COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-75/18(P)
14 Aprile 1975

E. Etim: ON THE CONNECTION BETWEEN PCAC AND PCDC ANOMALIES AND ITS IMPLICATIONS FOR THE NEW HADRONIC STATES. -

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ABSTRACT. -

It is argued that the relationship, $|S_\pi| = KR/4$, between the PCAC anomaly S_π, associated with the decay $\pi^o \to \gamma\gamma$, and the constant $R = \lim_{s\to\infty} (\sigma_{e^+e^- \to hadrons}(s) / \sigma_{e^+e^- \to \mu^+\mu^-}(s))$ defines the ratio $K = R_E/R$ between the yield of "normal" hadrons to the total in e^+e^- annihilation. Approximately equal production of "old" and "new" hadronic states at very high energies is predicted. Implications for the decays $\eta \to \gamma\gamma$, $\eta' \to \gamma\gamma$ and $\sigma \to \gamma\gamma$ are also reported.

From the singularity structure of the product of currents at short distances, Crewther has derived an interesting relation

(1) $$|S_\pi| = \frac{K}{4} R$$

between the PCAC[2] and PCDC[1, 3] anomalies, defined respectively by

(2a) $$S_\pi = -\frac{\pi^2}{12} \varepsilon_{\mu\nu\alpha\beta} \int d^4x\, d^4y\, x_\alpha\, y_\beta \langle 0 | T(\partial^\lambda J^{(3)}_{5\lambda}(y) J_\mu(x) J_\nu(0)) | 0 \rangle$$

(2b) $$\frac{R}{6} = \frac{\pi^2}{12} \int d^4x\, d^4y\, x_\mu\, x_\nu \langle 0 | T(\partial^\lambda D_\lambda(y) J_\mu(x) J_\nu(0)) | 0 \rangle$$

where $J^{(3)}_{5\lambda}(y)$ is the third isospin component of the axial-vector current, $J_\mu(x)$ the hadronic electromagnetic current, $\theta(y) = \partial^\lambda D_\lambda(y)$ the trace of the energy-momentum tensor with $D_\lambda(y) = y^\tau\, \theta_{\tau\lambda}(y)$ the dilatation current. The constant K is defined by the operator product expansion[1]

(3) $$J_\mu(x) J_\nu(0) = R(g_{\mu\nu} x^2 - 2x_\mu x_\nu)(\pi x^2)^{-4} + K \varepsilon_{\mu\nu\alpha\beta}\, x^\alpha \left(\frac{2}{9} J^{(0)}_{5\beta}(0) + \frac{1}{3} J^{(3)}_{5\beta}(0) + \frac{1}{3\sqrt{3}} J^{(8)}_{5\beta}(0)\right)(\pi x^2)^{-2} + \ldots$$

where the axial-vector currents transform under SU(3) like η_1, π^0 and η_8 respectively. Taking the discontinuity of both sides of (3) it follows at once from the commutator

(4) $$\left[J_\mu(0,\vec{x}),\, J_\nu(0) \right] = -2iK \varepsilon_{\mu\nu\alpha\beta}\, \xi^\alpha \left(\frac{2}{9} J^{(0)}_{5\beta}(0) + \frac{1}{3} J^{(3)}_{5\beta}(0) + \frac{1}{3\sqrt{3}} J^{(8)}_{5\beta}(0)\right) \delta^{(3)}(\vec{x})$$

where $\xi^\alpha \equiv (1, 0, 0, 0)$, that K is assumed to represent the modification of the usual U(6) X U(6) current algebra commutation rule[4]

$$(4') \quad \left[J_\mu(0,\vec{x}), J_\nu(0)\right] = -2i\varepsilon_{\mu\nu\alpha\beta}\,\xi^\alpha \left(\frac{2}{9}J^{(0)}_{5\beta}(0) + \frac{1}{3}J^{(3)}_{5\beta}(0) + \frac{1}{3\sqrt{3}}J^{(8)}_{5\beta}\right)\delta^{(3)}(\vec{x})$$

by interactions at short distances. One would expect from this, that, a priori, K could be any finite number, not necessarily less than unity. In fact if there is a rather large parallel - antiparallel asymmetry in polarized deep inelastic electroproduction[5], which measures the commutator in eq. (4), and if there is a large production rate for the process $e^+ + e^- \rightarrow \mu^+ + \mu^- + \pi^0$ at high energies[6], which measures its square, a natural explanation, albeit not the only, would be that K is greater than one. On the other hand making use of the experimental value of S_π[7] and the indications on R from SPEAR[8], it follows from eq. (1) that K is necessarily less than one and will be very much so if R comes ont eventually to be much greater than two. This circumstance is hard to understand, particularly so since hadronic symmetries, like the U(6) X U(6) current algebra abstraction, should become increasingly better approximations at high energies, with $K \simeq 1$. It is, consequently, not easy to miss the impression that, in a certain sense, the main function of the constant K is to modify the U(6) X U(6) commutator (4') in a such a way as to make it possible to accomodate a large value of R with the correct width of the decay $\pi^0 \rightarrow \gamma\gamma$. This arrangement, although it appears to work, is less than compelling. There are, as can easily be seen from eq. (3), other constraints on K, besides S_π, which must be considered. These come from the decays $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$, whose widths are related to the corresponding PCAC anomalies S_η and $S_{\eta'}$.

The purpose of this paper is to suggest a different interpretation of the connection between PCAC and PCDC anomalies, which does not require a modification of the U(6) X U(6) current algebra commutator (4'), and restricts only a component contribution to R, and hence can accomodate, in principle, an arbitrary large value of R. The further limitation on R comes from the decays $\eta \rightarrow \gamma\gamma$ and $\eta' \rightarrow \gamma\gamma$ whose anomalous constants

S_η and $S_{\eta'}$ can also be related to R as well as to S_π. The new interpretation of eq. (1), as will emerge in the sequel, does not imply that the Crewther alternative is invariably excluded. The point is that the value of K introduced in this particular way is very close to one. Our main claim is therefore that there is a strong indication in the relationship between PCAC and PCDC anomalies of a large contribution to R from hadronic states coupled to the newly discovered vector mesons[9].

Motivated by the recent discovery of these new vector mesons, of normal electromagnetic, but suppressed hadronic, couplings, we envisage an enlarged structure of the hadronic electromagnetic current

(5)
$$J_\mu \equiv J_\mu^{(E)} = J_\mu^{(3)} + \frac{1}{\sqrt{3}} J_\mu^{(8)} \longrightarrow J_\mu^{(E,M)} = (J_\mu^{(3,0)} + \frac{1}{\sqrt{3}} J_\mu^{(8,0)}) + + (J_\mu^{(0,3)} + \frac{1}{\sqrt{3}} J_\mu^{(0,8)})$$

from a U-spin scalar of $SU(3) \equiv SU(3)_E$ to a U-spin scalar of the direct product $SU(3)_E \times SU(3)_M$ with charge operators $Q(E) = I_3(E) + Y(E)/2$ and $Q(M) = I_3(M) + Y(M)/2$ respectively[10]. The new vector mesons are assigned to an $\underline{8}$ + $\underline{1}$ multiplet of $SU(3)_M$ and are singlets of $SU(3)_E$. Similarly the "normal" hadrons are singlets of $SU(3)_M$. With this proviso, one finds, on making use of asymptotic chiral symmetry and the operator product expansion

(6)
$$\begin{aligned} J_\mu^{(E,M)}(x)\, j_\nu^{(E,M)}(0) = {} & R_E(g_{\mu\nu} x^2 - 2x_\mu x_\nu)(\pi x^2)^{-4} + \\ & + R_M(g_{\mu\nu} x^2 - 2x_\mu x_\nu)(\pi x^2)^{-4} + \\ & + \varepsilon_{\mu\nu\alpha\beta}\, x^\alpha (\frac{2}{9} J_{5\beta}^{(0,0)} + \frac{1}{3} J_{5\beta}^{(3,0)} + \frac{1}{3\sqrt{3}} J_{5\beta}^{(8,0)} (\pi x^2)^{-2} + \\ & + \varepsilon_{\mu\nu\alpha\beta}\, x^\alpha (\frac{2}{9} J_{5\beta}^{(0,0)} + \frac{1}{3} J_{5\beta}^{(0,3)} + \frac{1}{3\sqrt{3}} J_{5\beta}^{(0,8)})(\pi x^2)^{-2} + \ldots \end{aligned}$$

in eq. (2), that

(7) $$|S_\pi| = R_E/4$$

whence on comparison with eq. (1) we get

(8) $$K = R_E/R, \qquad R = R_E + R_M$$

We have thus identified the constant K in eq. (1) with the fraction of hadronic yield in e^+e^- annihilation at high energies from the $SU(3)_E$ piece of the hadronic electromagnetic current. Since $R_M \neq 0$ the condition $K < 1$ is now both natural and obvious.

The immediate consequences of this new interpretation are of course that Bjorken's[5] estimate ($\gtrsim 20\%$) of the parallel-antiparallel asymmetry in polarized electroproduction and the analysis of Gross and Treiman[6] of the cross section of the process $e^+ + e^- \to \mu^+ + \mu^- + \pi^0$ remain unchanged. The rate of the inclusive process[6] $e^+ + e^- \to \mu^+ + \mu^- + x$ which receives contributions from both pieces of the electromagnetic current is increased by the factor $(1 + (1-K)/K)^2)$.

Now if the $SU(3)_E \times SU(3)_M$ singlet operators $\partial^\lambda D_\lambda(y)$ and $\partial^\lambda J^{(0,0)}_{5\lambda}$ are both soft, and if we assume that in the short distance limit they can be treated as chiral partners, then considering the decays $\sigma \to \gamma\gamma$, $\pi^0 \to \gamma\gamma$, $\eta \to \gamma\gamma$ and $\eta' \to \gamma\gamma$ together leads to a very simple derivation of the Crewther relation. Our assumption implies that the $SU(3)_E$ and $SU(3)_M$ type hadrons are equivalent with respect to their electromagnetic interactions. Not surprisingly the value of the ratio K is found to be 1/2 in this case.

Define for any pseudoscalar meson $P^{(a,b)}$ (e.g. $P^{(a,b)} \equiv \pi^0, \eta, \eta', \ldots$) the Green's function

(9) $$S_P(q^2) = -\frac{\pi^2}{12}\,\varepsilon_{\mu\nu\alpha\beta}\int d^4x\, d^4y\, e^{iqx} x_\alpha y_\beta \langle 0 | (T\partial^\lambda J^{(P)}_{5\lambda}(y) J_\mu(x) J_\nu(x)) | 0\rangle$$

where the interpolating field of $P^{(a,b)}$ is defined by

(10) $$P^{(a,b)}(y) = \frac{1}{f_P}\,\partial^\lambda J^{(a,b)}_{5\lambda}(y); \qquad a, b = 0, 1 \ldots . 8$$

Comparing eq. (9) for $\eta_1(y) = (1/f_{\eta_1})\,\partial^\lambda J^{(0,0)}_{5\lambda}(y)$ with the Green's function for its (assumed) chiral partner $\theta(y) = \partial^\lambda D_\lambda(y)$, that is

(11) $$\bar{\Delta}(q^2) = \frac{2}{12}\, g_{\mu\alpha}\, g_{\nu\beta} \int d^4x\, d^4y\, e^{iqx}\, x_\alpha\, x_\beta \langle 0 | T(\partial^\lambda D_\lambda(y) J_\mu(x) J_\nu(0)) | 0 \rangle$$

one reads off from eq. (6), at $q^2 = 0$[11]

(12) $$|S_{\eta_1}| = \frac{4}{3}\,|S_\pi| = \frac{R}{6}; \qquad |S_{\eta_8}| = \frac{1}{\sqrt{3}}\,|S_\pi|$$

The constants R and K are thus completely determined

(13) $$K = 1/2, \qquad R = 8|S_\pi| \simeq 4$$

where in the last line we have used $|S_\pi| \simeq 1/2$. Clearly the generation of $J^{(0,0)}_{5\lambda}$ from both pieces of $J^{(E,M)}_\mu$ with equal coefficients in eq. (6) is important for this result.

The coupling constants $g_{P\gamma\gamma}$ defined by

(14) $$\Gamma(P \to \gamma\gamma) = \frac{m_P^3}{16}\left(\frac{g^2_{P\gamma\gamma}}{4\pi}\right)$$

are related to the anomalies S_P by[2]

(15) $$g_{P\gamma\gamma} = -\frac{\alpha}{\pi}\, 2\sqrt{2}\, \frac{m_P^2}{f_P}\, S_P$$

Hence using the mixing

(16) $$\eta = \eta_8 \cos\theta - \eta_1 \sin\theta, \quad \eta' = \eta_8 \sin\theta + \eta_1 \cos\theta, \qquad \theta \simeq -10^\circ$$

we obtain the correct widths $\Gamma(\eta \to \gamma\gamma)$ and $(\eta' \to \gamma\gamma)$ in terms of $\Gamma(\pi^o \to \gamma\gamma)$ from eqs. (12) and (15), that is[12]

(17)
$$g_{\eta\gamma\gamma} = \frac{g_{\pi^o\gamma\gamma}}{\sqrt{3}} (\cos\theta - 2\sqrt{2}\sin\theta)$$
$$g_{\eta'\gamma\gamma} = \frac{g_{\pi^o\gamma\gamma}}{\sqrt{3}} (2\sqrt{2}\cos\theta + \sin\theta)$$

provided

(18)
$$\frac{m^2_{\eta_8}}{f_{\eta_8}} = \frac{m^2_\pi}{f_\pi}, \qquad \frac{m^2_{\eta_1}}{f_{\eta_1}} = \sqrt{\frac{3}{2}}\,\frac{m^2_\pi}{f_\pi}$$

the first of which is the exact SU(3) expectation while the second requires approximate overall symmetry in the pseudoscalar nonet. Thus, as noted previously, η_1 couples not only to pairs of isovector and isoscalar photons of $SU(3)_E$ but also to those of $SU(3)_M$. This is different from the conclusion of Kramer, Schildknecht and Steiner[13].

Consider now the decays $\sigma \to \gamma\gamma$ and $\sigma \to \pi\pi$. It is reasonable to expect, on the basis of eqs. (7) and (8) and from the PCDC relation

(19)
$$\sigma(y) = (m^2_\sigma F_\sigma)^{-1}\, \partial^\lambda D_\lambda(y)$$

that σ will not couple with its full strength ($\sim 1/F_\sigma$) to the E- or M- type hadrons. Assuming the same strength reduction as for the corresponding photons, one will have, for instance,

(20)
$$g_{\sigma\pi\pi} \simeq K m^2_\sigma / F_\sigma, \qquad g_{\sigma\gamma\gamma} = \left(\frac{2\alpha}{3\pi} R\right)/F_\sigma$$

where the coupling constants are defined by the interaction Lagrangians

(21)
$$L_{\sigma\pi\pi} = g_{\sigma\pi\pi}\, \sigma(\vec{\pi}\cdot\vec{\pi}), \quad L_{\sigma\gamma\gamma} = -g_{\sigma\gamma\gamma}\, F_{\mu\nu} F_{\mu\nu}$$

and $F_\sigma/K \simeq 150$ MeV is gotten from $m_\sigma \simeq 700$ MeV and the width $\Gamma(\sigma \to \pi\pi) \simeq 400$ MeV[7]. Eliminating F_σ from eq. (20) one finds for the decay $\sigma \to \gamma\gamma$ [3]

$$\Gamma(\sigma \to \gamma\gamma) = 0.2\left(\frac{R}{K}\right)^2 \simeq 12.8 \text{ KeV} \tag{22}$$

on making use of eq. (13). The above estimate for $\Gamma(\sigma \to \gamma\gamma)$ is of the same order of those obtained from finite energy sum rules[14].

It is interesting to compare the predictions for R and K in eq. (13) with those obtained by application of the extended vector meson dominance[15] to include the new vector mesons $\psi(3.1)$ and $\psi'(3.7)$. To this end we assume that $\Delta(q^2)$ (cf eq. (11)) satisfies an unsubtracted dispersion relation[16] so that using the anomalous trace identity[3]

$$\Delta(q^2) = -2q^2 \frac{\partial \Pi(q^2)}{\partial q^2} - \frac{2\alpha}{3\pi} R \tag{23}$$

one can write

$$\Delta(q^2) - \Delta(0) = -\frac{2}{\pi} q^2 \int_0^\infty ds \frac{\mathrm{Im}\Pi(s)}{(s-q^2)^2} \tag{24}$$

Saturating $\mathrm{Im}\,\Pi(s)$ with the contributions of vector meson peaks of both E- and M-types, we have

$$\mathrm{Im}\Pi(s) = 4\pi^2\alpha \sum_{n=0}^{\infty} \frac{m_n^2}{f_n^2} \delta(s-m_n^2) \tag{25}$$

where $(e\,m_n^2/f_n)$ is the coupling of the vector meson of mass m_n to the photon. Making use of the mass and coupling constant spectra $m_n^2 = m_o^2(1+2n)$ and $f_n^2 = f_o^2(1+2n)$[15], eqs. (24) and (25) give[16]

$$\begin{aligned}\Delta(q^2) - \Delta(0) = {} & \frac{16\pi\alpha}{3f_\rho^2}\left(-\frac{q^2}{2m_\rho^2}\right)\zeta\left(2, \frac{1}{2} - \frac{q^2}{2m_\rho^2}\right) + \\ & + \frac{16\pi\alpha}{3f_{\bar\rho}^2}\left(-\frac{q^2}{2m_{\bar\rho}^2}\right)\zeta\left(2, \frac{1}{2} - \frac{q^2}{2m_{\bar\rho}^2}\right)\end{aligned} \tag{26}$$

(33) $$KK' = \frac{1}{2}$$

It is now clear from the particular combination of K and K' in eq. (32) why it is not easy to suspect the presence of $K = R_E/R$ in the Crewther relation. However, since many models seem to agree on a value of R_E around 2[16,17], the experimental value of $|S_\pi| \simeq 1/2$ implies that K' cannot be very much different from one.

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with ϱ in $SU(3)_E$ and $\bar{\varrho}$ in $SU(3)_M$. We have multiplied the rhs of (26) by an overall factor of 4/3 to account for isoscalars. $\zeta(2,z)$ is the generalized Riemann Zeta function of order 2 and variable z. Equating the constant terms of both sides of (26) for large q^2 yields[16]

(27) $$R = 8\pi^2\left(\frac{1}{f_\varrho^2} + \frac{1}{f_{\bar{\varrho}}^2}\right) = R_E + R_M$$

To calculate R_M we shall identify $\psi(3.1)$ as the equivalent of $\omega(780)$. We consider two possibilities: i) $\psi(3.1)$ is not the lowest mass member of the family ii) $\psi(3.1)$ is the lowest mass member of the family. Corresponding to the first possibility and taking, as an indication the mass squared separation

(28) $$2m_o^2 = m_{\psi'}^2 - m_\psi^2 \simeq 4.1\ \text{GeV}^2$$

eq. (27) gives, on making use of the experimental decay width[9]

(29) $$\Gamma(\psi \to 1^+1^-) = \frac{4\pi\alpha^2}{3}\ \frac{m_\psi}{f_\psi^2} \simeq 5\ \text{KeV}$$

(30) $$R \simeq 5.18, \qquad K \simeq 0.48$$

where $R_E = 8\pi^2/f_\varrho^2 \simeq 2.5$[15,16].

If $\psi(3.1)$ is the lowest mass member of the family one finds with the mass spectrum $m_n^2 = m_\psi^2(1+bn)$ and width $\Gamma(\psi \to 1\bar{1}) \simeq 4.5$ KeV

(31) $$R \simeq 4\cdot 96, \qquad K \simeq 0\cdot 5$$

These numbers are very close to those in eq. (13).

To conclude we note that if the overall coefficient of the axial-vector terms in eq. (6) is K', our arguments still apply with the result

(32) $$|S_\pi| = \frac{K'}{4} R_E = \frac{KK'}{4} R$$

Substituting from here into (12) yields

(10) - The notation is that of J. Schwinger, University of California (Los Algeles) preprint (February 1975) and Science (to be published). Q(E) is the electric and Q(M) the magnetic charge.

(11) - The equality $|S_{\eta_1}| = R/6$ is obviously the maximal expectation from asymptotic chiral symmetry between $\partial^\lambda D_\lambda(y)$ and $\partial^\lambda J_{5\lambda}^{(0,0)}$. A weaker demand would be that the ratio of $|S_{\eta_1}|$ to $R/6$ is given by the squares of the projectors $\varepsilon_{\mu\nu\alpha\beta}$ and $g_{\mu\alpha}\, g_{\nu\beta}$ i.e. 4! to 4^2. In this case $|S_{\eta_1}| = (3/2)(R/6) = R/4$, $|S_\pi| = 3R/16$ and $K = 3/4$.

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