an upper bound to the value of the integral.

The lower bound in a) and b) is derived from the Jensen convexity inequality[6, 11]:

$$\int d\mu(s) \exp(f(s)) \geq M \exp\left[\frac{1}{M}\int d\mu(s)\, f(s)\right]; \quad M = \int d\mu(s) \quad (8)$$

Applying it to eq. (5) we get:

$$G(x) \geq \int_0^\infty dt \exp(-tm^2) K(x,t) \exp\left[-g \int_0^t d\sigma \int_0^t d\tau \int dy \int dz \cdot \right. \qquad (9)$$

$$\left. \cdot\ K(y,\sigma)\, K(z-y, \tau-\sigma)\, K(x-z, t-\tau)\, V(z-y)\right]$$

The short distance behaviour of the theory is investigated using lower and upper bounds for the Green functions : these bounds concide with the free field results in the short distance limit. We can use as lower bound the output of the Jensen inequality; the upper bound can be obtained or using saddle point integration or the generalized Holder inequality[6]:

$$\int d\nu(t) \exp\left[\int d\mu(s)\, g(s,t)\right] \leq$$

$$\leq \exp\left\{\frac{1}{M}\int d\mu(s) \ln\left[\int \exp\left[M g(s,t)\right] d\nu(t)\right]\right\} \qquad (10)$$

where, as in Jensen inequality, μ and ν are non negative measures. The application of this inequality to the propagator yelds the bound:

$$G(x) \leq \int dt \exp(-tm^2) \exp\left\{\frac{1}{t^2}\int d\tau\, d\sigma \ln\left[\int dy\, dz \cdot \right.\right. \qquad (11)$$

$$\left.\left. \cdot\ K(y,\sigma)\, K(z-y \quad \tau-\sigma)\, K(x-z, t-\tau) \exp\left[-gt^2 V(x-z)\right]\right]\right\}$$

The proof of point d) is rather trivial: the exponent is a quadratic form, the integral is gaussian and can be exactly performed. The existence of this soluble model may be used to check the validity of various approximation techniques and to investigate formal properties of the theory.

The generalization of these techniques to a current current interaction of bosons is straightforward; the extension to the fermionic case may present some difficulties. WKB techniques can also be used. The introduction of interal quantum numbers (SU(3), color) can be readly done. We think that following these lines it will be possible to construct a realistic theory of strong interactions; using few parameters it should be possible to obtain qualitative and quantitative predictions for the properties of strong interacting particles.

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