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G. Parisi: HIGH TEMPERATURE EXPANSION AND THE REGGEON CALCULUS. -

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ABSTRACT. -

We show that in the framework of Reggeon calculus the use of the "high temperature expansion" may produce an accurate determination of the asymptotic behaviour of total cross sections.

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In a very interesting series of papers Gribov and Migdal[1,2] put forward the theory of interacting Reggons. In principle their approach is able to predict the asymptotic behaviour of total cross sections for strong interacting particles. Unfortunately the exact solution of their equations is not known: the difficulties one has to face, are very similar to those found in the study of the properties of systems near second order phase transitions[2]. It has been shown that the total cross section increase like $(\lg s)^{\eta}$, the costant η being a "critical exponent" of the model.

The two problems, Reggeon calculus and second order phase transitions, are strongly connected: the same technique, namely the ε expansion[3], has been used to compute the critical exponents in both cases[4,5]. However the ε expansion for the Reggeon calculus seems to be not very convergent ($\varepsilon = 2$) and no reliable predictions can be extracted from the first two orders in ε[6].

It is well known that the ε expansion in not the best way to compute critical exponents in statistical mechanics: the only safe me

thod is the high temperature expansion[7]; it accounts to treate the "kinetical energy" as a perturbation. This expansion has the merit of being convergent also in the critical region; very high order terms can be computed: in the three dimensional Ising model all orders up to the 12^{th} are known[8].

Having these considerations in mind, we propose to use high temperature expansion to solve the Reggeon calculus for the Pomeranchuk, i.e. to compute the appropriate critical exponents.

We want to study the following problem: a system of interacting non relativistic particles; creation and distruction of particles is allowed. The Lagrangian density is in two space one time dimensions:

$$(1) \qquad \mathcal{L} = \alpha' \partial_\mu \psi^+ \partial_\mu \psi + M^2 \psi^+ \psi + ig(\psi^+ + \psi)\psi^+\psi + i\psi^+ \frac{\overleftrightarrow{\partial}}{\partial t} \psi$$

ψ is a complex field α', $1-M^2$ and g are parameters which play the rôle of slope of the trajectory, of bare intercept and of three Pomeranchuk coupling constant. It may be interesting to note that the interaction Lagrangian is purely antithermitian.

According the analysis of Gribov and Migdal the following formulae hold in the region of small energy gap m^2:

$$(2) \qquad \int \langle 0 | T[\psi(x,t)\psi^+(0,0)] | 0 \rangle \cdot \exp(i\omega t + ik\cdot x)\, d^2x\, dt \equiv$$

$$\equiv D(\omega, k, m) = m^{-2\lambda\mu} F\left(\frac{k^2}{\omega^\nu}, \frac{m^{2\lambda}}{\omega}\right) \qquad \eta = \mu - 1$$

Some care must be used to impose the requirement

$$(3) \qquad \langle 0 | \psi | 0 \rangle = \langle 0 | \psi^+ | 0 \rangle = 0$$

A violation of this condition would imply the existence of two poles in the correlation function D: one of them corrispond to particle propagation, the second to hole propagation. This condition may be easily implemented adding to the Lagrangian density an appropriate linear term:

$$(4) \qquad \Delta = h(\psi^+ + \psi)$$

Using Feynmann's path integral formulation[9], the correlation function

D(x,t) can be written as a functional integral:

$$Z\,D(x,t) = \int d\psi d\psi^{+} \exp\left[i \int \mathcal{L}(x',t')\, d^2x'\, dt'\right] \psi(x,t)\, \psi^{+}(0,0)$$

(5)

$$Z = \int d\psi d\psi^{+} \exp\left[i \int \mathcal{L}(x',t')\, d^2x'\, dt'\right]$$

At this formal level we are allowed to multiply the argument of the exponent in (5) by i without changing the value of the correlation functions, apart from trivial phase factors. We introduce a simple cubic lattice over space time. The fields φ are defined only at the lattice sites; $\varphi(k)$ denotes the field φ at the point k of the lattice. The interaction is nearest-neighbor; the potential is different is space and time directions.

The partition function can be written as:

$$Z = \int \Pi_k\, d\varphi(k)\, d\varphi^{+}(k)\, \varrho\left[\varphi(k), \varphi^{+}(k)\right] \quad x$$

$$x \exp\beta\left\{\frac{i}{2}\left[\varphi^{+}(k)\,\varphi(k+\delta_t) - \varphi(k)\,\varphi^{+}(k+\delta_t)\right] +\right.$$

(6)

$$- \alpha'\left[\varphi^{+}(k)\,\varphi(k+\delta_x) + \varphi(k)\,\varphi^{+}(k+\delta_x) +\right.$$

$$\left.\left. + \varphi^{+}(k)\,\varphi(k+\delta_y) + \varphi(k)\,\varphi^{+}(k+\delta_y)\right]\right\}$$

$$\varrho\left[\varphi, \varphi^{+}\right] = \exp\left\{-M^2\varphi^{+}\varphi - ig(\varphi+\varphi^{+})\,\varphi^{+}\varphi + h(\varphi+\varphi^{+})\right\}$$

where δ_t, δ_x, δ_y are respectively the unity vectors in the t, x and y directions.

The functional integrals in eq. (5) and (6) are clearly different, however in the region where m^2 is very small and the correlation lenght is much greater than the lattice spacing their behaviour must be similar: in particular the equality of the critical exponents for both systems follows from the celebrated universality principle[10]. The same principle implies that critical exponents are indipendent from the detailed form of the interaction. Further simplifications may be obtained changing the allowed range of integration over φ or changing the density function ϱ.

We decompose the φ field in its real and immaginary parts: φ_1, φ_2; we impose the constraint that both φ_1 and φ_2 may assume only the value -1, 0, 1.

We finally arrive to the partition function (7), where the sum is extended over all possible configurations of the fields φ_1 and φ_2.

$$
\begin{aligned}
Z = \Sigma_{\{\varphi\}} \Pi_k \varrho\,[\varphi_1(k), \varphi_2(k)] \exp -\beta \Big\{ \Sigma_k \big[& -\varphi_1(k)\varphi_2(k+\delta_t) + \\
& + \varphi_1(k+\delta_t)\varphi_2(k) + 2\alpha' \big[\varphi_1(k)\varphi_1(k+\delta_x) + \varphi_2(k)\varphi_2(k+\delta_x) + \\
& + \varphi_1(k)\varphi_1(k+\delta_y) + \varphi_2(k)\varphi_2(k)\varphi_2(k+\delta_y)\big]\big]\Big\}
\end{aligned}
\tag{7}
$$

$$
\varrho\,[\varphi_1, \varphi_2] = \exp\big[-M^2(\varphi_1^2+\varphi_2^2) - ig\,\varphi_1(\varphi_1^2+\varphi_2^2) - h\,\varphi_1\big]
$$

β is a parameter which can be varied during the computation; however its final value must be of order 1. In the limit $\beta = 0$ the model is trivial; formal development in powers of β via the linked-cluster expansion appears straightforward.

The high temperature expansion (development in powers of β) can be used to estimate the critical exponents: the following quantities may be computed:

$$
D(0,0,\beta); \quad \frac{\partial}{\partial k^2} D(0,k^2,\beta)\Big|_{k^2=0}; \quad \frac{\partial}{\partial \omega} D[0,\omega,\beta]\Big|_{\omega=0} \tag{8}
$$

All these functions must have a singularity at the same value of β; from the analysis of the singularities the value of the critical exponents can be obtained.

A test of the accuracy of the computation of the exponents may be done checking that the final results are reasonable independent from parameters like α', M^2, g.

We do not enter herein a detailed description of the techniques used in the high temperature expansion: they can be found on many excellent reviews. Also in this particular case the computation of the lower orders is quite easy; the difficulties increase strongly with the order. Some care must be used to treat properly the linear term of eq. (4) which is β dependent; this can be done changing the diagrammatical rules.

We have shown that the problem of finding the asymptotic behaviour of hadronic cross section can be studied using familiar techniques of statistical mechanics: the critical exponents of a peculiar spin system must be found.

Using the same techniques we propose in this paper, critical exponents have been computed in many spin systems with an accuracy of 1% or less.

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