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G. Matone and A. Tranquilli: MODE-LOOKING ON A LONG  
CAVITY TO OBTAIN HIGH POWER LASER PULSES.

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INTRODUCTION. -

As it has been reported elsewhere<sup>(1)</sup>, the Ladon project foresees the interaction region between photons and electrons in the Adone right cross section.

Since the monochromaticity of the gamma-beam is mainly controlled by the electron angular divergency, it is recommended to avoid the quadrupole region, where this quantity could be substantially worse.

In the present planning a suitable bending allows to shift the beam orbit in such a way as to limit the interaction region within the two quadrupoles<sup>(2)</sup>.

Aim of the present paper is to present an alternative solution which suggests to modify our CW Argon ion Laser into a pulsed Laser by using the mode locking technique.

The repetition frequency we want to obtain must be equal to the separation in time between two bunches in Adone which is 117 nsec.

1) Modes in a Laser cavity. -

A mode has been defined as a standard configuration of the electromagnetic field satisfying the Maxwell equation and the boundary conditions of the cavity.

For a stable resonator the mode frequency is given by<sup>(3)</sup>:

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$$(1) \quad \nu = \frac{c}{2L} \left[ (q+1) + \frac{m+n+1}{\pi} \cos^{-1} \sqrt{\left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right)} \right]$$

where:

L = cavity length

$R_1, R_2$  = curvature radii of the cavity mirrors

c = light velocity

From the relation (1) one can see that:

a) two modes with the same indices m, n and with q-value different by one unit, show the following difference in frequency:

$$(2) \quad \nu_q - \nu_{q-1} = c/2L = \nu$$

b) two modes with the same index q but with (m+n)-value different by one unit, differ in frequency by the following amount:

$$(3) \quad \nu_{m+n} - \nu_{m+n-1} = \frac{c}{2\pi L} \cos^{-1} \sqrt{\left(1 - \frac{L}{R_1}\right) \left(1 - \frac{L}{R_2}\right)}$$

2) Mode Locking. -

It is well known that many longitudinal modes oscillate in a normal gas Laser as due to the intrinsic width of the line, the Doppler effect, the broadening for thermal collisions and the gain hole effect(4).

It is easy to see that the number of the longitudinal modes is given by:

$$(4) \quad N = \frac{\Delta \nu}{\nu}$$

where  $\Delta \nu$  is the total width of the line.

Therefore, the wave electric field E(t) in a generic position can be considered as the superposition of these N longitudinal modes with amplitude  $E_n$ :

$$(5) \quad E(t) = \sum_{n=1}^N E_n e^{i[(\omega_0 + n\omega)t + \Phi_n]}$$

where  $\omega_0$  is a generic reference frequency and  $\Phi_n$  is the phase of the n-th mode.

In general the phase  $\Phi_n$  varies in a random way and this causes the known fluctuations in the output power.

A possible way to avoid this inconvenience is to lock among themselves the relative values of the phases.

Let us put in eq. (5)  $\Phi_n = 0$  and for simplicity  $E_n = 1$

$$(6) \quad E(t) = \sum_{n=1}^N e^{i(\omega_0 + n\omega)t} = e^{i\omega_0 t} \frac{\text{sen}(N\omega t/2)}{\text{sen}(\omega t/2)}$$

The mean output power is proportional to  $|E(t)|^2$  so that one has that:

$$(7) \quad P(t) \propto \frac{\text{sen}^2(N\omega t/2)}{\text{sen}^2(\omega t/2)}$$

From this expression for the output power one can deduce that:

- 1) This power is emitted in bunches with a period  $T = 2L/c$
- 2) On the peak the power is  $N$ -time greater than the mean value.
- 3) The time interval between the peak and the first zero is given by  $t_p = T/N$  (see Fig. 1). In particular:

$$(8) \quad t_p = T/N = 1/N\nu = 1/\Delta\nu$$

So that the pulse width in the mode locking operation is approximately the inverse of the line width.

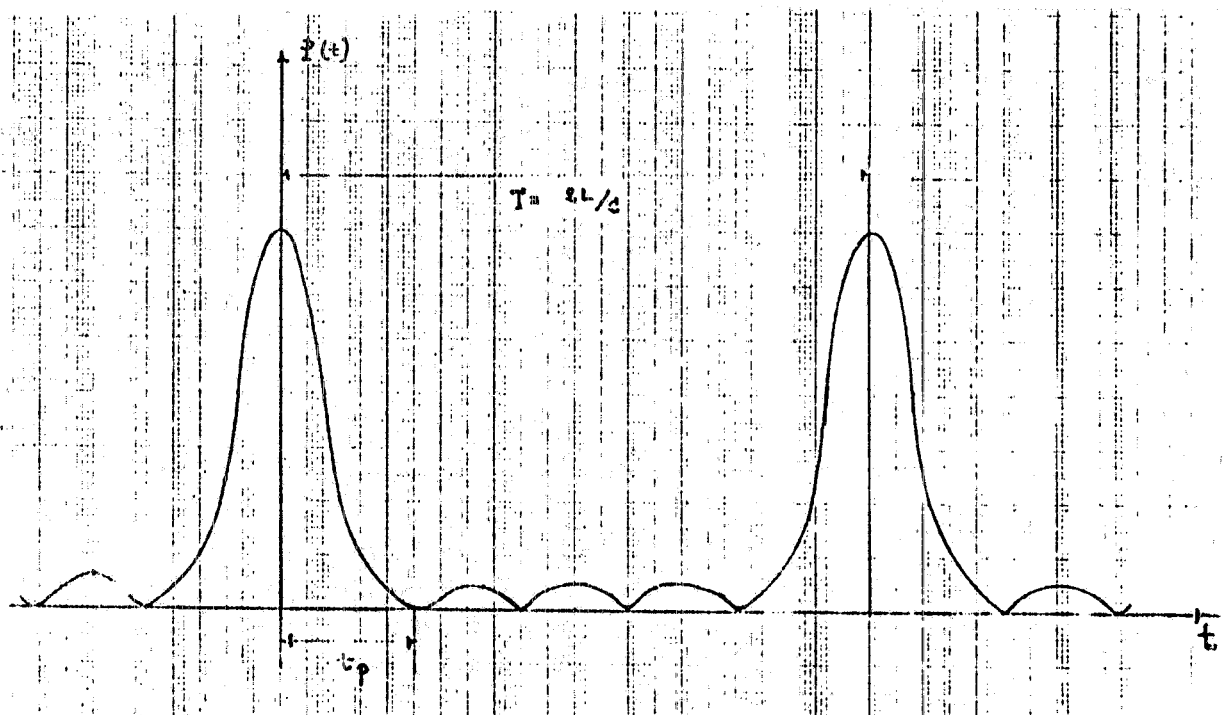


FIG. 1 - Typical behaviour of train pulses in a mode-locking operation with  $N = 5$ .

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For our Laser  $\Delta\nu = 5$  GHz corresponding to a pulse width of 0.2 nsec and since we want an interval of 117 nsec. between two pulses, the length cavity must be of:

$$L = cT/2 = 17.5 \text{ m}$$

From eq. (2) and eq. (4) the number of oscillating modes comes out to be  $N \sim 400$ .

### 3) Technique for mode locking. -

There are essentially two methods to obtain the mode locking: either by tuning the cavity losses or by introducing a saturable absorber into the cavity.

We intend to produce this phase modulation by using the transverse electrooptic effect in a synthetic crystal quartz.

The oscillation frequency of the crystal must be equal to the separation frequency between two adjacent axial modes, in our case  $\nu = 8.5585$  MHz.

For obvious reason its position inside the cavity must be very close to the mirrors and furthermore its length has to be much less than the total cavity length in order not to introduce any possible phase delay.

Finally its location at the Brewster angle will minimize the losses that certainly will be introduced.

### 4) Laser operation in the mode locking technique. -

During the time interval between two pulses the high losses of the cavity inhibits the Laser effect.

The standard rate equation for the inverted population in a four level Laser, is<sup>(4)</sup>:

$$(9) \quad \dot{n} = (M - n) \beta - Bqn - n/\tau$$

where:

$n = n_2 - n_1$  inverted population between two levels

$M = n_1 + n_2$

$B \propto \sigma$  constant proportional to the absorption cross section for stimulated emission ( $\sigma$ )

$\tau$  life time of the upper level (4 nsec)

$q(t)$  number of photons in the cavity

$\beta$  pumping rate ( $\text{sec}^{-1}$ )

In the interval between two pulses the Laser effect is switched off

and therefore the term  $\beta n$  is negligible in comparison with the others.

In this case the eq. (9) becomes:

$$(10) \quad \dot{n} = (M-n) \beta - n/\tau$$

and its solution is:

$$(11) \quad n(t) = \frac{\beta M}{\beta + \frac{1}{\tau}} - A e^{-(\beta + \frac{1}{\tau})t}$$

where A is a constant which has to be determined with the initial conditions.

Its asymptotic behaviour is given by:

$$(12) \quad n(t) = n_i = \frac{\beta \tau M}{1 + \beta \tau}$$

Moreover one can easily see that in our case  $\beta \tau \ll 1$  so that in conclusion one has:

$$n_i = \beta \tau M$$

The critic inverted population, which guarantees the Laser effect is given by  $n_c \sim \beta_c \tau M$ , and therefore to let the Laser operate one must have  $\beta/\beta_c \geq 1$ , condition which is satisfied in our case where this ratio is approximatively equal to  $\sim 14$ .

#### 5) Available output power. -

It is now available from the Coherent Radiation a mode locking device obtained with an Argon ion Laser on a cavity 1.5 m long (model 465 A. Q. mode-locker).

This technique provides pulses every  $10^{-8}$  sec with a peak power of  $\sim 40$  watt corresponding to a mean value of 1 watt which has to be compared with the 4 watt value obtained in CW operation.

The main reasons responsible for the power reduction are due essentially to the following effect:

- a) all the photons emitted in spontaneous decays during the interval between two pulses are lost (effect  $P_a$ ).
- b) the pulse width does not overlap the whole active medium (effect  $P_b$ ).

We can now evaluate the losses in our case by comparing our sy-

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stem with the result obtained with the 465 A.Q. mode locker.

As for as the second effect  $P_b$  is concerned there shouldn't be any difference because the pulse width is the same in both the cases.

Instead, we should expect a difference for the first effect  $P_a$  because the two repetition frequencies differ for a factor  $\sim 10 \div 8.55 \cdot 10^6$  Hz which has to be compared with  $10^8$  Hz.

It is easy to realize with simple arguments that the available output power can have been stored only during the time interval  $\Delta t_a$  of Fig. 2 since during the interval  $\Delta t_b$  all the pumped power will be lost through spontaneous decays.

Furthermore the interval  $\Delta t_a$ , which includes the light pulse and the recovery time is exactly the same in the two cases: consequently taking into account both the effects  $p_a$  and  $p_b$ , in first approximation the ratio between the two mean output powers will be given by the ratio between the 2 frequencies.

On the other hand the peak power, which is the most important thing for our purposes, will remain approximatively 40 Watt.

Therefore with the cavity described in ref. (1), the internal peak power will be  $\sim 2400$  Watt which provides a final photon intensity of the same order of that foreseen in ref. (1).

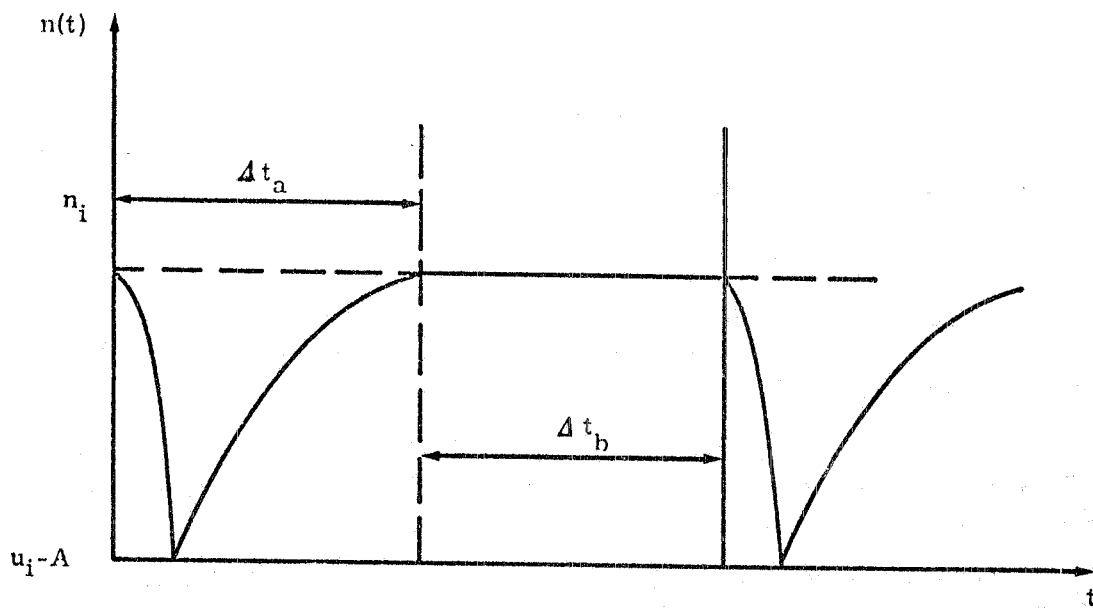


FIG. 2 - Sketch of the typical time behaviour of  $n(t)$ .

In conclusion we showed that this method can considerably improve the beam characteristics without affecting the total number of gamma.

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