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G. Parisi: STRONG COUPLING IN FIELD THEORY.-

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ABSTRACT -

We propose to use the high temperature expansion for performing computations in field theory for large coupling constant.

A serious difficulty in field theory is due to the absence of computational techniques for not small coupling constant; perturbation theory is useless: it is impossible to compute enough orders to have meanful results for strong coupling.

However the theory of interacting scalar fields in Minkowski space is intimately connected to the statistical mechanics[1, 2, 3] of a particular spin system in the limit of zero lattice spacing.[4] For spin systems the high temperature expansion[5] provides a tool to compute interesting quantites and is not restricted to small values of the coupling constant. In the high temperature expansion the kinetical energy is treated as a perturbation: the presence of a strong ultraviolet cutoff (i.e. lattice spacing) avoids the appearence of ultraviolet divergences.

High temperature expansion is superior to the standard perturbation in the coupling constant: its radius of convergence is very large and

the computations are reasonably simple: it has been possible to compute the first 12 orders for the two point function at zero momentum in the 3 dimensional Ising model[6].

In this note we want to discuss two points:

I) How the high temperature expansion can be used to extract informations on quantum field theory.

II) How the high temperature expansion may be modified to avoid the introduction of the lattice and how the whole computation can be done in the more familiar momentum space.

The partition function of the $\lambda\varphi^4$ theory in the Euclidean space can be written as:

$$Z = \lim_{\delta \to 0} \int \Pi_k d\varphi(k) \exp -\delta \left\{ \beta \sum_{k,i} \frac{\varphi(k) V(k-i) \varphi(i)}{\delta^2} + \right.$$

$$\left. + \Sigma_k \left(\frac{M^2}{2} \varphi^2(k) + \lambda \varphi^4(k) \right) \right\}$$

δ is the lattice spacing and V is a nearest neibor interaction.

Similar formulae hold for the correlation functions; the parameter M^2 must be chosen such to produce a finite limit. The correlation function of the equivalent theory in Minkowski space may be obtained by analytical continuation in the external points.

We denote by m the renormalized mass and by g the dimensionless bare coupling constant: $g = \lambda m^{(-4+D)}$; the local limit in (1) is reached sending δm to zero at fixed g. This can be done sending the mass to zero at fixed lattice spacing[7]. It may be convenient to expandem in powers of β ($\beta = 1/KT$) at fixed g, to determine the value β_c for which $m = 0$ and to compute any interesting dimensionless quantity from its expansion in powers of β. It is reasonable to suppose that the high temperature expansion is convergent for $\beta = \beta_c$.

Using this technique it is possible to compute correlation functions and their derivatives at zero external momentum. The positions of the nearest singularities in the two points function can be computed; introducing operators of definite spin and computing their correlation functions the whole low energy spectrum may be computed.

Unfortunately this last task seems to be too difficult for the techniques normally used in the high temperature expansion; the introduction of operators of definite angular momentum is not simple: the theory is not rotational invariant before the limit $m \to 0$; the computation of 4

point correlation function is quite complicated.

These difficulties may be avoided sending to zero the lattice spacing at fixed shape of the potential (not first neighbor) and only at this stage performing the high temperature expansion. The theory is now rotational invariant at any stage and it is possible to work directly in the more familiar momentum rappresentation.

The partition function for a spin system on the lattice can be written as

$$(2) \qquad Z = \int d\varphi(k)\, \varrho\, [\varphi(k)]\, \exp - \beta \sum_{i,l} \varphi(i)\varphi(l)\, V(i-l)$$

where $V(x)$ is the potential and $\varrho(\varphi)$ the bare spin distribution; the sum in (2) runs over all the lattice sites.

Following Poliakov (2) can be written as:(8)

$$(3) \qquad = \exp\left[\sum_K \tilde{\varrho} \left(\frac{\delta}{\delta J_K}\right)\right] \exp - \beta \sum_{il} J(i)J(l)V(i-l) \Bigg|_{J=0}$$

where

$$(4) \qquad \exp \tilde{\varrho}(X) = \int d\varphi \exp(X\varphi)\cdot \varrho(\varphi)$$

The zero spacing limit can be easyly done: the output is:

$$(5) \qquad \exp \int dx\, \varrho \left(\frac{\delta}{\delta J(x)}\right) \cdot \exp - \beta \int dxdy\, J(x)\, J(y)\, V(x-y) \Bigg|_{J=0}$$

The expansion of (5) in powers of β does not provide any difficulties. If the potential is taken equal to $\exp - x^2$ all integrals can be reconduced to gaussian ones: they can be easyly evaluated in the momentum space.

We hope that the zero mass limit of (5) and (2) are identical; the infinite cutoff limit should not be dependent from the way the cutoff is introduced.

We think that it may be possible to compare the outputs of this calculation with the informations coming from standard perturbation theory and to obtain new informations in the large coupling region.

It is interesting to note that a different limit can also be studied: $m \to 0$, $\lambda = \text{cost.}$, $g \to \infty$.

In this case m must be proportional to $(\beta_c - \beta)^{\nu}$, ν being a critical exponent of the model; the value of ν may be extracted from the analysis of the high temperature expansion: it should be equal to the one obtained in the Ising case. The whole computational scheeme can be extended also to non integer dimensions, where the only sorces of informations are or the ε expansion[7] or the 1/N expansion[9, 10].

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