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M. Greco, G. Pancheri-Srivastava and Y. Srivastava:  
RADIATIVE EFFECTS FOR RESONANCES WITH APPLICATIONS  
TO COLLIDING BEAM PROCESSES. -

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M. Greco, G. Pancheri-Srivastava<sup>(x)</sup> and Y. Srivastava<sup>(x)(+)</sup>;  
RADIATIVE EFFECTS FOR RESONANCES WITH APPLICATIONS TO  
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ABSTRACT -

New expressions are presented for radiative corrections to colliding beam experiments in presence of narrow resonances, including interference effects. These are applied to  $\psi(3.1)$  and  $\psi(3.7)$  to obtain the values  $(\Gamma_e \Gamma_h / \Gamma) = (4.0 \pm 0.2)$  keV for  $\psi(3.1)$  and  $(2.8 \pm 0.3)$  keV for  $\psi(3.7)$ .

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This letter is devoted to present explicit expressions for the radiative correction factors in  $e^+e^-$  scattering when there is a resonance present. The recent discovery of two very narrow resonances<sup>(1, 2, 3)</sup> in this channel has made mandatory a precise evaluation of these factors, in order to be able to extract from the experimental data the actual widths (total and partial) of  $\psi(3.1)$  and  $\psi(3.7)$ .

The method adopted here stems from a set of papers<sup>(4, 5, 6)</sup> initiated a few years ago by B. Touschek for the specific need of colliding beam experiments. The present work complements the above references in two respects: i) it includes interference from a smooth QED background with a rapidly varying resonance term, ii) it takes into account interference effects due to soft photon emission from the initial and final legs. Neither of these effects were dealt with in earlier works<sup>(7, 8)</sup>.

The general expression for the product of two matrix elements summed over the emission of an arbitrary number of real soft photons can be written as

$$(1) \quad M^{(1)} M^{(2)+} = \frac{1}{(2\pi)^2} \int_0^{\Delta\omega} d\omega M_{\lambda}^{(1)}(W-\omega) \int_0^{\Delta\omega} d\omega' M_{\lambda}^{(2)+}(W-\omega') \times \\ \times \int_{-\infty}^{+\infty} dt e^{-i\omega t} \int_{-\infty}^{+\infty} dt' e^{i\omega' t'} e^{h(t, t')}$$

where

$$h(t, t') = \beta_i \int_{\lambda}^{\Delta\omega} \frac{dk}{k} e^{i(t-t')k} + \beta_{int} \int_{\lambda}^{\Delta\omega} \frac{dk}{k} (e^{itk} + e^{-it'k}) + \beta_f \int_{\lambda}^{\Delta\omega} \frac{dk}{k},$$

and

$\Delta\omega$  = energy resolution of the experiment,

$\lambda$  = infrared energy cut-off,

$W = \text{c.m. energy,}$

$$\beta_{i,f} = \frac{4\alpha}{\pi} \left[ \ln \frac{W}{m_{i,f}} - \frac{1}{2} \right],$$

$$\beta_{\text{int}} = \frac{4\alpha}{\pi} \ln (\text{tg } \theta/2),$$

$\theta = \text{c.m. scattering angle.}$

$M_{\lambda}^{(1,2)}$  is a matrix element without any real photons, containing an infrared divergence in  $\lambda$  which cancel exactly a similar divergence present in  $h(t, t')$ .

As the derivation of eq. (1) is a rather straightforward extension of refs. (5) and (6), we defer to a forthcoming publication the details of the calculation and rather concentrate on the consequences of eq. (1) and its implications for the present experiments.

The processes we want to discuss are

$$(I) \quad e^+e^- \rightarrow e^+e^-,$$

$$(II) \quad e^+e^- \rightarrow \mu^+\mu^-,$$

$$(III) \quad e^+e^- \rightarrow \text{hadrons,}$$

via a narrow resonance.

As far as the hadrons are concerned, all interference effects can be neglected to a good ( $\sim 5\%$ ) approximation and, by putting  $\beta_f = \beta_{\text{int}} = 0$ , we get for the radiatively corrected cross section<sup>(6)</sup>

$$(2) \quad \sigma_h(W) = \beta_i \int_0^{\Delta\omega} \frac{d\omega}{\omega} \left(\frac{2\omega}{W}\right)^{\beta_i} \sigma_o(W-\omega)(1+\delta_v)$$

where  $\delta_v$  is the genuine ultraviolet correction,

$$\delta_v = \frac{13}{12} \beta_i + \frac{\alpha}{\pi} \left( \frac{\pi^2}{3} - \frac{17}{18} \right) \text{ for hadrons}^{(9)}, \text{ and } \sigma_o(W-\omega) \text{ is the usual Breit-Wigner cross section peaked at } W = M + \omega.$$

For very narrow

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resonances like  $\psi$  (3.1) and  $\psi$  (3.7), eq. (2) yields

$$\sigma_h(W) = \left[ \frac{(W-M)^2 + (\Gamma/2)^2}{M^2/4} \right]^{\beta_i/2} \cdot \left[ 1 + 2\beta_i \frac{W-M}{\Gamma} \left( \frac{\pi}{2} + \text{tg}^{-1} \left( \frac{W-M}{\Gamma/2} \right) \right) \right] \cdot (1 + \delta_v) \sigma_o(W) \quad (3)$$

where  $\Gamma$  is the total width and we have evaluated the  $\omega$ -integral in the limit  $\Delta\omega \rightarrow \infty$ . The error involved is of order  $\beta_i [(W-M)^2 + (\Gamma/2)^2] / \Delta\omega^2$ , hence negligible. This is very interesting as it shows that the soft photon emission is regulated by the width of the resonance, i. e. by an intrinsic physical quantity, rather than by the experimental resolution (as long as  $\Delta\omega \gg \Gamma$ ). The second term in the square bracket in eq. (3) shows the expected radiative tail.

We now wish to obtain the correction formula for processes (I) and (II). What is observed here is a differential cross-section  $d\sigma/d\cos\theta$ , which we shall also call  $\hat{\sigma}(W, \theta)$  to simplify the notation. The cross section is now made of three terms, the purely resonant  $\hat{\sigma}_{\text{RES}}$ , the QED-resonance interference  $\hat{\sigma}_{\text{INT}}$  and the QED background  $\hat{\sigma}_{\text{QED}}$ . Applying eq. (1) to the three different terms we get

$$\begin{aligned} \hat{\sigma}_{1+1-}(W, \theta) = & \left[ \left( \frac{2\Delta\omega}{M} \right)^{\beta_f} \beta_i \int_0^{\Delta\omega} \frac{d\omega}{\omega} \left( \frac{2\omega}{W} \right)^{\beta_i} \hat{\sigma}_{\text{RES}}(W-\omega, \theta) + \right. \\ (4) \quad & + \left( \frac{2\Delta\omega}{M} \right)^{\beta_f + \beta_{\text{int}}} (\beta_i + \beta_{\text{int}}) \int_0^{\Delta\omega} \frac{d\omega}{\omega} \left( \frac{2\omega}{W} \right)^{\beta_i + \beta_{\text{int}}} \hat{\sigma}_{\text{INT}}(W-\omega, \theta) + \\ & \left. + \left( \frac{2\Delta\omega}{M} \right)^{\beta_i + \beta_f + 2\beta_{\text{int}}} \hat{\sigma}_{\text{QED}}(W, \theta) \right] (1 + \delta_v) \end{aligned}$$

where  $\delta_v$ , as before, is the ultraviolet correction<sup>(8)</sup>. Performing the  $\omega$ -integral as for the hadron case, the differential cross-section for  $e^+e^- \rightarrow$  lepton pairs can be written as

$$(5) \quad \frac{d\sigma_{1^+1^-}(W, \theta)}{d\cos\theta} = \left[ C_{\text{RES}} \frac{d\sigma_{\text{RES}}(W, \theta)}{d\cos\theta} + C_{\text{INT}} \frac{d\sigma_{\text{INT}}(W, \theta)}{d\cos\theta} + C_{\text{QED}} \frac{d\sigma_{\text{QED}}(W, \theta)}{d\cos\theta} \right] \cdot (1 + \delta_V)$$

where

$$(6) \quad C_{\text{RES}} = \left( \frac{2\Delta\omega}{M} \right)^{\beta_f} \left[ \frac{(W-M)^2 + (\Gamma/2)^2}{M^2/4} \right]^{\frac{\beta_i + 2\beta_{\text{int}}}{2}} \left[ 1 + 2\beta_i \frac{W-M}{\Gamma} \times \right. \\ \left. \times \left( \frac{\pi}{2} + \text{tg}^{-1} \left( \frac{W-M}{\Gamma/2} \right) \right) \right]$$

$$(7) \quad C_{\text{INT}} = \left( \frac{2\Delta\omega}{M} \right)^{\beta_f + \beta_{\text{int}}} \left[ \frac{(W-M)^2 + (\Gamma/2)^2}{M^2/4} \right]^{\frac{\beta_i + \beta_{\text{int}}}{2}} \left[ 1 - (\beta_i + \beta_{\text{int}}) \times \right. \\ \left. \times \frac{\Gamma}{2(W-M)} \left( \frac{\pi}{2} + \text{tg}^{-1} \left( \frac{W-M}{\Gamma/2} \right) \right) \right]$$

$$(8) \quad C_{\text{QED}} = \left( \frac{2\Delta\omega}{W} \right)^{\beta_i + \beta_f + 2\beta_{\text{int}}}$$

In eq. (5) we have explicitly separated out the infrared factors from the uncorrected differential cross section  $d\sigma_{\text{RES}}/d\cos\theta$ , etc. The above expressions (6) and (7) have also been obtained using an alternative method based on the coherent states formalism, developed in ref. (5), the details of which shall be presented elsewhere.

One might notice that eq. (6) shows a dependence from  $\beta_{\text{int}}$  not present in the first term of eq. (4). The reason lies in the fact that eq. (1), when used for the purely resonant term, cannot be integrated analytically unless  $\beta_{\text{int}} = 0$ . However the coherent states formalism quoted above gives us the exact  $\beta_{\text{int}}$  dependence of this term as it appears in eq. (6). In an experiment in which the charge of the final particles is not observed,  $\beta_{\text{int}}$  may safely be set equal to zero.

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To obtain the experimentally observed cross-sections one has to integrate eqs. (3) and (5) upon the machine resolution and the experimental apparatus as well. An excellent estimate for the reduction of the hadronic peak value when radiatively corrected and integrated upon the machine can be obtained as follows. Assuming the machine resolution function to be

$$G(W'-W) = \frac{1}{\sqrt{2\pi\sigma}} e^{-\frac{(W'-W)^2}{2\sigma^2}}$$

we find for the observed peak cross-section  $\tilde{\sigma}(M)$

$$(9) \quad \tilde{\sigma}(M) \approx \frac{6\pi^2 \Gamma_e \Gamma_h}{\sqrt{2\pi\sigma} M^2 \Gamma} \left(\frac{\Gamma}{M}\right)^{\beta_i} \exp\left(-\frac{\Gamma}{2\sqrt{2}\sigma}\right)^2 \left[ \operatorname{erfc}\left(\frac{\Gamma}{2\sqrt{2}\sigma}\right) + \frac{\beta_i}{2} E_1\left(\frac{\Gamma^2}{8\sigma^2}\right) \right] (1 + \delta_v)$$

where  $\operatorname{erfc}(z)$  and  $E_1(z)$  are defined as in ref. (10),  $\Gamma_e$  and  $\Gamma_h$  are the partial decay widths into electrons and hadrons respectively and  $\sigma$  is the machine parameter, such that  $(\Delta W)_{\text{FWHM}} = 2.3548 \sigma$ .

In eq. (9) the second term in the square bracket represents the contribution from the radiative tail. For resonances whose total width is smaller than the machine resolution, a simple expansion of eq. (9) in powers of  $\Gamma/(2\sqrt{2}\sigma)$  leads to

$$(9) \quad \tilde{\sigma}(M) \approx \frac{6\pi^2 \Gamma_e \Gamma_h}{\sqrt{2\pi\sigma} M^2 \Gamma} \left(\frac{\Gamma}{M}\right)^{\beta_i} \left[ 1 + \left(\frac{\Gamma}{2\sqrt{2}\sigma}\right)^2 \right] \left\{ 1 - \frac{\Gamma}{\sqrt{2\pi\sigma}} + \beta_i \times \right. \\ \left. \times \left[ \ln \frac{2\sqrt{2}\sigma}{\Gamma} - \frac{\gamma}{2} \right] \right\} (1 + \delta_v)$$

where  $\gamma = .5772$  is Euler's constant.

Again we find that, for a very narrow resonance like  $\psi(3.1)$ , the width of the resonance is the parameter which dictates the correction. Our expression (9') disagrees with Yennie's result<sup>(9)</sup>, however.

The observed values for  $\tilde{\sigma}(3.1)$  and  $\tilde{\sigma}(3.7)$  can now be used in eq. (9') to determine the ratio  $\Gamma_e \Gamma_h / \Gamma$ , once the machine parameter  $\sigma$  is known. In table I we give the result of this calculation, both for SPEAR<sup>(1)</sup> as well as the Frascati  $\gamma\gamma$ -group<sup>(2)</sup>, for suitable values of  $\sigma(x)$ . For  $\psi(3.1)$  the result is rather insensitive to the exact value of  $\Gamma$ . For  $\psi(3.7)$ , whose total width, as indicated by experiments, might be comparable with  $\sigma$ , we present in table I our results for two rather different sets of values of  $\Gamma$  and  $\sigma$ . As is clear from fig. 1 more data for  $\psi(3.7)$  are needed to pinpoint the parameters. The thrustworthiness of the peak approximation can be judged by comparing table I with the detailed fits to the experimental data shown in fig. 1. The curves shown have been obtained integrating eq. (3) upon the machine resolution.

Our final estimates are:

$$\psi(3.1): \quad \frac{\Gamma_e \Gamma_h}{\Gamma} = (4.0 \pm 0.2) \text{ keV},$$

$$\psi(3.7): \quad \frac{\Gamma_e \Gamma_h}{\Gamma} = (2.8 \pm 0.3) \text{ keV}.$$

Regarding the leptonic modes of  $\psi(3.1)$ , the extraction of the dynamical parameters is more complicated since the details of the experimental set ups have to be taken into account. Moreover unlike the hadronic case, interference effects are in principle important. For  $e^+e^- \rightarrow e^+e^-$  we show explicitly in fig. 2 the excitation curve obtained through eq. (5) to (8) and upon integration over the apparatus of the  $\gamma\gamma$

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(x) - The values of  $\sigma$  have been chosen to fit the observed excitation curves. For ADONE our values of  $\sigma$  agree with its expected value whereas for SPEAR there seems to be a discrepancy of about 30%.



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group<sup>(2)</sup> at Frascati (and the machine resolution as well). We assumed  $\psi(3.1)$  to be a vector particle,  $J^P = 1^-$ , and as one can see from fig. 2(a), there is practically no interference left upon machine integration. This is doubly checked by fig.2(b), where we have arbitrarily changed the sign of the interference term. We thus reach the conclusion that the dip observed before the resonance in the  $e^+e^-$  spectrum at SPEAR<sup>(1)</sup> as well as at Frascati<sup>(2)</sup> cannot be explained away as a radiative effect. We may remind the reader that if  $\psi(3.1)$  has  $J < 2$  no dip results before the resonance. Thus we are left with two basic questions: where does the dip come from in the original cross section and even if one did obtain it, why is it not washed out upon machine integration? It could be therefore interesting to know if the  $e^+e^-$  decay mode of  $\psi(3.7)$  shows a similar behaviour.

For the muonic decay of  $\psi(3.1)$  our formulae (eqs. (5) - (7)) give a rather interesting charge asymmetry solely as a radiative effect. This asymmetry is a very sensitive function of the distance from the resonance and can be sizeable in magnitude. However this effect could again be washed out by machine integration. This problem is presently under study by the MEA group at Frascati.

During the writing of this paper we become aware of ref. (11) where a completely different asymmetry factor due to infrared radiation is presented, which depends solely on  $\Delta\omega$ . As stated earlier, for very narrow resonances the relevant parameter is not  $\Delta\omega$  but  $\Gamma$  (so long as  $\Gamma \ll \Delta\omega$ ). We also disagree with a conclusion in ref. (11) which states that even upon machine integration much of the asymmetry (due to interference alone) survives.

We are greatly indebted to our colleagues G. Capon and G. Penso for helping us in analyzing the  $\gamma\gamma$  group data and providing us with the computer programs. Thanks are also due to the members of  $\overline{B\overline{B}}$ ,  $\gamma\gamma$  and MEA groups at Frascati for many helpful discussions. Finally, we wish to thank E. Etim and B. Touschek for sharing with us their insight into the theoretical aspects of radiative problems.

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TABLE I

Comparison of  $(\Gamma_e \Gamma_h / \Gamma)$  obtained by peak method (eqn. 9' of text) and by detailed fit to data (see curves).

$\psi(3.1)$			$\psi(3.7)$		
	$(\frac{\Gamma_e \Gamma_h}{\Gamma})$ Peak method	$(\frac{\Gamma_e \Gamma_h}{\Gamma})$ Curves		$(\frac{\Gamma_e \Gamma_h}{\Gamma})$ Peak method	$(\frac{\Gamma_e \Gamma_h}{\Gamma})$ Curves
(SPEAR) <sup>(1)</sup> $\sigma = 0.96$ MeV $\Gamma = 80$ keV $\tilde{\sigma}(M) = 2.28 \mu\text{b}$	3.86 keV	3.81 keV	(SPEAR) <sup>(1)</sup> $\sigma = 1.15$ MeV $\Gamma = 1.0$ MeV $\tilde{\sigma}(M) = 730$ nb	2.74 keV	2.75 keV
(SPEAR) <sup>(1)</sup> $\sigma = 1.05$ MeV $\Gamma = 80$ keV $\tilde{\sigma}(M) = 2.31 \mu\text{b}$	4.18 keV	4.17 keV	(SPEAR) <sup>(1)</sup> $\sigma = 1.4$ MeV $\Gamma = 0.2$ MeV $\tilde{\sigma}(M) = 762$ nb	2.71 keV	2.78 keV
(Frascati group $\gamma\gamma 2$ ) <sup>(2)</sup> $\sigma = 1.2$ MeV $\Gamma = 80$ keV $\tilde{\sigma}(M) = 1.87 \mu\text{b}$	3.81 keV	3.81 keV			

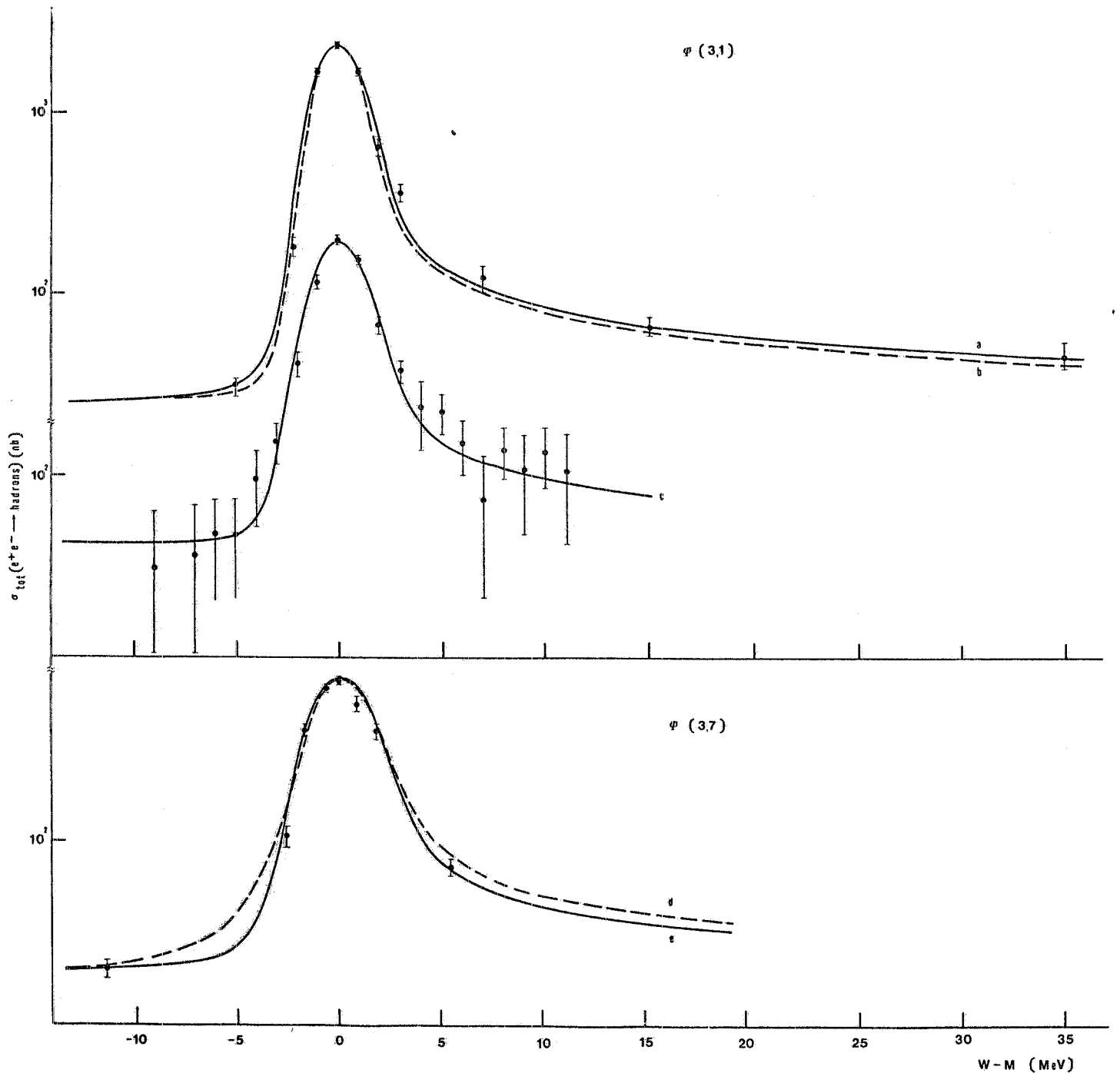


FIG. 1 - Hadronic excitation curves for  $\psi(3.1)$  and  $\psi(3.7)$ . Data are taken from SPEAR (ref. 1) and Group  $\gamma\gamma$  at Frascati (ref. 2). The various parameters are:  $\psi(3.1)$ : a)  $\Gamma_e \Gamma_h / \Gamma = 4.17$  keV;  $\sigma = 1.05$  MeV;  $\Gamma = 80$  keV; b)  $\Gamma_e \Gamma_h / \Gamma = 3.81$  keV;  $\sigma = 0.96$  MeV;  $\Gamma = 80$  keV; (c)  $\Gamma_e \Gamma_h / \Gamma = 3.81$  keV;  $\sigma = 1.2$  MeV;  $\Gamma = 80$  keV.  $\psi(3.7)$ : d)  $\Gamma_e \Gamma_h / \Gamma = 2.75$  keV;  $\sigma = 1.15$  MeV;  $\Gamma = 1$  MeV; e)  $\Gamma_e \Gamma_h / \Gamma = 2.78$  keV;  $\sigma = 1.4$  MeV;  $\Gamma = 0.2$  MeV.

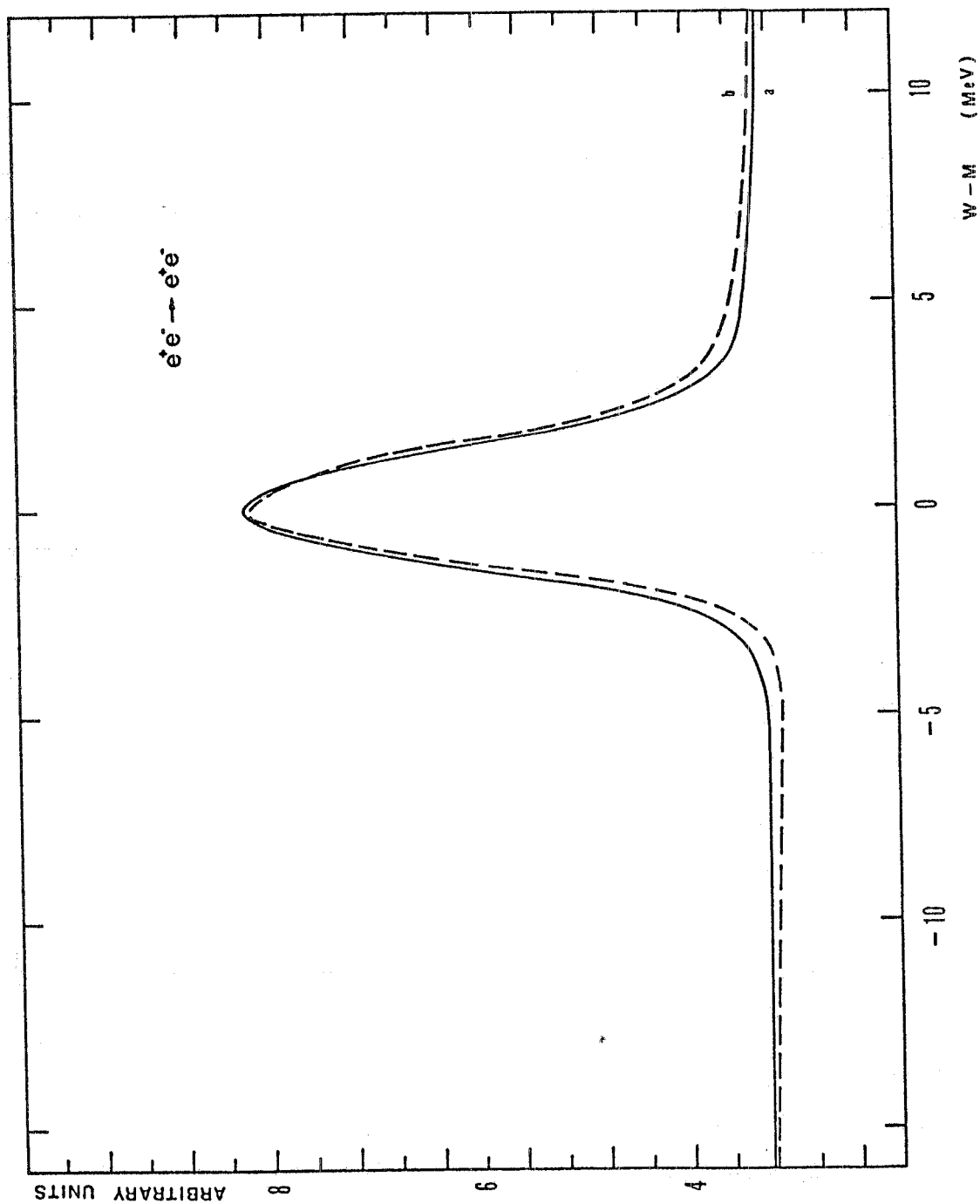


FIG. 2 -  $e^+e^-$  final state excitation curve (eq. 5) for Frascati group geometry (ref. 2). Curve a) corresponds the interference term for a vector particle, while b) has the opposite sign for this term.