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Technical Notes

Evaluation of Integrals by Differentiation with Application to Collision Methods

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ABSTRACT

The main difficulty which arises in the use of collision methods consists in integrating over functions having a pole at some point. This fact implies the need for very careful and extensive numerical calculations to obtain good results.

An integration method, which also has general validity, is presented here to overcome such a difficulty. It is shown that this method can be applied in collision problems connected with the physics of nuclear reactors.

INTRODUCTION

Often, to evaluate physical quantities connected with nuclear reactors, it is necessary or convenient to use collision methods instead of diffusion or transport methods.

We consider a region, supposed to have monodimensional geometry, which is extended in the interval $(0,R)$. Let $S(\rho)$ be the neutron source defined in this region representing the number of neutrons per unit volume that are created at the point ρ.

The number of neutrons per unit volume which have a first collision at the point r is given by

$$N(r) = \int_0^r S(\rho)\, P_c^+(\rho,r)\, d\rho + \int_r^R S(\rho)\, P_c^-(\rho,r)\, d\rho \qquad (1)$$

where $P_c^+(\rho,r)$ and $P_c^-(\rho,r)$, when multiplied by dr, are the probabilities that a neutron born in a point ρ suffers its first collision in the interval $(r, r+dr)$. Whichever geometry is adopted (slab, cylindrical, or spherical), the functions $P_c^+(\rho,r)$ and $P_c^-(\rho,r)$ have a pole at the point $\rho = r$.

The numerical values of the integrals appearing in Eq. (1), in general, cannot be deduced analytically and must be calculated numerically. This fact, because of the discontinuity, implies the need for very careful and extensive calculations. But, out of necessity, the neutron flux and the importance distribution have been evaluated directly by Eq. (1) (Refs. 1 and 2).

In this paper, an integration method, which also has a general validity, is presented; by the use of this method, numerical calculations can be drastically reduced.

THE INTEGRATION METHOD

In order to evaluate the integral

$$N(r) = \int_a^r S(\rho)\, P_c(\rho,r)\, d\rho \;, \qquad (2)$$

[1]G. BITELLI and M. SALVATORES, *Nucl. Sci. Eng.*, **36**, 309 (1969).

[2]G. BITELLI and M. SALVATORES, "Utilization of Generalized Perturbation Techniques and of a Collision Method for Localized Experiments in Fast Facilities," *Proc. Intern. Conf. Institution of Civil Engineers*, London (June 1969).

we consider the function

$$J(r) = \int_a^r S(\rho)\, P_f(\rho,r)\, d\rho \quad , \tag{3}$$

where

$$\frac{\partial P_f(\rho,r)}{\partial r} = - P_c(\rho,r) \quad . \tag{4}$$

We differentiate $J(r)$ with respect to r, obtaining

$$\frac{dJ}{dr} = S(r)\, P_f(r,r) + \int_a^r S(\rho)\, \frac{\partial P_f(\rho,r)}{\partial r}\, d\rho \quad . \tag{5}$$

The integral in Eq. (5) is, by Eqs. (4) and (2), equal to $-N(r)$, so we get the result

$$N(r) = S(r)\, P_f(r,r) - \frac{dJ}{dr} \quad . \tag{6}$$

Formula (5) may be used instead of Eq. (2), provided dJ/dr is easily expressed. Obviously the method presented is convenient if the function $P_c(\rho,r)$ has a pole in the point $\rho = r$ and if we obtain the function $P_f(\rho,r)$. If, otherwise, the point r lies within the interval of integration, $N(r)$ can be expressed as follows

$$N(r) = N^+(r) + N^-(r) \quad ,$$

where

$$N^+(r) = \int_a^r S(\rho)\, P_c^+(\rho,r)\, d\rho, \quad N^-(r) = \int_r^b S(\rho)\, P_c^-(\rho,r)\, d\rho \quad .$$

We consider the functions

$$J^+(r) = \int_a^r S(\rho)\, P_f^+(\rho,r)\, d\rho$$

and

$$J^-(r) = \int_r^b S(\rho)\, P_f^-(\rho,r)\, d\rho \quad ,$$

where

$$\frac{\partial P_f^+(\rho,r)}{\partial r} = - P_c^+(\rho,r)$$

and

$$\frac{\partial P_f^-(\rho,r)}{\partial r} = + P_c^-(\rho,r) \quad . \tag{7}$$

Using the procedure presented above, we obtain for the function $N(r)$ the expression

$$N(r) = S(r)\, [P_f^+(r,r) + P_f^-(r,r)] - \frac{dJ}{dr} \quad ,$$

where $J(r)$ is given by

$$J(r) = J^+(r) - J^-(r) \quad .$$

APPLICATION TO THE COLLISION METHOD

Let us consider, for the sake of simplicity, a homogeneous slab system extending in the interval $(0,R)$. In this system the number of neutrons per unit volume which have a first collision at the point r and coming from the total neutron source is given by

$$N(r) = \int_0^r S(\rho)\, P_c^+(\rho,r)\, d\rho + \int_r^R S(\rho)\, P_c^-(\rho,r)\, d\rho \quad .$$

For an isotropic source $P_c^+(\rho,r)$ and $P_c^-(\rho,r)$ are given by

$$P_c^+(\rho,r) = +\int_0^{\pi/2} \Sigma_t\, \frac{\exp\left(-\Sigma_t \frac{r-\rho}{\cos\theta}\right)}{\cos\theta}\, \frac{\sin\theta\, d\theta}{2}$$

$$P_c^-(\rho,r) = +\int_0^{\pi/2} \Sigma_t\, \frac{\exp\left(-\Sigma_t \frac{\rho-r}{\cos\theta}\right)}{\cos\theta}\, \frac{\sin\theta\, d\theta}{2}$$

where Σ_t is the total macroscopic cross section. These functions have a pole at the point $\rho = r$.

We now consider the functions $J^+(r)$ and $J^-(r)$ defined as follows

$$J^+(r) = \int_0^r S(\rho)\, P_f^+(\rho,r)\, d\rho$$

$$J^-(r) = \int_r^R S(\rho)\, P_f^-(\rho,r)\, d\rho \quad ,$$

where $P_f(\rho,r)$ is the probability that a neutron born at ρ passes through the interval (ρ,r) without suffering collisions. The function $J^+[J^-]$ represents the number of neutrons coming from the interval $(0,r)\,[(r,R)]$ and passing the plane at $\rho = r$. The functions $P_f^+(\rho,r)$ and $P_f^-(\rho,r)$ in slab geometry are given by

$$P_f^+(\rho,r) = \int_0^{\pi/2} \frac{\exp\left(-\Sigma_t \frac{r-\rho}{\cos\theta}\right)}{2}\, \sin\theta\, d\theta$$

$$P_f^-(\rho,r) = \int_0^{\pi/2} \frac{\exp\left(-\Sigma_t \frac{\rho-r}{\cos\theta}\right)}{2}\, \sin\theta\, d\theta \quad . \tag{8}$$

Note that the functions $P_f^+(\rho,r)$ and $P_f^-(\rho,r)$ satisfy relationships (7) and, therefore, we can apply the integration method explained above.

Putting $b^+ = \Sigma_t(r - \rho)$, $b^- = \Sigma_t(\rho - r)$, and $t = \cos\theta$, Eqs. (8) become

$$P_f^+(\rho,r) = \frac{1}{2}\int_0^1 \exp\left(-\frac{b^+}{t}\right) dt = \frac{1}{2}\, E_2(b^+)$$

$$P_f^-(\rho,r) = \frac{1}{2}\int_0^1 \exp\left(-\frac{b^-}{t}\right) dt = \frac{1}{2}\, E_2(b^-) \quad .$$

The functions $E_2(b^+)$ and $E_2(b^-)$ can be easily calculated by methods described in Ref. 3. Since these functions are continuous, we can calculate without difficulties the functions $J(r) = J^+(r) - J^-(r)$ and its derivative dJ/dr.

At last we obtain the function

$$N(r) = -\frac{dJ}{dr} + S(r)\left[\frac{E_2(0)}{2} + \frac{E_2(0)}{2}\right] \quad .$$

The procedure presented here in the multigroup formulation has been applied to obtain the effective neutron multiplication factor, the neutron fluxes, and the importance distributions for nuclear reactors.[4] The results obtained are in very good agreement with other methods of transport theory,[5] and the times involved in the calculations are of the same order of magnitude. Moreover, as expected, the calculation times using the escape probabilities, as presented here, instead of the collision probabilities are of several orders of magnitude smaller and give the same precision.

[3] *Handbook of Mathematical Functions*, M. ABRAMOWITZ and I. A. STEGUN, Eds., p. 227, Dover Publications, Inc., New York (1964).

[4] G. BITELLI and L. TORELLI, "Fluxes and Effective Neutron Multiplication Factor by Escape Probabilities," to be published as a Comitato Nazionale per l'Energia Nucleare report.

[5] K. D. LATHROP, "DTF-IV—A Fortran-IV Program for Solving the Multigroup Transport Equation with Anisotropic Scattering," LA-3373, Los Alamos Scientific Laboratory (1965).