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G. Matone and D. Prosperi: PROPOSAL FOR THE MEASUREMENT OF THE PROTON ELECTRIC AND MAGNETIC POLARIZABILITIES.

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1. - INTRODUCTION. -

As it is well known from classical electrodynamics, the interaction of photons with a system of charges may be described by a fairly small number of real parameters, provided that the wavelength of the photon considerably exceeds the dimension of the system and the photon frequency is essentially less than its resonance frequencies. These constants fully determine the behaviour of the system in static or slowly changing fields and are expressed through its charge, magnetic moment and polarizabilities(1).

To illustrate this point, let us briefly examine the expression of the elastic (Compton) scattering amplitude of low-energy photons by a non relativistic and spherically symmetric system of charge (Ze) and mass (Am). At the order ω^2, we have(2):

$$f(\omega) = -\frac{(Ze)^2}{Am} + \omega^2 \left[\frac{(Ze)^2}{Am} \frac{\langle r_{ch}^2 \rangle}{3} + \alpha\right] (\vec{\varepsilon}_i \cdot \vec{\varepsilon}_f) + \tag{1}$$
$$+ \omega^2 (\vec{\varepsilon}_i \wedge \vec{k}_i)(\vec{\varepsilon}_f \wedge \vec{k}_f) \left[\chi - \frac{1}{6} \frac{Ze^2}{m} \langle r_{ch}^2 \rangle + \frac{\langle \vec{D}^2 \rangle}{6\,Am}\right]$$

where $\langle r_{ch}^2 \rangle$ is the charge mean square radius of the system, $\vec{D}$ is its magnetic dipole operator, ω is the incoming photon frequency, ($\vec{\varepsilon}_i$, $\vec{k}_i$; $\vec{\varepsilon}_f$, $\vec{k}_f$) are polarizations and momentum vectors of the incoming and outgoing photons. Moreover, α and χ are constants, respectively called "electric polarizability" and "magnetic susceptivity" of the system .

The origin of the first three terms of $f(\omega)$ can be understood in the following way[2]. The electric field $\vec{E}(\vec{r},t) = \vec{E}_o\, e^{i(\vec{k}\cdot\vec{r}-\omega t)}$ of the photon produces two effects:

1) It generates an oscillation of the center of mass coordinate $\vec{R}(t)$ of the system and an associate time-varying dipole moment

$$\vec{D}(t) = Ze\, F_{ch}(\omega^2)\vec{R}(t),$$

where $F_{ch}(k^2=\omega^2)$ is the charge form factor of the system. Moreover, $\vec{R}(t)$ can be obtained by the motion equation

$$(Am)\,\ddot{\vec{R}}(t) = \vec{F}(t),$$

where $\vec{F}(t) = (Ze)F_{ch}(\omega^2)\vec{E}_o\, e^{-i\omega t}$ is the total electric force acting on the system.

2) It polarizes the internal structure of the system inducing an additive contribution $\vec{d}(t)$ to the electric dipole moment, given by:

$$\vec{d}(t) = \alpha < \vec{E}(\vec{r},t) >,$$

where $< \vec{E}(\vec{r},t) > \simeq \vec{E}_o\left[1+0(\omega^2 < r^2_{ch} >)\right]e^{-i\omega t}$ is the average field over the system.

Finally, the total dipole moment $[\vec{D}(t)+\vec{d}(t)]$ acts like a classical antenna and radiates an energy whose intensity is proportional to

$$|\ddot{\vec{D}}(t)+\ddot{\vec{d}}(t)|^2 = |(Ze)F_{ch}(\omega^2)\ddot{\vec{R}}(t)+\ddot{\vec{d}}(t)|^2 \propto \left|\frac{(Ze)^2}{Am}F^2_{ch}(\omega^2) - \omega^2\alpha\right|^2.$$

We can now develop $F^2_{ch}(\omega^2)$ into a Taylor series obtaining

$$\left[F^2_{ch}(\omega)\right]^2 \simeq 1 - \frac{1}{3}\omega^2 < r^2_{ch} > + \ldots .$$

By putting $F^2_{ch}=1$, we generate the first term of eq. (1), that is the usual "Thomson amplitude". The corrective term $F^2_{ch} = -1/3\,\omega^2 < r^2_{ch} >$ generates the second contribution of eq. (1), generally called "retardation term". Moreover, the sum of the terms proportional to $\omega^2(\vec{\varepsilon}_i\cdot\vec{\varepsilon}_f)$ is known as "electric Rayleigh amplitude".

The origin of the residual terms of eq. (1) can be understood in a similar ray. The magnetic field of the photon $\vec{H}(\vec{r},t)$ induces a magnetic dipole moment $\vec{m}(t) = \chi < \vec{H}(\vec{r},t) >$. By applying the classical antenna formula we obtain once more an irradiated energy whose intensity is proportional to $\omega^2(\chi + \ldots)$. This term is called "Rayleigh magnetic amplitude".

Several points should be noted: first the electric and magnetic terms of eq. (1) can be easily separated thanks to their different dependence on polarization vectors and angles. Second, Thomson and electric Rayleigh scattering contributions can be separated only by their different energy behaviour. Moreover, in a photon scattering experiment it is impossible to physically separate the various contributions to the Rayleigh amplitudes, both by varying angles and by varying the energy at this order; we can only measure parameters of complex structure, as

(2)
$$\begin{cases} \bar{\alpha} = \alpha + \frac{1}{3}\frac{(Ze)^2}{Am} \langle r_{ch}^2 \rangle , \\ \bar{\beta} = \chi - \frac{1}{6}\frac{Ze^2}{m} \langle r_{ch}^2 \rangle + \frac{1}{6\,Am} \langle \vec{D}^2 \rangle \; ^{(x)} . \end{cases}$$

Finally, in the case of particles with spin, the scattering amplitude acquires additional terms whose contribution to the cross sections will be explicitely given in the next section.

2. - THE CROSS SECTION FOR PHOTON COMPTON SCATTERING. -

Aim of the present note is to briefly analyze the experimental information now available on the proton polarizabilities and to present the proposal of a new precise determination of both parameters.

Let us start with a short discussion of the cross section for proton Compton scattering. It can be specialized according to whether the polarization vector $\vec{\varepsilon}_i$ is parallel or perpendicular to the scattering plane, as follows[3]:

(3)
$$\frac{d\sigma^{\perp}}{d\Omega} = \left(\frac{e^2}{m}\right)^2 \left\{ 1 - 2\left(\frac{\omega}{m}\right)^2 (A_E + A_M \cos\theta) + \left(\frac{\omega}{m}\right)^2 \left(\frac{1}{4}\beta_1 - \frac{1}{2}\beta_2\cos\theta - \frac{1}{4}\beta_3\cos^2\theta\right) \right\} ,$$

$$\frac{d\sigma^{\shortparallel}}{d\Omega} = \left(\frac{e^2}{m}\right)^2 \left\{ \cos^2\theta + 2\frac{\omega}{m}\cos\theta(1-\cos^2\theta) - 2\left(\frac{\omega}{m}\right)^2 (A_M + A_E\cos\theta)\cos\theta + \left(\frac{\omega}{m}\right)^2 \left(\frac{1}{4}\beta_4 - \frac{1}{2}\beta_2\cos\theta + \frac{1}{4}\beta_5\cos^2\theta + \cos^4\theta\right) \right\}$$

(x) - The last contribution to $\bar{\beta}$ is present only in ref. (2).

where $\lambda = 1,7896$ is the proton anomalous magnetic moment and

$$\beta_1 = 2(1+\lambda)^4 + \lambda^2, \quad \beta_2 = 2\lambda^3 + 5\lambda^2 + 4\lambda + 1, \quad \beta_3 = \lambda^4 + 4\lambda^3 + 3\lambda^2,$$

$$\beta_4 = (1+\lambda)^4 + \lambda^2 + 5, \quad \beta_5 = 3\lambda^2 + 4\lambda - 8$$

Moreover, the dimensionless factors A_E and A_M are the parameters $\bar{\alpha}$ and $\bar{\beta}$ expressed in units of $(e^2/\hbar c)(\hbar/mc)^3 = 6.8 \times 10^{-5}\ \mathrm{fm}^3$.

The cross section for unpolarized γ rays is given by the usual expression

$$\frac{d\sigma}{d\Omega} = \frac{1}{2}\left(\frac{d\sigma^{\perp}}{d\Omega} + \frac{d\sigma^{\parallel}}{d\Omega}\right) \tag{4}$$

which has been plotted in Fig. 1 as a function of θ for different values of A_E and A_M.

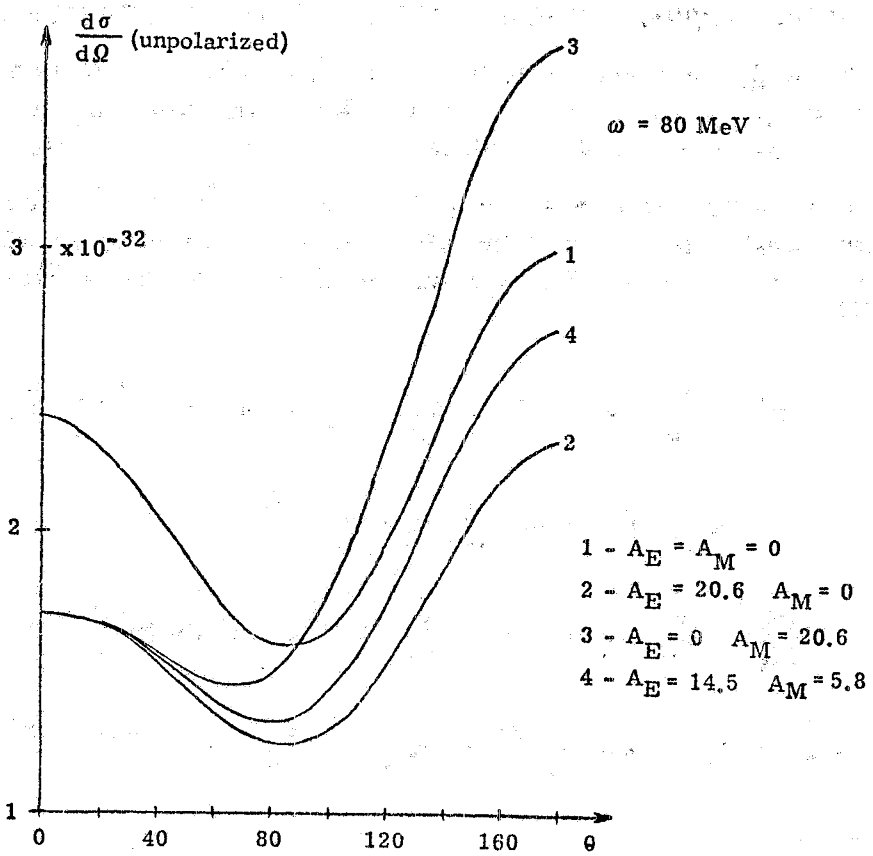

FIG. 1 - Cross section with unpolarized photons in function of the scattering angle θ and for different values of A_E, A_M.

In particular one has:

$$\frac{d\sigma}{d\Omega}(0^o) = \left(\frac{e^2}{m}\right)^2 \left\{ \left[1-2(A_E+A_M)\left(\frac{\omega}{m}\right)^2\right] + 2.72\left(\frac{\omega}{m}\right)^2 \right\} ,$$

(5) $$\frac{d\sigma}{d\Omega}(90^o) = \left(\frac{e^2}{m}\right)^2 \frac{1}{2} \left\{ 1+\left[-2A_E+48.6\right]\left(\frac{\omega}{m}\right)^2 \right\} ,$$

$$\frac{d\sigma}{d\Omega}(180^o) = \left(\frac{e^2}{m}\right)^2 \left\{ 1+\left[38.4-2(A_E-A_M)\right]\left(\frac{\omega}{m}\right)^2 \right\}$$

As one can see, the cross section is determined by (A_E+A_M) in the forward direction and by (A_E-A_M) in the backward direction. Separate information on A_E can be obtained only at 90^o where $(d\sigma/d\Omega)$ doesn't depend on A_M.

3. - PROTON POLARIZABILITIES. -

As clearly shown by Fig. 1, the Compton scattering of photons is a sensitive toool for measuring the electric and magnetic polarizabilities of a proton. In general α and χ measure the ability of the elementary charges determining the internal structure of the studied system, to be deformed by the outer electric and magnetic fields. In the nucleon case, they measure the polarizability of the "meson cloud" or, in a modern language, the polarizability of its "quarks structure". To clarify this point let us present in Fig. 2 a selection of graphs contributing to generate in $f(\theta)$ quadratic terms of the kind previously considered.

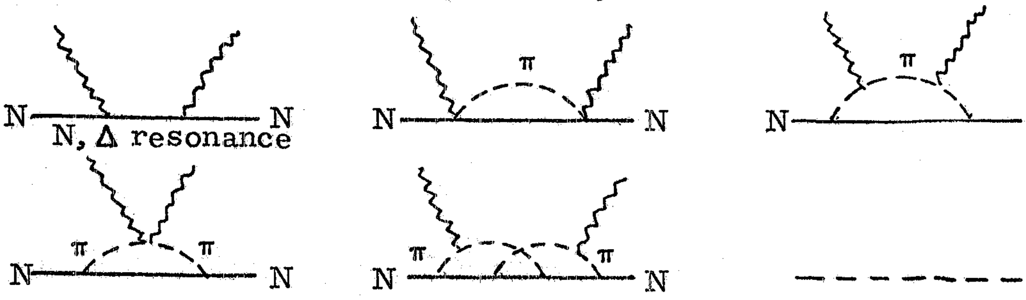

FIG. 2 - A selection of graphs contributing to the electric and magnetic nucleon polarizabilities.

An important question is whether the knowledge of the nucleon polarizabilities can be considered a sensitive test of the various models proposed to describe its internal structure (similar arguments can be applied to other elementary particles). This problem has not yet been clarified from the theoretical point of view; in addition, as it will be shown in the following sections, the presently available experimental results seem to be too uncertain to allow definite conclusions.

a) - The Goldansky experiment. -

The only direct experimental determination of such fundamental parameters was made a decade ago by Goldansky et al. (4) who investigated the Compton scattering cross section by an umpolarized bremsstrahlung photon beam in the backward hemisphere; the energy ranged from 40 to 70 MeV with a mean value of 56 MeV.

It should be noted at once that in this energy range the Rayleigh scattering contribution doesn't exceed about 10% of the Thomson energy independent contribution and this figure is comparable to the systematic uncertainties in the cross section normalization ($\sim 8\%$) quoted by the authors(4). However, by relying only on the measured value $d\sigma(90^o)/d\Omega = (1.1 \pm 0.05) 10^{-32}$ cm^2, Goldansky et al. gave the estimate $A_E = (16 \pm 5.8)$ or, equivalently, $\bar{\alpha} = (11 \pm 4)\ 10^{-4}$ fm^3.

A more precise evaluation of $\bar{\alpha}$ as well as an estimate of $\bar{\beta}$ was obtained by a fit of all experimental data. To constrain the fit, Goldansky and coworkers fixed the sum $(\bar{\alpha} + \bar{\beta})$ by the exact dispersion relation

$$(6) \qquad \bar{\alpha} + \bar{\beta} = \frac{1}{2\pi^2} \int_0^\infty \frac{1}{\omega^2}\, \sigma_T(\omega)\, d\omega$$

where $\sigma_T(\omega)$ is the total photo-absorption cross section. For proton, the experimental photo-production data existing at the time allowed to obtain $(\bar{\alpha}_p + \bar{\beta}_p) = 11 \times 10^{-4}$ fm^3 or $(A_E + A_M) = 16$.

Since eq. (6) is obtained from the most general properties of the scattering matrix, there are no grounds to question it in any degree at all. Therefore, since $\bar{\alpha}$ and $\bar{\beta}$ are positive, eq. (6) provides an upper limit of both constants.

By taking into account this constraint, Goldasnky and coworkers obtained

$$(7) \qquad \begin{cases} \bar{\alpha}_p = (9 \pm 2) \times 10^{-4}\ \text{fm}^3, \\ \bar{\beta}_p = (2 \pm 2) \times 10^{-4}\ \text{fm}^3. \end{cases}$$

This value of $\bar{\alpha}_p$ is in remarkable agreement with the estimate obtained at 90^o. Moreover, the quoted error is only of statistical origin; for an estimate of the overall error, remarkably higher, see the next section.

To conclude, let us compare $\bar{\alpha}_p$ with the neutron corresponding parameter $\bar{\alpha}_n$. Limits to the neutron electric polarizability have been given as $-50 \times 10^{-4}\ \mathrm{fm}^3 < \bar{\alpha}_n < 60 \times 10^{-4}\ \mathrm{fm}^3$ by Aleksandrov et al. (5) in an experiment drifting slow neutrons in the Coulomb field of lead nuclei. A further estimate of $\bar{\alpha}_n$ can be obtained by analyzing experimental cross sections of Compton scattering by deuterons. This was done by Tenore and Verganelakis(3) in the framework of the impulse approximation with some corrections which strongly improved the theory. They found a value $\bar{\alpha}_n \simeq 12 \times 10^{-4}\ \mathrm{fm}^3$ somewhat larger than $\bar{\alpha}_p$.

b) - Estimate of the proton polarizabilities by sum rules. -

In the Goldansky experiment the proton magnetic polarizability seems to be smaller than the electric one. Recently Bernabeu et al. (6) pointed-out that this finding must be considered as "extremely surprising". In fact, one knows from experiments that the $\Delta(1236)$ barion resonance is the dominant contribution to the sum rule (6) and this excitation is of magnetic nature. So one expects that the nucleon should be essentially a good paramagnetic object and then $\chi > \alpha$. Furthermore, in a naive quark model, the resonance corresponds to a spin reorientation of the quarks, i.e. to the effect an external magnetic field would have on the nucleon. However, no corresponding simple electric excitation exists, which again argues for an important value of χ as compared to α.

To clarify to what extent the result $\bar{\alpha} > \bar{\beta}$ is compelling, Bernabeu et al. (6) attempted to critically reanalyze Goldansky experiment. They adopted a more precise determination of the sum rule (6) by the modern photoproduction data giving $(\bar{\alpha}_p + \bar{\beta}_p) = (14.2 \pm 0.3) \times 10^{-4}\ \mathrm{fm}^3$(7). By taking into account also the systematic error they obtained

(8) $$\bar{\alpha}_p = (10 \pm 5) \times 10^{-4}\ \mathrm{fm}^3, \qquad \bar{\beta}_p = (4 \mp 5) \times 10^{-4}\ \mathrm{fm}^3.$$

It is evident that, on the basis of the estimate (8), a value $\bar{\beta}_p > \bar{\alpha}_p$ cannot be ruled-out, so that further experiments are needed to clarify if $\bar{\beta}_p > \bar{\alpha}_p$ or $\bar{\beta}_p < \bar{\alpha}_p$.

To further examine this point the same authors(6) derived the following "backward sum rule" for $(\bar{\alpha} - \bar{\beta})$:

$$\bar{\alpha} - \bar{\beta} = -\left(\frac{e^2}{4\pi}\right)\frac{1}{2M^3}\,\lambda\,(2Z+\lambda) + \frac{1}{2\pi^2}\int_0^\infty \frac{d\omega}{\omega^2}$$

(9) $$\times\left(1+2\frac{\omega}{m}\right)^{1/2}\left[\sigma\,(\Delta\pi = \mathrm{yes}) - \sigma\,(\Delta\pi = \mathrm{no})\right] +$$

$$+\left[\text{annihilation channel } \gamma\gamma \to N\bar{N} \text{ contribution}\right]$$

They evaluated the integrand in the r. h. s. with the same photoproduction data used to evaluate ($\bar{\alpha}_p + \bar{\beta}_p$) together with the multipolarity analysis of Lüke[8]. By neglecting the annihilation channel and by combining the two sum rules for ($\bar{\alpha}_p + \bar{\beta}_p$) and ($\bar{\alpha}_p - \bar{\alpha}_n$) they obtained the new estimate

(10) $$\bar{\alpha}_p \simeq 4 \times 10^{-4}\ \mathrm{fm}^3, \qquad \bar{\beta}_p \simeq 10 \times 10^{-4}\ \mathrm{fm}^3$$

which disagrees with the Goldansky result. However, it must be noted that this result is subjected to the uncertainties deriving from the annihilation channel which presently cannot be easily estimated. The same method gives for the neutron $\bar{\alpha}_n \simeq 1,2\ \bar{\alpha}_p$ leaving unchanged the sum of the two polarizabilities ($\bar{\alpha}_n + \bar{\beta}_n = \bar{\alpha}_p + \bar{\beta}_p$).

Very recently, Bernabeu et Tarrach[9] have shown that the nucleon electric polarizability can be connected to the longitudinal Compton cross--section by writing an unsubtracted dispersion relation for the longitudinal Compton amplitude. They obtained

(11) $$\bar{\alpha} = \frac{e^2}{4\pi}\frac{\lambda^2}{4m^2} + \frac{1}{2\pi^2}\int_{\nu_{th.}}^{\infty} d\nu \lim_{q^2 \to 0} \frac{\sigma_L(\nu, q^2)}{-q^2},$$

where $\sigma_L(\nu, q^2)$ is related to the two usual nucleon structure functions $W_{1,2}$ by means of the relation

$$\sigma_L(\nu, q^2) = \left(1 - \frac{\nu^2}{q^2}\right) W_2(\nu, q^2) - W_1(\nu, q^2).$$

The main importance of this relation is that it gives a model independent method allowing to estimate a lower limit for $\bar{\alpha}_p$. In fact, in order to get a numerical estimate of eq. (11) one needs the slope of $\sigma_L(\nu, q^2)$ at $q^2 \to 0$ as a function of ν. On the other hand, we have now experimental data only in the range $0.5 < \nu < 7.5$ GeV. If we observe that the integrand in the r. h. s. of eq. (11) is positive definite, we can immediately conclude that a cut in the integration reduces the estimate of $\bar{\alpha}_p$ below its real value. By using the data quoted in the Bloom report[10], we obtain the limit $\bar{\alpha}_p \gtrsim 2 \times 10^{-4}\ \mathrm{fm}^3$ which agrees with all previous estimates.

A calculation of the proton electric polarizability achieved by calculating the inelastic contribution of the two photon exchange graph (see Fig. 3), to the electron-proton scattering amplitude has been given by Schröder[11]. This line of approach is very similar to that of Bernabeu and Tarrach[9]: after introducing several approximations, he was able to give the following result

$$\bar{\alpha}_p \simeq \frac{2}{\pi}(\bar{\alpha}_p + \bar{\beta}_p).$$

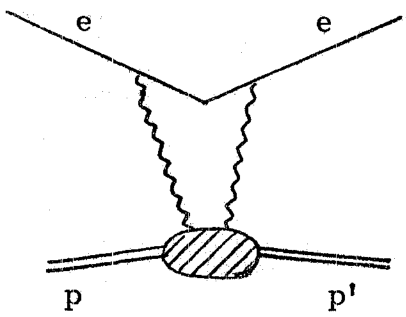

FIG. 3 - Feynman graph for the two photon exchange amplitude in electron proton scattering.

By using eq. (6), one gets

$$\bar{\alpha}_p = 9 \times 10^{-4}\ \mathrm{fm}^3$$

which implies $\bar{\alpha}_p > \bar{\beta}_p$, in disagreement with the Ericson's result.

c) - Other theoretical estimates. -

A theoretical expression for the nucleon polarizabilities can be easily obtained by second order perturbation theory. If one neglects exchange effects one has(1, 2)

$$(12) \qquad \alpha = 2 \sum_n \frac{|\langle n | d_z | 0 \rangle|^2}{E_n - E_o}, \qquad \chi = 2 \sum_n \frac{|\langle n | m_z | 0 \rangle|^2}{E_n - E_o},$$

where $\vec{d}$ and $\vec{m}$ are, respectively, the dipole electric and magnetic operators defined in the usual way; moreover, the sum must be extended over all the nucleon excited states.

A rough estimate of the previous expression for α_p can be obtained by substituting $(E_n - E_o)$ with an average denominator $\langle \Delta E \rangle$ and by using the completeness relation $\sum_n | n \rangle \langle n | = 1$. For A non relativistic fermion constituents in a s-state of the relative motion we find:

$$(13) \qquad \alpha_p \simeq \frac{2}{\langle \Delta E \rangle} \langle p | \left(\sum_{1}^{A}{}_j\, e_j z_j \right)^2 | p \rangle = \frac{2}{\langle \Delta E \rangle} \sum_{1}^{A}{}_j\, e_j^2 \langle p | z_j^2 | p \rangle = \frac{2}{3} \frac{1}{\langle \Delta E \rangle} \sum_{1}^{A}{}_j\, e_j^2 \langle r_{ch}^2 \rangle_p .$$

It is immediate to check that, for the usual symmetric quark model, we have $\Sigma e_j^2 = e^2(4/9+4/9+1/9) = e^2$ and $\alpha_p \simeq 2/3\,(e^2/\langle \Delta E\rangle)\langle r^2_{ch}\rangle_p$. By putting $\langle r^2_{ch}\rangle_p = 0.65\ \mathrm{fm}^2$ and $\langle \Delta E\rangle \simeq 500$ MeV according to a recent work by Cannata[12], we obtain $\alpha_p \simeq 12.4 \times 10^{-4}\ \mathrm{fm}^3$; a similar approach gives $\alpha_n \simeq (2/3)\,\alpha_p$.

On the other hand, to estimate β_p, we can exploit the fact that the photo-absorption cross section is dominated by the $\Delta(1236)$ resonance. So we can write

$$\chi_p \simeq \frac{2}{\langle \Delta E(1236)\rangle} \left|\langle p|m_z|1236\rangle\right|^2 . \tag{14}$$

According to the symmetric quark model we have (13)

$$\left|\langle p|m_z|1236\rangle\right|^2 = \left(\frac{2\sqrt{2}}{3}\ \mu_p\right)^2 = 7.7 \times 10^{-2}\ \mathrm{MeV\ fm}^3$$

and, finally, $\chi_p \simeq 5.2 \times 10^{-4}\ \mathrm{fm}^3$.

Coming now to calculate $\bar{\alpha}_p$ and $\bar{\beta}_p$ we obtain

$$\begin{cases} \bar{\alpha}_p \simeq (\alpha + 3.3 \times 10^{-4})\,\mathrm{fm}^3 = 15.7 \times 10^{-4}\ \mathrm{fm}^3 , \\ \bar{\beta}_p \simeq (\chi + 0 \times 10^{-4})\ \mathrm{fm}^3 = 5.2 \times 10^{-4}\ \mathrm{fm}^3 . \end{cases} \tag{15}$$

The value $(\bar{\alpha}_p + \bar{\beta}_p) \simeq 2.1 \times 10^{-3}\ \mathrm{fm}^3$ calculated from the previous estimate exceeds the value given by the sum rule (6) by about 50%, so that at least one of the two parameters must be strongly overestimated.

We can now conclude this section by stressing the fact that presently the situation is so confused that new experimental data are seriously needed.

4.- POLARIZED PHOTON BEAM.-

a) - Experimental set-up.-

Let us now briefly discuss the possibilities offered by a monochromatic and plane-polarized photon beam of the kind provided by the "Ladon project[14]". First of all the monochromaticity of the photon beam ($\Delta E_\gamma / E_\gamma \simeq 1\%$) removes all the usual difficulties one has with bremsstrahlung beams. Moreover, a polarized photon beam allows to measure linear combinations of A_E adn A_M which cannot be reached by unpolarized gamma rays and therefore provides extra constraints, making more reliable the data analysis.

A plot of the differential cross sections $(d\sigma''/d\Omega)_\theta$ and $(d\sigma^\perp/d\Omega)_\theta$ given by eqs. (3) is shown in Fig. 4. Let us now introduce the two following linear combinations of $(d\sigma''/d\Omega)_\theta$ and $(d\sigma^\perp/d\Omega)_\theta$

(16) $$f_1(\theta) = \frac{1}{2}\left[\frac{d\sigma^\perp}{d\Omega} - \frac{d\sigma''}{d\Omega}\right]_\theta, \qquad f_2(\theta) = \frac{1}{2}\left[\frac{d\sigma}{d\Omega}\cos^2\theta - \frac{d\sigma''}{d\Omega}\right]_\theta$$

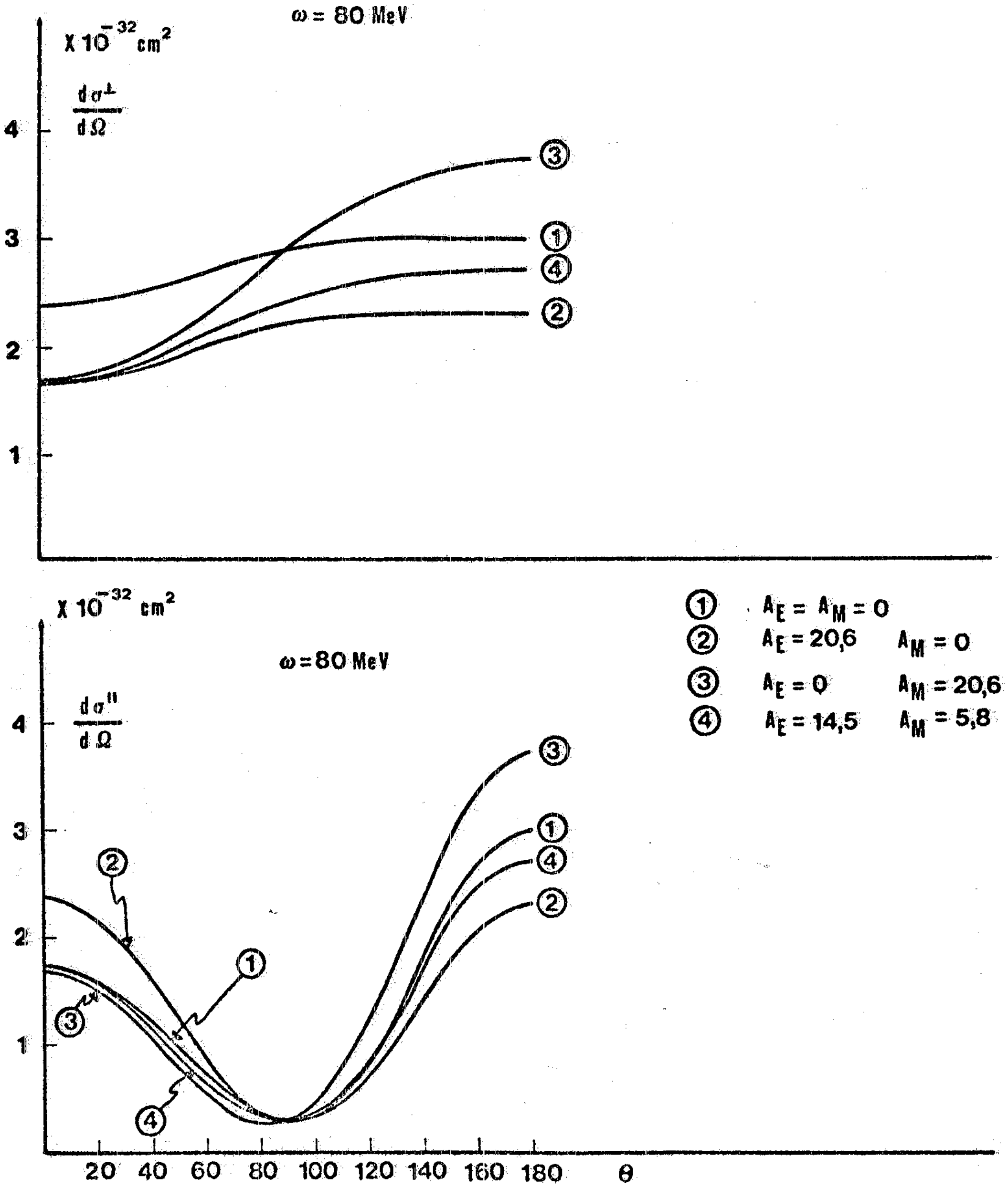

FIG. 4 - $d\sigma^\perp/d\Omega$ and $d\sigma''/d\Omega$ as a functions of θ for different values of A_E, A_M.

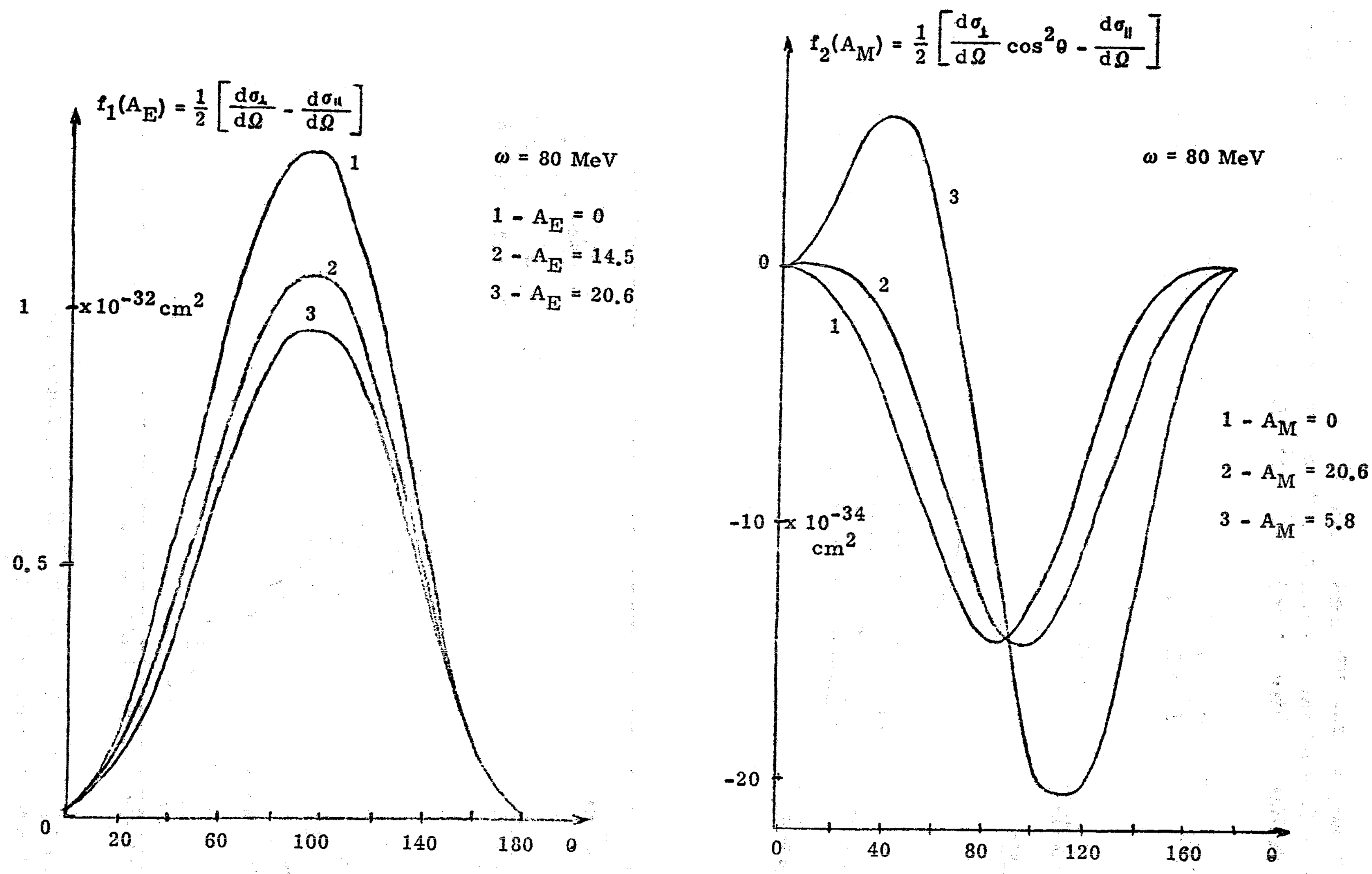

FIG. 5 - $f_1(A_E)$ and $f_2(A_M)$ as a function of θ at different values of A_E, A_M.

It is straightforward to check that they depend respectively only on A_E and A_M, so that $f_1 \equiv f_1(A_E, \theta)$, $f_2 \equiv f_2(A_M, \theta)$. The functions $f_1(A_E, \theta)$ and $f_2(A_M, \theta)$ are plotted in Fig. 5, in the cases

$$A_E = 0,\ 14.5,\ 20.6 \qquad \text{and} \qquad A_M = 0,\ 5.8,\ 20.6$$

A schematic sketch of the experimental set-up planned by us to measure $(d\sigma^{\parallel}/d\Omega)_\theta$ and $(d\sigma^{\perp}/d\Omega)_\theta$ is shown in Fig. 6. The photon beam passes through a collimator system and hits a liquid H_2 target. The scattered photons are detected at the same time by six counters placed at three different angles ($\theta = 60^o, 90^o, 120^o$) on both the scattering planes parallel and perpendicular to the initial polarization vector $\vec{\varepsilon}_i$.

For the sake of defiteness let us assume the detectors to be NaI(4"x 6") counters placed at 60 cm from the center of the target and covering a solid angle $\Delta\Omega \simeq 9 \times 10^{-2}$ ster. Their efficiency (ε) will be essentially determined by the width of the energy window of the electronic analysis chain; in the following we will assume a typical value $\varepsilon = 50\%$, somewhat higher than the standard "full-peak efficiency". Moreover their energy resolution, of the order of 10%, will be certainly enough for our purposes.

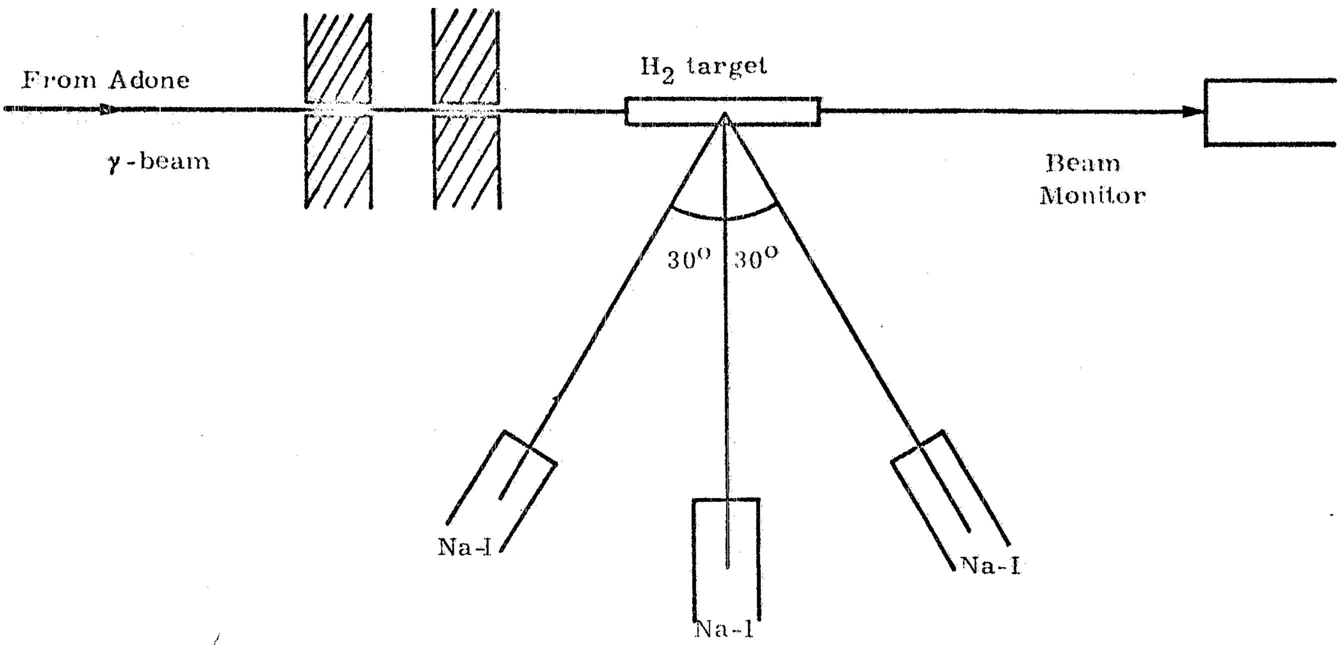

FIG. 6 - Plant view of the experimental apparatus.

b) - Counting rates. -

The counting rate of every gamma detector can be obtained by the standard expression

$$C_\gamma = \frac{d\sigma}{d\Omega} (\Delta\Omega_\gamma \cdot \varepsilon)\phi_\gamma (T_\varrho) A^{-1}, \tag{17}$$

where $(\Delta\Omega_\gamma \cdot \varepsilon) \simeq 4.5 \times 10^{-2}$. Assuming a beam intensity $\phi_\gamma \simeq 1.5 \times 10^7$ photons/sec(14) and a liquid hydrogen cell 30 cm long ($T_\varrho \simeq 2.1$ gr/cm^2) one has:

$$C_\gamma \simeq 8.5 \times 10^{29} \left(\frac{d\sigma}{d\Omega}\right) \text{counts/sec.}$$

At an incident energy $E_\gamma = 80$ MeV, the typical counting rates one can obtain at the selected angles are:

(18)
$$\theta = 90^\circ: \quad C_\gamma^{\parallel} \simeq 9 \text{ counts/h}, \quad C_\gamma^{\perp} \simeq 76 \text{ counts/h};$$
$$\theta = 60^\circ, 120^\circ: \quad C_\gamma^{\parallel} \simeq 21 \text{ counts/h} \quad C_\gamma^{\perp} \simeq 70 \text{ counts/h}$$

Thus for our apparatus the trigger are should be of the order of 270 counts/h.

Remembering eqs. (16), we can write:

$$f_1(A_E, \theta) \propto (C_\gamma^{\perp} - C_\gamma^{\parallel})_\theta, \qquad f_2(A_M, \theta) \propto (C_\gamma^{\perp} \cos^2\theta - C_\gamma^{\parallel})_\theta. \tag{19}$$

If we extract A_E and A_M from eqs. (19) we obtain statistical errors given by the following expressions:

$$\frac{|\Delta A_E|}{A_E} = \left| \frac{f_1(A_E)}{f_1(A_E) - f_1(0)} \right| \frac{|\Delta f_1|}{f_1}, \qquad \frac{|A_M|}{A_M} = \left| \frac{f_2(A_M)}{f_2(A_M) - f_2(0)} \right| \frac{|\Delta f_2|}{f_2} \tag{20}$$

where:

$$\frac{|\Delta f_1|}{f_1} = \frac{\sqrt{C_\gamma^{\perp} + C_\gamma^{\parallel}}}{C_\gamma^{\perp} - C_\gamma^{\parallel}}, \qquad \frac{|\Delta f_2|}{f_2} = \frac{\sqrt{C_\gamma^{\perp} \cos^4\theta + C_\gamma^{\parallel}}}{C_\gamma^{\perp} \cos^2\theta - C_\gamma^{\parallel}}.$$

Guessing the real values of A_E and A_M to be not too far from the present experimental determination[4] ($A_E \simeq 14$, $A_M \simeq 6$), eqs. (20) give:

$$\frac{|\Delta A_E|}{A_E}(90^o) \simeq \frac{58}{\sqrt{n}}\ \%, \qquad \frac{|\Delta A_E|}{A_E}(60^o, 120^o) \simeq \frac{68}{\sqrt{n}}\ \%,$$

$$\frac{|\Delta A_M|}{A_M}(60^o, 120^o) \simeq \frac{3.1}{\sqrt{n}}\ \%,$$

where n is the number of counting hours.

Assuming n to be of the order of 300, we obtain

$$\frac{|\Delta A_E|}{A_E}(90^o) \simeq 3.4\%, \qquad \frac{|\Delta A_E|}{A_E}(60^o, 120^o) \simeq 4\%,$$

$$\frac{|\Delta A_M|}{A_M}(60^o, 120^o) \simeq 18\% .$$

Finally, by combining these results together, we obtain the following estimate of the statistical precision we can get with all measurements:

(20') $$\frac{|\Delta A_E|}{A_E} \simeq 2\%, \qquad \frac{|\Delta A_M|}{A_M} \simeq 13\%.$$

Let us remember that the corresponding statistical errors in the Goldansky experiment[4] are 20% and 100% respectively.

c) - Background. -

Since our apparatus doesn't select the scattered photons which are in coincidence with the recoil protons, we are forced to carry-out an accurate analysis of all possible causes of background.

Let us start with some preliminary remarks. As it is well known, in the elastic scattering two-body processes there is a fixed relation between the scattering angle θ and the final energy ω':

$$\omega' \simeq \omega - \frac{q^2}{2M} \simeq \omega\,(1 - 2\,\frac{\omega}{M}\,\mathrm{sen}^2\,\frac{\theta}{2}).$$

All scattered photons have energies not too far from the beam energy ($\omega' \gtrsim 68$ MeV). Moreover, it is clearly convenient to discriminate the photons with energy windows centered around ω' whose width should be tipically $\Delta\omega'/\omega' \simeq \pm 15\%$. It is now obvious that the existence of these windows greatly helps to reduce the background.

The main background sources are due to the Delbrück elastic scattering and the radiative production of electron pairs. The cross section for the first process in hydrogen is very small ($\sigma_{tot} \simeq 10^{-34}$ cm^2) and besides is very strongly forward peaked ($d\sigma/d\Omega \sim \theta^{-4}$), so that we can neglect it completely for all angles $\theta \gtrsim 60^o$. The same argument applies also to the second process. Its contribution to the background is lowered further by the fact that only a small part of the gamma rays following the production of pairs possesses energies inside the energy windows.

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