COMITATO NAZIONALE PER L'ENERGIA NUCLEARE
Laboratori Nazionali di Frascati

LNF-74/63(L)
Novembre 1974

(Lezioni)

L. A. Kondratyuk: COLLISION THEORY FOR NUCLEAR INTERACTIONS AT INTERMEDIATE AND HIGH ENERGY.

(Lezioni tenute nel Novembre 1974)

Servizio Documentazione
dei Laboratori Nazionali di Frascati del CNEN
Casella Postale 70 - Frascati (Roma)

Laboratori Nazionali di Frascati del CNEN
Servizio Documentazione

LNF-74/63(L)
Novembre 1974

L. A. Kondratyuk[(x)]: COLLISION THEORY FOR NUCLEAR INTERACTIONS AT INTERMEDIATE AND HIGH ENERGY.

1. - INTRODUCTION. -

Every year a number of papers devoted to the nuclear interactions at intermediate and high energy increases very much. In these lectures we do not want to give a review of all the schemes and models which were developed up to now. Our main aim is to collect here the most important results, to give their derivations (may be not finest but rather transparent), and to answer on the question: "What do we really must know to calculate the cross-section of some nuclear reaction?"

We shall deal with particles of a short wave length $kR \gg 1$ (where k is the momentum per particle in the lab. system, R is the radius of nucleus, $\hbar = c = 1$) and discuss two types of reactions:

i) interactions of elementary particles with nuclei

$$a_i + A_i \rightarrow a_f + A_f \tag{A}$$

where a_i, a_f may be electrons, protons, pions, kaons, hyperons, etc.; A_i and A_f are nuclear systems;

ii) interactions of heavy ions

$$A_i + B_i \rightarrow A_f + B_f \tag{B}$$

(without nucleon transfer from A to B).

General theoretical foundations of the collision theory may be found in the famous book of M. Goldberger and K. Watson[(1)]. Here main attention will be given to the application of the collision theory to the description of reactions (A) and (B). When $kR \gg 1$ then usually for calculation of the cross-sections of these reactions it is necessary to know only the elementary amplitudes (the a-N or N-N scattering amplitudes) and wave functions of the initial and final nuclei. The main topics which we shall discuss are: derivation of the Glauber approximation (GA) for the scattering amplitude of reaction (A) and its connection with the fixed scatterer approximation (FSA) and the optical model (OM); the validity of the FSA and the GA; the GA for collision of heavy ions (reaction (B)); the comparison of the predictions of the GA with experimental data; the inelastic screening for hadron scattering on nuclei at high energy; peculiar polarization phenomena for knock-out reactions

---

(x) - From Institute for Theoretical and Experimental Physics, Moscow (USSR).

(p, 2p) on nuclei; the interference of the electromagnetic and strong interaction; new theorem for final state interaction in deuteron break up and reaction $\pi^- p \to p\pi^-$ for forward protons. A part of the material is based on the lectures given by the author at the 1972 Summer School on Theoretical Nuclear Physics in Moscow Ingeneering Physical Institute(2) (Sects. 2 and 6).

2. - MANY-CHANNEL POTENTIAL MODEL, WATSON THEORY AND GLAUBER APPROXIMATION. -

2. 1. - Many-channel potential model. -

The scattering of the point particle a on the compound system A may be described by the Hamiltonian (see, for example, ref. (1)).

$$H(\vec{R}, \vec{r}_1, .., \vec{r}_A) = k_a(\vec{R}) + h_A(\vec{r}_1, .., \vec{r}_A) + \sum_i V_i(\vec{R} - \vec{r}_i) , \tag{1}$$

where $\vec{R}$ is the coordinate of the a particle, $\vec{r}_i$ are the coordinates of the nucleons in the A nucleus, $v_i(\vec{R} - \vec{r}_i)$ is the potential of the aN_i interaction, $k_a(\vec{R})$ is the operator of the kinetic energy of the a particle, h_A is the target Hamiltonian:

$$\left[h_A(\vec{r}_1, .., \vec{r}_A) - \varepsilon_\nu\right] \varphi_\nu(\vec{r}_1, .., \vec{r}_A) = 0 , \tag{2}$$

where φ_ν and ε_ν are the eigen function and energy of the ν level (the energy of the ground state ε_o is equal to 0).

Expanding the wave function of the a+A system in the series in φ_ν

$$\psi(\vec{R}, \vec{r}_1, .., \vec{r}_A) = \sum_\nu \psi_\nu(\vec{R}) \varphi_\nu(\vec{r}_1, .., \vec{r}_A) \tag{3}$$

we get the set of coupled equations for the wave functions $\psi_\nu(\vec{R})$ (see, for example, ref. (3))

$$\left[k_a(\vec{R}) - E + \varepsilon_\nu\right] \psi_\nu(\vec{R}) = - \sum_\mu v_{\mu\nu}(\vec{R}) \psi_\mu(\vec{R}) \tag{4}$$

where

$$v_{\mu\nu}(\vec{R}) = \langle \mu \Big| \sum_i V_i(\vec{R} - \vec{r}_i) \Big| \nu \rangle = \int \varphi^*_\mu(\vec{r}_1, .., \vec{r}_A) \sum_i V_i(\vec{R} - \vec{r}_i) \varphi_\nu(\vec{r}_1, .., \vec{r}_A) \left\{d^3 r\right\}_A \tag{5}$$

$$\left\{d^3 r\right\}_A = d^3\vec{r}_1, .., d^3\vec{r}_A \; \delta\left(\frac{r_1, ..., r_A}{A}\right). \tag{5b}$$

The outgoing wave function $\psi^{(+)}_\nu(\vec{R})$ which describes the a system with the ν excited state of the target is the solution of the set of coupled equations

$$\psi^{(+)}_\nu(R) = e^{i\vec{k}_\nu \vec{R}} + \int G^{(+)}_{k\nu}(\vec{R} - \vec{R}')\left(\sum_\mu v_{\mu\nu}(\vec{R}') \psi^{(+)}_\mu(\vec{R}')\right) d^3\vec{R}' \tag{6}$$

where the Green function $G^{(+)}_{k\nu}(\vec{R})$ may be represented in the form

$$G^{(+)}_{k\nu}(\vec{R}) = \left[K_a(\vec{R}) + E_k - \varepsilon_\nu\right] \frac{1}{(2\pi)^3} \int d^3\vec{k}' \frac{e^{i\vec{k}'\vec{R}}}{(E_k - \varepsilon_\nu)^2 - E^2_{k'} + i\delta} \tag{7}$$

(where $E_k = \sqrt{\vec{k}^2 + m^2_a}$, $k_a(\vec{R}) e^{i\vec{k}\vec{R}} = E_k e^{i\vec{k}\vec{R}}$) or

$$G^{(+)}_k(\vec{R}) = - \frac{E^{(\nu)}_k}{2\pi} \frac{e^{ik_\nu R}}{R} \tag{8}$$

where $E_k^{(\nu)} = E_k - \varepsilon_\nu$, $k_\nu = \sqrt{(E_k^{(\nu)})^2 - m_a^2}$.

We can solve system (6) by the iteration procedure. After this the amplitude of the reaction $a + A_i \rightarrow a + A_f$ can be written in the form of the finite series

$$F_{if}(E_k, \vec{\Delta}) = -\frac{E_k^{(\nu)}}{2\pi} \sum_{n=1}^{\infty} \int d^3\vec{k}_1, \ldots, d^3\vec{k}_n \, e^{-i\vec{k}_f \vec{R}_n} v_{f\nu}(\vec{R}_n) G_{k\nu}^{(+)}(\vec{R}_n - \vec{R}_{n-1}) \cdot$$
$$\cdot \, v_{\nu\mu}(\vec{R}_{n-1}) \ldots v_{\mu\sigma}(\vec{R}_2) G_{k\sigma}^{(+)}(\vec{R}_2 - \vec{R}_1) v_{\sigma i}(\vec{R}_1) e^{i\vec{k}_i \vec{R}_1}, \qquad (10)$$

where the sum is performed over all the repeating indexes μ, ν, σ; $\vec{k}_i$ and $\vec{k}_f$ are the momenta of the incident and scattered particle; $\vec{\Delta} = \vec{k}_i - \vec{k}_f$ is the momentum transfer.

Formula (10) gives the general solution for the amplitude of the reaction

$$a + A_i \rightarrow a + A_f \quad . \qquad (11)$$

But it is rather difficult to use this general solution for applications because exp. (10) is dependent not only on the wave functions of initial and final nuclear states, but also on the wave functions of the all excited states.

It is much more convenient for applications to use approximated expressions. The most con venient are: the fixed scattered FSA (it may be also called as adiabatic approach or closure approximation for intermediate states), the Glauber approach and the optical model. The main aim of the all these approach is to evaluate the amplitude of reaction (11) without any information about intermediate states.

2. 2. - Fixed scattered approximation (FSA), Glauber approach, and optical model, DWBA. -

The FSA can be used when the incident particle is fast and its velocity v_o is much higher than the mean velocity of nucleons in nucleus $v_o \gg \langle v_N \rangle$ [4]. In the framework of the FSA we neglect the excitation energy ε_ν in exp. (7) for the Green function as compared with the energy difference $E_k - E_{k'}$. We shall see latter why these two inequalities $v_o \gg \langle v_N \rangle$ and $\varepsilon_\nu \ll |E_k - E_{k'}|$ are equivalent. If the value of ε_ν in exp. (7) can be neglected, we need not information about the wave functions $\varphi_\nu(\vec{r}_1, \ldots, \vec{r}_A)$ for calculation $F_{if}(E_k, \vec{\Delta})$. Indeed, in this case the Green function (7)-(8) is not dependent on the ν index and we can perform the summation over all the intermediate states in exp. (11) with the help of the completeness condition for the wave functions

$$\sum_\nu \varphi_\nu(\vec{r}_1, \ldots, \vec{r}_A) \, \varphi_\nu^*(\vec{r}'_1, \ldots, \vec{r}'_A) = \delta(\vec{r}_1 - \vec{r}'_1) \ldots \delta(\vec{r}_A - \vec{r}'_A) \, . \qquad (12)$$

After this summation the scattering amplitude (10) may be written in the form

$$F_{if}(E_k, \vec{\Delta}) = -\frac{E_k}{2\pi} \sum_{n=1}^{\infty} \int d^3\vec{R}_1 \ldots d^3\vec{R}_n \, e^{-i\vec{k}_f \vec{R}_n} \, \phi_f^*(\vec{r}_1, \ldots, \vec{r}_A) v(\vec{R}_n, \vec{r}_1, \ldots, \vec{r}_A) \cdot$$
$$\cdot \, G_k^{(+)}(\vec{R}_n - \vec{R}_{n-1}) v(\vec{R}_{n-1}, \vec{r}_1, \ldots, \vec{r}_A) \ldots v(\vec{R}_2, \vec{r}_1, \ldots, \vec{r}_A) G_k^{(+)}(\vec{R}_2 - \vec{R}_1) \cdot \qquad (13)$$
$$\cdot \, v(\vec{R}_1, \vec{r}_1, \ldots, \vec{r}_A) e^{i\vec{k}_i \vec{R}_1} \, \phi_i(\vec{r}_1, \ldots, \vec{r}_A) \left\{ d^3 r \right\}_A$$

where $v(\vec{R}, \vec{r}_1, \ldots, \vec{r}_A) = \sum_i v_i(\vec{R} - \vec{r}_i)$; the δ-function takes into account a center-off-mass motion and the wave functions ϕ_i and ϕ_f depend only on the relative coordinates of the nucleons.

In the FSA we consider nucleons in a nucleus to be "frozen" during the passage of the fast particle thorugh a nucleus.

Further transition from the FSA to the Glauber approximation[5] can be performed by using of the eikonal approximation for the Green function

$$G_k^{eik}(\vec{R}_2-\vec{R}_1) = -\frac{i}{v_k} e^{ik(z_2-z_1)}\,\theta(z_2-z_1)\,\delta(\vec{\rho}_2-\vec{\rho}_1) \tag{14}$$

where $\vec{\rho}_i$ are the transverse components of the vectors $\vec{R}_i$ ($\vec{\rho}_i$ are in the plane normal to the momentum $\vec{k}$); z_i are the longitudinal components of the vectors $\vec{R}_i$.

After substitution of exp. (14) for the Green function into exp. (13) we find

$$F_{if}(E_k, \vec{\Delta}) = \langle f \Big| \sum_{n=1}^{\infty} F^{(n)}(E_k, \vec{\Delta}) \Big| i \rangle \tag{15a}$$

$$F^{(n)}(E_k, \vec{\Delta}) = -\frac{E_k}{2\pi}\frac{1}{v_k^{n-1}}(-i)^{n-1}\int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}}\, d\vec{z}_1 \ldots d^3 z_n\, v(\vec{\rho}, \vec{z}_n; \vec{r}_1, \ldots, \vec{r}_A)\,\cdot$$

$$\cdot\ \theta(z_n - z_{n-1})\, v(\vec{\rho}, z_{n-1}; \vec{r}_1, \ldots, \vec{r}_A) \ldots v(\vec{\rho}, z_2; \vec{r}_1, \ldots, \vec{r}_A)\,\theta(z_2 - z_1)\ \cdot \tag{15b}$$

$$.\ v(\vec{\rho}, z_1; \vec{r}_1, \ldots, \vec{r}_A) = -\frac{ik}{2\pi}\frac{(-i)^n}{n!\, v_k^n}\int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}}\left[\int_{-\infty}^{\infty} v(\vec{\rho}, z; \vec{r}_1, \ldots, \vec{r}_A)\right]^n$$

where $\langle f | F^{(n)} | i \rangle = \int \{d^3 r\}_A\, \phi_f^*(\vec{r}_1, \ldots, \vec{r}_A)\, F^{(n)}\, \phi_i(\vec{r}_1, \ldots, \vec{r}_A)$

where

$$\{d^3 r\}_A = d^3\vec{r}_1 \ldots d^3\vec{r}_A\, \delta\left(\frac{\vec{r}_1, \ldots, \vec{r}_A}{A}\right). \tag{15c}$$

To transform exp. (15) to a more convenient form let us introduce the phase shifts

$$\delta_i(\vec{\rho} - \vec{s}_i) = -\frac{1}{2v_k}\int_{-\infty}^{\infty} v_i\left(\sqrt{(\vec{\rho} - \vec{s}_i)^2 + z^2}\right) dz\,. \tag{16}$$

We have

$$-\int_{-\infty}^{\infty} dz\, v(\vec{\rho}, z; \vec{r}_1, \ldots, \vec{r}_A) = -\int_{-\infty}^{\infty} dz \sum_i v_i\left(\sqrt{(\vec{\rho} - \vec{s}_i)^2 + (z - \zeta_i)^2}\right) =$$

$$= 2\sum_i \delta_i(\vec{\rho} - \vec{s}_i) = 2\,\delta(\vec{\rho}, \vec{s}_1, \ldots, \vec{s}_A) \tag{17}$$

where $\vec{s}_i$ and $\vec{\zeta}_i$ are respectively the transverse and longitudinal components of the vectors $\vec{r}$.

Performing in (15) the summation over n we get

$$F_{if}(E_k, \vec{\Delta}) = \frac{ik}{2\pi}\langle f \Big| \int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}}\left[1 - e^{2i\delta(\vec{\rho}, \vec{s}_1, \ldots, \vec{s}_A)}\right] \Big| i \rangle\,. \tag{18}$$

Exp. (18) is the Glauber approximation for the scattering amplitude[5].

As the diagonal matrix elements $v_{\mu\mu}$ are usually much bigger then the nondiagonal ones $v_{\mu\nu} << v_{\mu\mu}$ it is also useful the perturbative solution of system (4). For the amplitude of elastic scattering the first approximation is equivalent to the neglection in (10) by all the excited states

$$F_{ii}^{pot}(E_k, \vec{\Delta}) = -\frac{1}{4\pi} \sum_{n=1}^{\infty} \int d^3\vec{R}_1 \ldots d^3\vec{R}_n \, e^{-i\vec{k}'\vec{R}_n} v_{ii}(\vec{R}_n) \, G_k^{(+)}(\vec{R}_n - \vec{R}_{n-1}) \cdot$$
$$\cdot \, v_{ii}(\vec{R}_{n-1}) \ldots v_{ii}(\vec{R}_2) \, G_n^{(+)}(\vec{R}_2 - \vec{R}_1) \, v_{ii}(\vec{R}_1) \, e^{i\vec{k}'\vec{R}_1} \tag{19}$$

or in the symbolic form

$$F_{ii}^{pot} \sim v_{ii} + v_{ii} G v_{ii} + v_{ii} G v_{ii} G v_{ii} + \ldots \tag{20}$$

The amplitude of inelastic scattering has the following form in the first approximation

$$F_{fi}^{pot} \sim v_{fi} + v_{fi} G_i v_{ii} + v_{ff} G_f v_{fi} + v_{fi} G_i v_{ii} + v_{ff} G_f v_{fi} G_i v_{ii} + v_{ff} G_f v_{ff} G_f v_{fi} + \ldots \tag{21}$$

In the eikonal approximation for the Green function exps.(19-21) can be transformed to the following very known forms

$$F_{ii}^{OM}(E_k, \vec{\Delta}) = \frac{ik}{2\pi} \int e^{i\vec{\Delta}\vec{\rho}} \, d^2\vec{\rho} \, (1 - e^{2i\delta(\vec{\rho})}) , \tag{22}$$

where $\delta(\vec{\rho}) = -\frac{1}{2v_k} \int_{-\infty}^{\infty} v_{ii}(\vec{\rho}, z) \, dz$.

$$F_{fi}^{DWBA}(E_k, \vec{\Delta}) = -\frac{1}{4\pi} \int e^{i\vec{q}\vec{\rho}} \, d^2\vec{\rho} \, e^{-i \int_z^{\infty} v_{ff}(\vec{\rho}, z') dz'} \cdot$$
$$\cdot \, v_{fi}(\vec{\rho}, z) \, e^{-i \int_{-\infty}^{z} v_{ii}(\vec{b}, z') dz'} \, dz . \tag{23}$$

The former expression corresponds to the optical model, the latter to the result of DWBA, in which it was neglected by a small contribution coming from non zero difference $\varepsilon_f - \varepsilon_i$ and non zero value of the longitudinal component Δ_z of momentum transfer.

It is important to note that exps. (18) and (22) contain the phase shifts but not the potential which was introduced only on intermediate steps. The phases $\delta_i(\vec{\rho} - \vec{s}_i)$ and $\delta(\vec{\rho})$ may be expressed through the a-N_i scattering amplitude $f_{aN_i}(\vec{q})$. The relation between the a-N_i scattering amplitude and the phase shift $\delta_i(\vec{\rho} - \vec{s}_i)$ is given by

$$f_{aN_i}(E_k, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\rho \, e^{i\vec{\Delta}\vec{\rho}} \, (1 - e^{2i\delta_i(\vec{\rho})}) . \tag{24}$$

It is useful to introduce the nucleon profile function

$$\Gamma_i(\vec{\rho}) = 1 - e^{2i\delta_i(\vec{\rho})} \tag{25}$$

which is related with the scattering amplitude f_{aN_i} by the expression

$$\Gamma_i(\vec{\rho}) = \frac{1}{2\pi i k} \int e^{-i\vec{\Delta}\vec{\rho}} d^2\vec{\Delta} \, f_{aN_i}(E_k, \vec{\Delta}) . \tag{26}$$

Then the scattering amplitude F_{fi} in the GA may be expressed through the nucleon profile functions

$$F_{fi}(E_k, \vec{\Delta}) = \frac{ik}{2\pi} \langle f \Big| \int e^{i\vec{\Delta}\vec{\rho}} d^2\vec{\rho} \left[1 - \prod_{i=1}^{A} (1 - \Gamma_i(\vec{\rho} - \vec{s}_i)) \right] \Big| i \rangle . \tag{27}$$

The phase $\delta(\vec{\rho})$ in the exp. (22) and the potentials v_{ii}, v_{fi}, and v_{ff} may also be expressed through the amplitude $f_{aN_i}(\vec{q})$. Usually we may neglect by the radius of the aN_i-interaction as compared with the radius of the nucleus. Then the optical potential may be written in the form

$$v_{fi}(\vec{r}) = - \frac{2\pi}{E_k} f_{aN_i}(0) \, A \, S_{fi}(\vec{r}) , \tag{28}$$

where $S_{fi}(\vec{r})$ is the single particle density matrix, which is connected with the form factor $S_{fi}(\vec{q})$:

$$S_{fi}(\vec{r}) = \int e^{-i\vec{q}\vec{r}} S_{fi}(\vec{q}) \frac{d^3\vec{q}}{(2\pi)^3} \tag{29a}$$

$$S_{fi}(\vec{q}) = \frac{1}{A} \int \left(\sum_{j=1}^{A} e^{i\vec{q}\vec{r}_j} \right) \psi_f^*(\vec{r}_1, \ldots, \vec{r}_A) \, \psi_i(\vec{r}_1, \ldots, \vec{r}_A) \left\{ d^3 r \right\}_A = \frac{1}{A} \langle f \Big| \sum_{j=1}^{A} e^{i\vec{q}\vec{r}_j} \Big| i \rangle . \tag{29b}$$

If we assume that the densities of initial and final nuclei are not very different $S_{ii}(\vec{r}) \approx S_{ff}(\vec{r})$ then exp. (27) can be written in the form[6]

$$F_{if}^{DWBA}(\vec{q}) = A f_{aN_i}(0) \left[S_{if}(\vec{q}) - \frac{1}{2\pi k i} \int d^2\vec{q}' \, S_{if}(\vec{q}') \, F_{ii}(\vec{q} - \vec{q}') \right] . \tag{30}$$

Exp. (30) determines the amplitude of the transition $i \to f$ through the transition form factor $S_{if}(\vec{q})$ and the amplitude of elastic scattering $F_{ii}(\vec{q})$.

2.3. - Watson theory. -

It is also possible from the beginning in exps. (6) and (10) to introduce two-body scattering amplitudes f_{aN_i} instead of two-body potentials V_i. In operator form exp. (6) may be written as

$$\psi^{(+)} = \chi + G U \psi^{(+)} \tag{31}$$

where $U = \sum_i V_i$, $\chi = e^{i\vec{k}_i\vec{R}}$. Two-body scattering matrix is related to two-body potential by the integral equation (the Lippman-Schwinger equation):

$$T_i = v_i + v_i G T_i . \tag{32}$$

Here it is also some problem with the off-shell effects, because interaction of an incident particle with free and bound nucleon may be different. We neglect for a moment this problem assuming that these interactions are equal (this corresponds to the impulse approximation of ref. (1)). The validity of this approximation we shall discuss in Sect. 2.4.

The solution of eq. (31) may be written in the form (see refs. (7, 1))

$$\psi^{+} = \chi + \sum_{i=1}^{A} GT_i\chi + \sum_{ij(i\neq j)} GT_iGT_j\chi + \sum_{ij\ell(i\neq j\neq \ell)} GT_iGT_jGT\chi + \dots \tag{33}$$

and the scattering amplitude is given by the series with contains A terms

$$F_{fi}(E_k,\vec{\Delta}) = \langle f \Big| \sum_j T_j + \sum_{j\ell(j\neq\ell)} T_jGT_\ell + \sum_{j\ell m(j\neq\ell\neq m)} T_jGT_\ell GT_m + \dots \Big| i\rangle \tag{34}$$

or in a more detail form

$$\begin{aligned}F_{fi}(E_k,\vec{\Delta}) = &\int d^3\vec{R}\, e^{-i\vec{k}_f\vec{R}} \sum_j T_j^{fi}(\vec{R})\, e^{i\vec{k}_i\vec{R}} + \int d^3\vec{R}_1 d^3\vec{R}_2 \sum_{j\ell(j\neq\ell)} e^{-i\vec{k}_f\vec{R}_2}\, T_j^{fv}(\vec{R}_2)\,\cdot\\ &\cdot \frac{2\pi}{E_{k_v}} G^{(+)}_{k_v}(\vec{R}_2-\vec{R}_1)\, T_\ell^{vi}(\vec{R}_1)\, e^{i\vec{k}_i\vec{R}_1} + \int d^3\vec{R}_1 d^3\vec{R}_2 d^3\vec{R}_3\, e^{-i\vec{k}_f\vec{R}_3}\,\cdot\\ &\cdot \sum_{j\ell m(j\neq\ell\neq m)} T_j^{fv}(\vec{R}_3)\frac{2\pi}{E_{k_v}} G^{(+)}_{k_v}(\vec{R}_3-\vec{R}_2)\, T_\ell^{v\sigma}(\vec{R}_2)\frac{2\pi}{E_{k\sigma}} G^{(+)}_{k_\sigma}(\vec{R}_2-\vec{R}_1)\, T_m^{\sigma i}(\vec{R}_1)\, e^{i\vec{k}_i\vec{R}_1} + \dots\end{aligned} \tag{35}$$

where

$$T_i^{\mu\nu}(\vec{R}) = \int \varphi^{*}_{\mu}(\vec{r}_1,\dots,\vec{r}_A)\, T_i(\vec{R}-\vec{r}_i)\, \varphi_\nu(\vec{r}_1,\dots,\vec{r}_A)\, \{d^3r\}_A \tag{36}$$

$$T_i(\vec{R}) = \int e^{i\vec{q}\vec{R}} f_{aN_i}(\vec{q}) \frac{d^3\vec{q}}{(2\pi)^3} \tag{37}$$

Exps. (33) and (35) are the Watson multiple scattering series. In the GA (see exps. (26), (27)) it was neglected by the longitudinal components of the momentum transfer Δ_z. In exp. (37) this dependence on q_z is taken into account but is necessary to be careful with inclusion of this dependence, because it is not Lorenz invariant and exp. (37) may be written only in definite system. We suggest that this expression is good in the center of mass system. The profile function $\Gamma_i(\vec{\rho})$ is related to the two--body scattering matrix by the expression

$$\Gamma_i(\vec{\rho}) = \frac{2\pi}{ik} \int T_i\left(\sqrt{\vec{\rho}^{\,2}+z^2}\right) dz\,. \tag{38}$$

In the FSA the Watson multiple scattering series (35) may be represented in the form

$$\begin{aligned}F_{fi}^{FSA}(E_k,\vec{\Delta}) = &\int d^3R_1 \{d^3r\}_A\, \phi_f^{*}(\vec{r}_1,\dots,\vec{r}_A) \Big\{ \int d^3\vec{R}\, e^{-i\vec{k}_f\vec{R}} \sum_j T_j(\vec{R}-\vec{r}_j)\, e^{i\vec{k}_i\vec{R}} +\\ &+ \int d^3\vec{R}_1 d^3\vec{R}_2 \sum_{j\ell(j\neq\ell)} e^{-i\vec{k}_f\vec{R}_2}\, T_j(\vec{R}_2-\vec{r}_j)\frac{2\pi}{E_k} G_k^{(+)}(\vec{R}_2-\vec{R}_1)\, T_\ell(\vec{R}_1-\vec{r}_\ell)\, e^{i\vec{k}_i\vec{R}_1} +\\ &+ \int d^3\vec{R}_1 d^3\vec{R}_2 d^3\vec{R}_3 \sum_{j\ell m(j\neq\ell\neq m)} e^{-i\vec{k}_f\vec{R}_3}\, T_j(\vec{R}_3-\vec{r}_j)\frac{2\pi}{E_k} G_k^{(+)}(\vec{R}_3-\vec{R}_2)\,\cdot\end{aligned} \tag{39}$$

$$\cdot \; T_{\ell}(R_2 - r_{\ell}) \frac{2\pi}{E_k} G_k^{(+)}(\vec{R}_2 - \vec{R}_1) T_m(\vec{R}_1 - \vec{r}_m) e^{i\vec{k}_i\vec{R}_1} + \ldots \Big\} \phi_i(\vec{r}_1, \ldots, \vec{r}_A) .$$

This formula may be regarded as the most suitable for the description of the interaction of elementary particles with nuclei at intermediate energies. But it is a more complicated than the Glauber formula (27). The transition from exp. (39) to exp. (27) can be done by using the eikonal approximation for the Green function $G_k^{(+)}$ (see exp. (14)). This approximation can be used for scattering on small angles for $k|\vec{R}_i - \vec{R}_j| \gg 1$, where $|\vec{R}_i - \vec{R}_j|$ is the effective distance between two nucleons in nucleus. If the correlations between the nucleons in nucleus are not important (this is usually realized), then $|\vec{R}_i - \vec{R}_j| \sim R$ and the eikonal approximation for the Green function $G_k^{(+)}$ is appliable for

$$kR \gg 1 . \tag{40}$$

It is very important to note that we can derive the Glauber expression (27) from (19) using only the eikonal approximation for the Green function (which is appliable for $kR \gg 1$) without the assumption on the validity of this approximation for two-body amplitude $T_i(\vec{R})$ (which is appliable for $ka \gg 1$, where a is the radius of the a-N interaction).

2. 4. - Applicability of the fixed scatterer approximation and the Glauber approximation. -

Let us estimate the order of magnitude of the corrections to the FSA. These corrections are connected with the term ε_ν in exp. (7) for the Green function and also with the off-shell effects. We shall start from the estimation of the former correction. For small momentum transfer $|\vec{k}' - \vec{k}''|^2 \ll k^2$ the Green function may be represented in the form[8]:

$$G_{k_\nu}^{(+)}(\vec{r}) = -\frac{E_k}{2\pi} \frac{\exp\left[i(k - \varepsilon_\nu/v_k) r\right]}{r} \approx -\frac{E_k}{2\pi} \frac{e^{ikr}}{r} (1 - i \frac{\varepsilon_\nu}{v_k} r) \tag{41}$$

where $v_k = k/E_k$ (we use the system of units $\hbar = c = 1$).

If we put $r \approx R$ then the correction in the Green function will be of the order of

$$|\eta| \sim \bar{\varepsilon}_\nu R/v_k \tag{42}$$

where $\bar{\varepsilon}_\nu$ is the average value of the excitation energy. For estimation we may put $\bar{\varepsilon}_\nu \sim \langle p \rangle^2/2m$, where $\langle p \rangle \sim R^{-1}$ is the mean momentum in the framework of the independent particle model. Then we have

$$|\eta| \sim \frac{1}{mRv_k} \sim \frac{\langle v_N \rangle}{v_k} \tag{43}$$

where $\langle v_N \rangle = \langle p \rangle/m$. Therefore the parameter $|\eta|$ is of the order of the ratio $\langle v_N \rangle / v_k$ which was stated above.

The value of η may increase with the transfer momentum Δ. For example, in the double scattering term every nucleon receives the momentum $\vec{\Delta}/2$ and the kinetic energy $\vec{\Delta}^2/8m$. This additional energy leads to the correction

$$|\eta_2(\vec{\Delta})| \sim \vec{\Delta}^2 R/8m\, v_k . \tag{44}$$

It is also important to note that the phase of η is different on $\pi/2$ from the phase of the main term. So this correction is real if the main term is imaginary. This means that the correction in the differential and total cross-sections (when spin effects are not detected) will be of the order of η^2.

In the case of hadron scattering the estimations (42) and (44) are rather upper values for η, because it is more reasonable to put $r \sim \ell$, where ℓ is the mean free part of hadron in nucleus. Then only for electron we shall have the estimations (42) and (44). For nuclei with $R \gg \ell$ the corresponding correcti ons will be much smaller.

To demonstrate why we can put the value $\langle p \rangle^2/2m$ or $(\vec{\Delta}/2)^2/2m$ for ε_ν let us consider the double scattering term for the amplitude (35). In the momentum representation we have

$$F_{fi}^{(2)}(E_k, \vec{\Delta}) \sim \int \frac{d^3\vec{q}}{(2\pi)^3} \frac{\sum_{j \neq \ell} f_j^{f\nu}(\vec{q})\, f_\ell^{\nu i}(\vec{\Delta}-\vec{q})}{E_k - E_{|\vec{k}-\vec{q}|} - \varepsilon_\nu + i\delta} . \tag{45}$$

The value of ε_ν is equal to the difference between the energies of the excited ν-state and the ground state:

$$\varepsilon_\nu = \langle \nu | v_A + T_A | \nu \rangle - \langle i | v_A + T_A | i \rangle \tag{46}$$

where v_A and T_A are the operators of the potential and kinetic energies of the A system. If the main part of the elementary amplitude $f_i^{f\nu}(\vec{q})$ does not depend on spins (as usually for high energy), this amplitude commutes with v_A and we have

$$\langle f | f_j | \nu \rangle \varepsilon_\nu \langle \nu | f_\ell | i \rangle = f_j^{f\nu} f_\ell^{\nu i} (\langle \nu | v_A + T_A | \nu \rangle - \langle i | v_A + T_A | i \rangle) . \tag{47}$$

After collision some of nucleons in nucleus began to move with different velocities as compared with their movement before collision. But nuclear forces are not dependent very much on the velocity of nucleons. Therefore, we can put

$$\langle f | v_A | f \rangle \approx \langle i | v_A | i \rangle$$

and ε_ν will be equal to the difference between the kinetic energies of the nucleons before and after collision

$$\varepsilon_\nu = \langle \nu | T_A | \nu \rangle - \langle i | T_A | i \rangle . \tag{48}$$

In the double scattering term (45) only one nucleon in the intermediate state receives the recoil. If its momentum befor the collision is $\vec{p}_i$ then after the collision it has the momentum $\vec{p}_i + \vec{\Delta}_i \approx \vec{p}_i + (\vec{\Delta}/2)$, where $\vec{\Delta}$ is the total momentum transfer. So we have

$$\varepsilon_\nu \sim \langle (\vec{p}_i + \frac{\vec{\Delta}}{2})^2 \rangle / 2m - \langle p_i^2 \rangle / 2m \tag{49}$$

which is in agreement with the previous estimations.

What may we say about the off-shell effects, which describe the difference between interactions of an incident particle with free and bound nucleon? If we have in mind that the nucleus consists from nucleons this means that properties of the nucleons inside the nucleus are not very different from properties of free nucleons. Really a nucleon inside nucleus can be deformed or polarized and it will be a superposition of the states $|N\rangle$, $|N\pi\rangle$, $|N2\pi\rangle$ and so on. In the first approximation the excitation spectrum of nucleon in nucleus may be represented by the nucleon resonances N^*. But due to weak boundness of nucleons in nucleus the probability of finding a nucleon resonance in nucleus is rather small (see, for example, ref. (9) and (10)). It is of the order of magnitude $k = B_N/(m^* - m)$, where B_N is the binding energy per nucleon, m^* is the mass of the resonance N^*, m is the nucleon mass. This ratio is of the order of 1%. So we may think that the off-shell effects connected with the difference interactions of free and bound nucleons can not be big.

The transition from the FSA to the Glauber approximation is connected with the validity of the eikonal approximation for the Green function. The corrections arising from the deviation from geometrical optics (from eikonal) are usually called as Fresnel corrections. These corrections were discussed in many papers (see, for example, refs. (11) and (8)).

For forward scattering these corrections are of the order of $1/kR$. If the momentum transfer is not equal to zero thay have an additional contribution proportional to $\Delta^2\ell/8k$ (for deuteron $\Delta^2R/8k$). The Fresnel corrections are real if the main term is imaginary, so for many cases the contribution of these corrections is much smaller than $1/kR$ or $\vec{\Delta}^2\ell/8k$ ($\Delta^2R/8k$ for deuteron) due to small value of the ratio $R_e f_{aN}/I_m f_{aN}$. This will take place, for example, for differential and total cross-sections. But for polarization phenomena we may expect corrections of the order of $1/kR$ or $\vec{\Delta}^2\ell/8k$ ($\Delta^2R/8k$ for deuteron).

For high-energy scattering of hadrons on nuclei there exist corrections to the GA connected with the possibility of the excitation of the projectile hadron. This inelastic screening effect appear for energy[12-16]

$$E \gtrsim m^2 R . \tag{50}$$

We shall discuss these effects in the following sections.

It is also necessary to remember about relativistic corrections to nuclear wave function. These corrections may be important for $\vec{\Delta}^2/8m^2 \gtrsim 1$.

3. - COLLISIONS OF HEAVY IONS. -

The formalism described in the previous sections may be easily generalyzed on the collisions of two nuclei. This generalization have been done in the framework of Glauber theory in papers (17)-(20). The Watson's form of multiple scattering series was investigated in paper (21). The scattering amplitude for the reaction (ii) may be derived from the solution of Schrödinger equation with the hamiltonian

$$H = H_A(\vec{R}_\alpha) + H_B(\vec{r}_j) + \Sigma\, v_{\alpha j}(\vec{R}_\alpha - \vec{r}_j) \tag{51}$$

where H_A and H_B are the free projectile and target hamiltonians, $v_{\alpha j}(\vec{R}_\alpha - \vec{r}_j)$ is the potential of the interaction between the N_α nucleon from projectile and the N_j nucleon from target.

The matrix describing the $N_\alpha - N_j$ scattering is defined by the expression

$$T_{\alpha j} = v_{\alpha j} + v_{\alpha j} G T_{\alpha j} \quad . \tag{52}$$

The final answer for the scattering amplitude is very similar to exp. (34)

$$F_{fi}^{BA}(E_k, \vec{\Delta}) = \langle f | \sum_{\alpha j} T_{\alpha j} + \sum_{\alpha j \neq \beta \ell} T_{\alpha j} G T_{\beta \ell} + \sum_{\alpha j \neq \beta\ell \neq \gamma m} T_{\alpha j} G T_{\beta \ell} G T_{\gamma m} + \ldots | i \rangle \tag{53}$$

In the eikonal approximation for the Green function we have the following representation (of the Glauber type) for the scattering amplitude in the c.m. system

$$F_{fi}^{AB}(\vec{\Delta}) = \frac{ik}{2\pi} \int d^2\vec{B}\, e^{+i\vec{\Delta}\vec{B}} \langle f | \Gamma(\vec{B}) | i \rangle \ , \tag{54}$$

$$\langle f | \Gamma(\vec{B}) | i \rangle = \int d^3\vec{R}_1 .. d^3\vec{R}_A d^3\vec{r}_1 .. d^3\vec{r}_B\, \delta\!\left(\frac{\vec{R}_1 + .. + \vec{R}_A}{A}\right) \delta\!\left(\frac{\vec{r}_1 + .. + \vec{r}_B}{B}\right) \cdot$$

$$\cdot\ \psi_f^{(B)*}(\vec{r}_1, .., \vec{r}_B)\, \psi_f^{(A)*}(\vec{R}_1 .. \vec{R}_A)\, \Gamma(\vec{B})\, \psi_i^{(B)}(\vec{r}_1, .., \vec{r}_B)\, \psi_i^{(A)}(\vec{R}_1 .. \vec{R}_A)$$

where the profile function is given by

$$\Gamma(\vec{B}) = 1 - \prod_{\alpha j}(1-\Gamma_{\alpha j}),$$
$$\Gamma_{\alpha j} = \Gamma_{\alpha j}(\vec{B}-\vec{S}_\alpha+\vec{S}_j) = \frac{1}{2\pi i k}\int e^{-i(\vec{B}-\vec{S}_\alpha+\vec{S}_j)\vec{q}} f_{\alpha j}(\vec{q})\, d^2\vec{q}\ . \tag{55}$$

If the atomic numbers A and B are big then it is convenient to use instead of the generalyzed Glauber model (54) the optical model. In the optical model the amplitude of the reaction $A_i + B_i \rightarrow A_f + B_f$ may be written as

$$F_{fi}(E_k,\vec{\Delta}) = \frac{ik}{2\pi}\int d^2\vec{B}\, e^{+i\vec{\Delta}\vec{B}}\left(1 - e^{2i\delta_{AB}(\vec{B})}\right) \tag{56}$$

where

$$\delta_{AB}(\vec{B}) = \frac{\pi}{k} f_{\alpha j}(0)\, T_{AB}(\vec{B}), \tag{57}$$
$$T_{AB}(\vec{B}) = AB\int d^2\vec{S}\int_{-\infty}^{\infty} dz_A \int_{-\infty}^{\infty} dz_B\ S_A(\vec{B}-\vec{S}, z_A)\, S_B(\vec{S}, z_B),$$

$S_A(\vec{r})$ and $S_B(\vec{r})$ are the single particle density functions (compare (57) with (22) and (27)).

4. - COMPARISON WITH EXPERIMENTAL DATA. -

In Figs. 1-7 we illustrate the agreement of the theoretical predictions based on the GA with experimental data.

In Fig. 1 the theoretical predictions of Glauber and Matthiae[23] on the reaction $p + {}^{12}C \rightarrow p +$ anything at 21.5 GeV/c are compared with the experimental data of Bellettini et al.[22]. The scattering cross-section $d\sigma/d\Omega = \sum_f |F_{fi}(E_k,\vec{\Delta})|^2$ was calculated in the framework of the GA with the factorized nuclear density

$$|\psi(\vec{r}_1,\ldots,\vec{r}_A)|^2 = \prod_{i=1}^{4}\rho_s(\vec{r}_i)\prod_{i=5}^{12}\rho_p(\vec{r}_i), \tag{58}$$

$$\rho_s(\vec{r}) = (1/\pi^{3/2}a_o^3)\exp(-r^2/a_o^2),$$
$$\rho_p(\vec{r}) = (2r^2/3\pi^{3/2}a_o^3)\exp(-r^2/a_o^2),\quad a_o = 1.6\ \text{fm}.$$

The nucleon-nucleon scattering amplitude was parametrized in the form

$$f_{NN}(\vec{q}) = \frac{ik}{4\pi}\,\sigma_{tot}(1-i\alpha)\,e^{-\beta^2 q^2/2}, \tag{59}$$

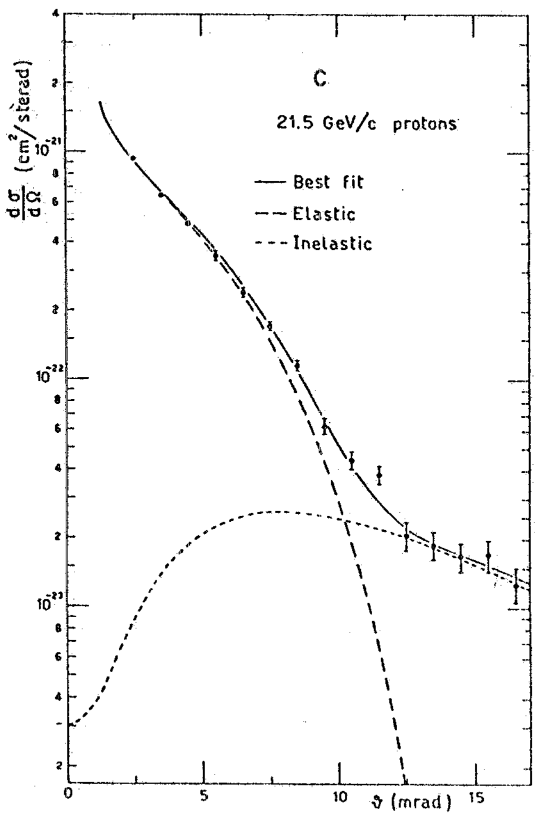


FIG. 1 - The results of Glauber and Matthiae[23] on the scattering cross-section of the reaction $p + {}^{12}C \rightarrow p +$ anything in comparison with experimental data of Bellettini et al.[22] at 21.5 GeV/c.

where σ_{tot} is the total cross-section of the N-N interaction, $\alpha = \mathrm{Re}\, f_{NN}(0)/\mathrm{Im}\, f_{NN}(0)$. The experimental data are fitted very well.

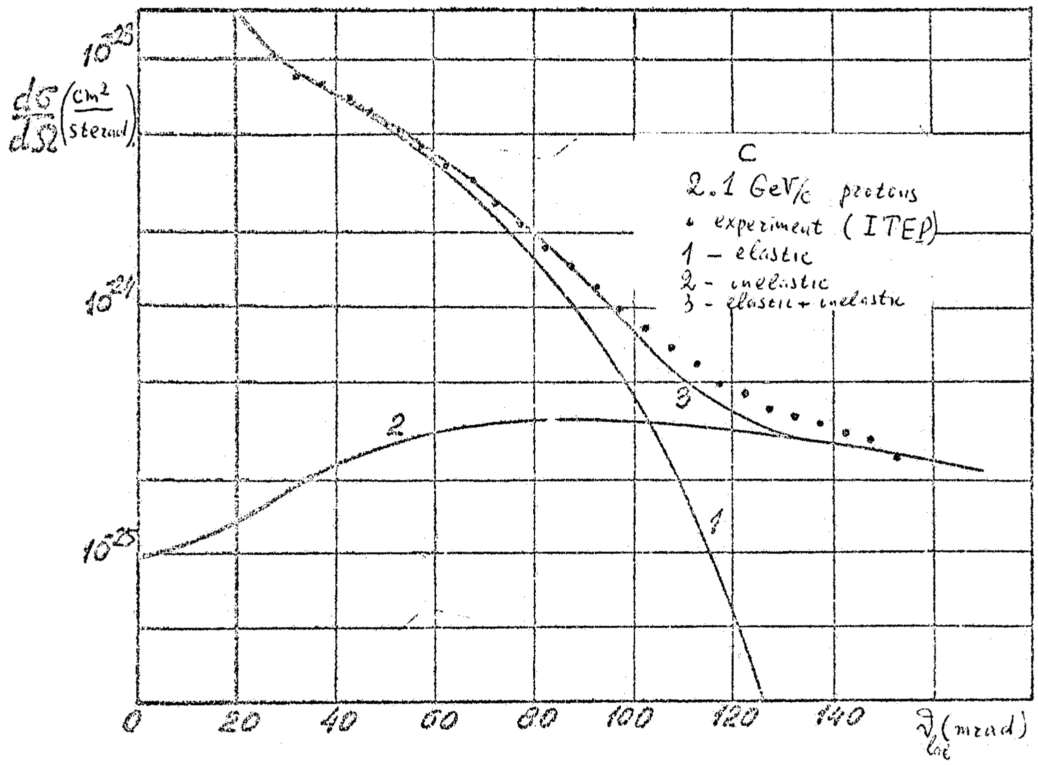


FIG. 2 - The results of theoretical calculations based on the GA in comparison with experimental data of ITEP group(24) on the reaction $p + {}^{12}C \rightarrow$ $\rightarrow p +$ anything at 2.1 GeV/c.

In Fig. 2 similar calculations(24) are compared with the experimental data of the ITEP group(24) on the same reaction at 2.1 GeV/c. The experimental data as in the former case are described very well. Some discrepancy (of about $\lesssim 10\%$) for $\theta \approx 100$ mrad may be connected with an uncertainty of the slope parameter β^2 for the p-n scattering amplitude.

In Fig. 3 the theoretical predictions(25) of the optical model on the elastic p-^{40}Ca scattering at T_p = 1 GeV are compared with the experimental data of the LIYaP group(25). The nuclear density was parametrized in two different forms:

$$\text{a)} \quad \rho(\vec{r}) = \rho_o(1 + W\frac{r^2}{R^2})(1 + \exp\frac{r - R}{a})^{-1}$$

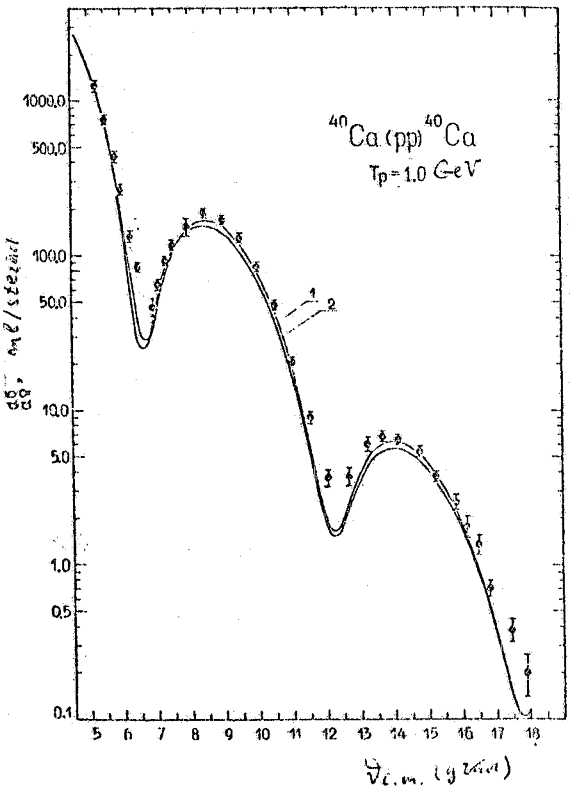
with R = 3.725 fm, a = 0.591 fm, W = -0.169 (curve 1);

$$\text{b)} \quad \rho(\vec{r}) = \rho_o(1 + \exp\frac{r - R}{a})^{-1}$$

with R = 3.6 fm, a = 0.576 fm (curve 2).

Almost everywhere the agreement is very good. Only near diffraction minima there are some discrepancy which may be connected with neglection of the angular resolution of the spectrometer, as well as the spin effects and the Fresnel corrections. Every small effect may be essential in the region of the minimum.

In Fig. 4 it is shown the comparison of the theoretical predictions(26) on the reaction $p + {}^{12}C \rightarrow p + {}^{12}C^*$ (4.43 MeV) at T_p = 1 GeV with the data of the Brookhaven group(27) and the LIYaP group(28). A good description of the data for angles $\theta \geq 10^o$ is achieved due to the inclusion of the nuclear deformation effect. The density distribution of the deformed ^{12}C nucleus was para-



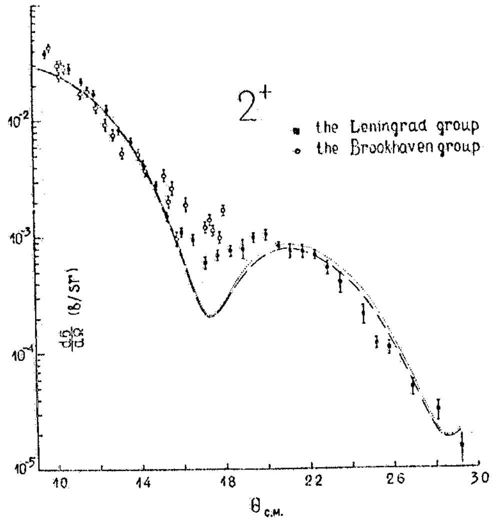


FIG. 4 - The results of Starodubsky and Domchenkov[26] on the reaction $p+{}^{12}C \rightarrow p+{}^{12}C^*$ (4.43 MeV) at T_p = 1 GeV in comparison with the data of the Brookhaven group[27] and LIYaP group[28].

FIG. 3 - The experimental results of LIYaP group[25] on the elastic p - ^{40}Ca scattering at T_p = 1 GeV in comparison with the theoretical predictions based on the GA[25].

metrized in the form:

$$\rho = \rho_o \left[1 + \exp\left\{(r - R)/a\right\}\right]^{-1} \tag{60}$$

where $R = R_o\left[1 + \beta P_2(\cos\theta)\right]$ in the target rest frame. The parameter of deformation β was put equal to -0.44.

In Fig. 5 it is plotted the cross-section of the elastic ^{12}C-^{16}O scattering at E_{LAB} = 168 MeV. The theoretical calculations performed in ref. (18) in the framework of the optical model (see, exps. (56) and (57) in these lectures) with the inclusion of the Coulomb interaction (this inclusion we shall discuss in following sections) agree very well with the experimental data[29].

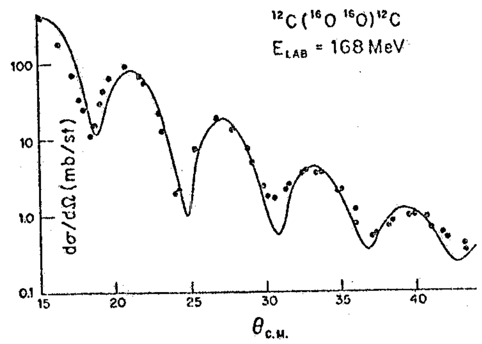


FIG. 5 - The theoretical description of the reaction $^{12}C(^{16}O, {}^{16}O)^{12}C$ (ref. (18)) and experimental data at E_{LAB} = 168 MeV (ref. (29)).

In Figs. 6-7 there are presented some results of Alberi, Bertocchi and Gregorio[30] who calculated differential cross-section and polarization for p-d elastic scattering at E_p = 582 MeV. They used the Glauber approximation and took into account spin structure of the N-N scattering amplitude. As we can see, the data[31-32] are not fitted very well by the solid curves for momentum transfer squared $|t| \gtrsim 0.3$ $(GeV/c)^2$. To reach a better fit the authors included (ad hoc) in all ten N-N amplitudes the phase factor

$$\exp\left[i\,\frac{\pi}{4}\left[\exp\left(-\frac{10}{t_o^2}(t - t_o/4)^2\right) - \exp(-10/16)\right]\right]$$

(the dashed curves), $t_o = 0.5\ (GeV/c)^2$.

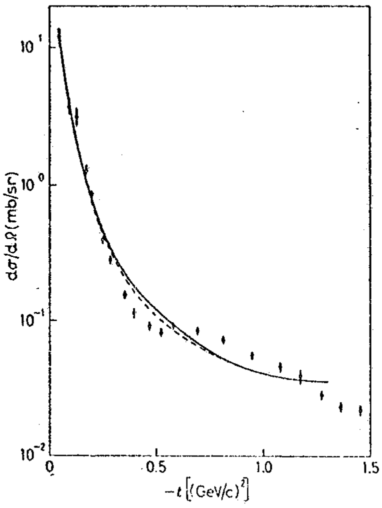


FIG. 6 - The results of Alberi et al.[30] on the differential cross-section of the reaction pd → → pd at 582 MeV in comparison with experimental data[31-32].

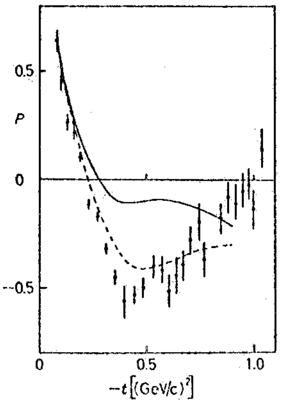


FIG. 7 - The results of Alberi et al.[30] on the polarization in the elastic p-d scattering at 582 MeV in comparison with experimental data.

This may be done because the absolute phase of the N-N amplitude may be strictly determined only for $t = 0$ (through the optical theorem).

But let us remember the Fresnel corrections. For $\vec{\Delta}^2 \gtrsim 0.3$ (GeV/c)^2 the parameter $\Delta^2 R/8k$ (see Section 2.4) for $k \sim 0.6$ GeV is of the order of 1. So we may expect that the deviation of the results of ref. (31) from the data is also connected with neglection of the Fresnel corrections[8].

Therefore it may be seen that when the conditions mentioned in Section 2.4, are satisfied the Glauber approximation works very well, and when these conditions are not satisfied there may appear noticeable corrections.

It is also useful to do some remarks concerning terminology. Usually in papers both expressions (22) and (27) are called as the Glauber approximations. It means that for both these expressions the particle-nucleus scattering phase is equal to the sum of the individual particle-nucleon phases. And the Glauber approximation is usually identified with the additive prescription for scattering phases. Here we connect termin "the GA" with the detail form of exp. (27), using for exp. (22) termin "the OM". For scattering on small angles $\Delta R \sim 1$ when $kR \gg 1$ the GA and the OM give practically equivalent results (with accuracy $\sim 1/A$). But for $A \sim 1$ (deuteron, helium) it is necessary to use the GA (exp. (27)).

5. - INELASTIC SCREENING. -

As it was mentioned in Sect. 2.4, for high energy $E \gtrsim m^2 R$ it may be noticeable the inelastic shadow correction connected with the possibility of the excitation of the incident hadrons[12-16]. Let us discuss the scattering of high-energy proton on nucleus

$$p + A_i \rightarrow p + A_f \quad . \qquad (61)$$

The incident proton may produce some hadronic state on one nucleon in the nucleus

$$p + N_1 \rightarrow \chi + N_1 \qquad (62)$$

and after this χ system may disappear after collision with another nucleon

$$\chi + N_2 \rightarrow p + N_2 \, . \qquad (63)$$

The cross-section of the reaction (61) is rather big, but for high-energy $E \gg m$ it is dominated by the contributions of big masses of the X system $M^2 \sim S$, $S = 2mE+m^2$. For scattering on small angles $\Delta R \lesssim 1$ the coherence condition implies the following limitation on the masses M which may contribute into the cross-section of reaction (61)

$$M^2/2E \lesssim R^{-1} . \tag{64}$$

For big masses $M^2/2E \gg R^{-1}$ the cross-section will be small due to big value of the longitudinal component of the momentum transfer from the initial to the intermediate state and from the intermediate to the final state.

So we see that the problem of the inelastic shadow correction is related to the dynamics of the inclusive reaction (61) in the region of $M^2/2E \lesssim R^{-1}$, i.e. in the triple-Regge limit $M^2/S \lesssim \lesssim 1/m_R \ll 1$.

In refs. (15a-15b) the inelastic shadow correction for scattering on deuteron was analyzed in the framework of the triple-Regge model. As it was shown in refs. (15a-15b) for calculations of the inelastic shadow corrections in nuclei for small momentum transfer $\Delta^2 R^2 \lesssim 1$ it is necessary to take into account only the part of the inclusive amplitude for reaction (61), which is connected with the diffraction dissociation (DD). The DD part of the cross-section for inclusive reaction $p+p \rightarrow \rightarrow X+p$ in the region $M \lesssim 2$ GeV has a resonance form (see exp. (15a)). In the region $M > 2$ GeV, $E \gg m$ the DD cross-section is well approximated by the expression(33)

$$\frac{d^2\sigma^{DD}(t=0)}{dt dM^2} = \frac{C}{M^2} \tag{65}$$

where $C = 2.4 \pm 0.5$ mb/GeV^2 is not dependent on E and M^2, $-t = \vec{\Delta}^2$.

The contribution of the inelastic shadow correction $\Delta\sigma$ to the total N-nucleus cross-section was calculated in ref. (16):

$$\Delta\sigma = -4\pi \int d^2\vec{b} \int_{(m+m_\pi)^2}^{(\sqrt{s}-m)^2} dM^2 \, \frac{d^2\sigma^{DD}(t=0)}{dt\, dM^2} \exp\left[-\tfrac{1}{2}\sigma T(b)\right] \left|F(q_L, b)\right|^2 \tag{66}$$

where $T(b) = A \int_{-\infty}^{\infty} \rho(b,z)dz$, ρ is the nuclear density function, q_L is the longitudinal component of the momentum transfer in the reaction (62):

$$q_L = \frac{M^2 - m^2}{S} \quad ; \text{ the form-factor}$$

$$F(q_L, b) = \int_{-\infty}^{\infty} \rho(b,z)\, e^{iq_L z}\, dz \tag{67}$$

gives the restriction of the integration over the mass M: $\frac{M^2}{S} mR \lesssim 1$.

The numerical estimation of the correction $\Delta\sigma$ are presented in Fig. 8. These estimations were performed in ref. (16) for the Gaussian density $\rho(r) = \pi^{-3/2} R^{-3} e^{-r^2/R^2}$, $R = 0.7\, A^{1/3}$ for ^{12}C and ^{16}O, and in the model of the black sphere with the radius $R = 1.1\, A^{1/3}$ fm for heavier nuclei.

The correction $\Delta\sigma$ gives the contribution ~ 2-$3\,\%$ into the total N-nucleus cross-section σ_{tot} and leads to the decreasing of σ_{tot} with energy. The value and the energy dependence of $\Delta\sigma$ given by exp. (65) are in agreement with the experimental data on the neutron-nucleus scattering received by ITEP-Moscow University group at Serpukhov and University of Maryland group at Batavia.

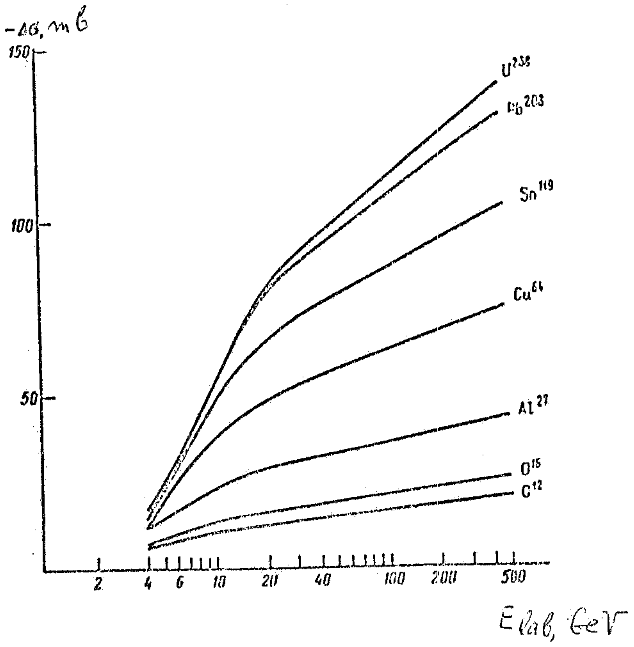


FIG. 8 - Inelastic shadow correction to the total nucleon-nucleus cross-section as a function of energy(16).

6. - COULOMB-NUCLEAR INTERFERENCE. -

6. 1. - Additivity of nuclear and Coulomb phase shifts. -

To include the Coulomb interaction into the Glauber formula (18) it is necessary to change the nuclear phase shifts of the a-N scattering on the sums of the nuclear and Coulomb phase shifts(5)

$$\delta(\vec{\rho}; \vec{S}_1, \ldots \vec{S}_A) = \sum_{j=1}^{z} (\delta^p(\vec{\rho} - \vec{S}_j) + \delta^c(\vec{\rho} - \vec{S}_j)) + \sum_{j=z+1}^{A} \delta^n(\vec{\rho} - \vec{S}_j) \tag{68}$$

where $\delta^p(\vec{\rho} - \vec{S}_j)$ and $\delta^n(\vec{\rho} - \vec{S}_j)$ are the phase shifts of the a-p and a-n scatterings, $\delta^c(\vec{\rho} - \vec{S}_j)$ is the Coulomb phase shift

$$\delta^c(\vec{\rho}) = -\frac{\alpha}{2\pi} \int \frac{d^2\vec{q}_\perp}{\vec{q}_\perp^{\,2} + \lambda^2} \rho_a(\vec{q}_\perp)\, \rho_p(\vec{q}_\perp)\, e^{-i\vec{\rho}\vec{q}_\perp} \tag{69}$$

where $\alpha = 1/137$, $\rho_a(\vec{q}_\perp)$ and $\rho_p(\vec{q}_\perp)$ are the charge form-factors of the particles a and p, λ is the photon mass. Introduction of λ corresponds to the choice of the Coulomb potential in the form $V(\vec{r}) = e_1 e_2 \frac{e^{-\lambda r}}{4\pi r}$, $\alpha = e^2/4\pi$. In the observable quantities λ must be put equal to 0. The method of elimination of this infrared divergency depends on experimental conditions. We shall discuss some examples here.

The principle of the additivity of the nuclear and Coulomb phase shifts follows from the additivity of the potentials - strong and Coulomb. If we introduce into Exp. (16) the potential

$$V_i = V_i^{(strong)} + V_i^{(Coulomb)}$$

then we get

$$\delta_i(\vec{\rho}) = \delta_i^{(strong)}(\vec{\rho}) + \delta_i^{(Coulomb)}(\vec{\rho}) \; . \tag{70}$$

This may be done only in the eikonal approximation for the a-N scattering amplitude. In the Watson multiple scattering theory we must add the amplitudes - strong and Coulomb

$$f_{aN_i}(\vec{q}) = f^{(s)}_{aN_i}(\vec{q}) + f^{(c)}_{aN_i}(\vec{q}) . \tag{71}$$

For the Coulomb interaction the eikonal approximation is valid also at low energy. So we may write for example

$$f^{(c)}_{ap}(\vec{q}) = \frac{ik}{2\pi} \int d^2\vec{\rho}\, e^{i\vec{q}\vec{\rho}} (1 - e^{2i\delta^c(\vec{\rho})}) \tag{72}$$

where $\delta^c(\vec{\rho})$ is given by Exp. (69).

Exps. (71)-(72) may also be used in the description of heavy ions collisions. The proton profile function (55) in this case must be calculated with the help of Exp. (71). To include the Coulomb interaction into the optical model (56) we must add to the phase $\sigma_{AB}(B)$ (See Exp. (57)) the term

$$\sigma^c_{AB}(\vec{B}) = - \frac{z_A z_B \alpha}{2\pi} \int \frac{d^2\vec{q}_\perp}{\vec{q}_\perp^{\,2} + \lambda^2} \rho_A(\vec{q}_\perp) \rho_B(\vec{q}_\perp) e^{-i\vec{\rho}\vec{q}_\perp} . \tag{73}$$

Note that for high energy the strong phase shift $\delta_i^{(s)}$ is practically imaginary and has a small real part. But the Coulomb phase shift δ^c is pure real. So in the first order in α the nuclear-Coulomb interference is proportional to the real part of the strong amplitude. This is the reason why the nuclear-Coulomb interference is used for the determination of the ratio $\mathrm{Re}f_{ap}/\mathrm{Im}f_{ap}$ at small scattering angles.

6. 2. - Bethe formula. -

Let us discuss the nuclear-Coulomb interference for the π-p scattering. Using the additivity principle for nuclear and Coulomb phase shifts we can write the following expression for the π^+-p scattering amplitude in the eikonal approximation

$$f_{\pi^+p}(E, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}} \left[1 - e^{2i\left[\delta^p(\vec{\rho}) + \delta^c(\vec{\rho})\right]} \right] \tag{74}$$

where $\delta^p(\vec{\rho})$ is the nuclear phase shift and $\delta^c(\vec{\rho})$ is the Coulomb phase shift

$$\delta^c(\vec{\rho}) = - \frac{\alpha}{2\pi} \int \frac{d^2\vec{q}_\perp}{\vec{q}_\perp^{\,2} + \lambda^2} \rho_\pi(\vec{q}_\perp) \rho_p(\vec{q}_\perp) e^{-i\vec{\rho}\vec{q}_\perp} . \tag{75}$$

The amplitude (74) may be represented in the form

$$f_{\pi^+p}(E, \vec{\Delta}) = f^{(c)}_s(E, \vec{\Delta}) + f_c(E, \vec{\Delta}) \tag{76}$$

where

$$f_c(E, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}} (1 - e^{2i\delta^c(\vec{\rho})})$$

is the pure Coulomb amplitude, and the amplitude

$$f^{(c)}_s(E, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\rho\, e^{i\vec{\Delta}\vec{\rho}} e^{2i\delta^c(\vec{\rho})} (1 - e^{2i\delta^p(\vec{\rho})}) \tag{77}$$

is equal to the pure strong amplitude $f_s(E, \vec{\Delta})$ in the absence of the Coulomb interaction

$$f_s(E, \vec{\Delta}) = \frac{ik}{2\pi} \int d^2\vec{\rho}\, e^{i\vec{\Delta}\vec{\rho}} (1 - e^{2i\delta^p(\vec{\rho})}) . \tag{78}$$

For a point charge

$$\delta^{c}(\vec{\rho}) = -\frac{\alpha}{2\pi}\int \frac{d^2\vec{q}_\perp}{\vec{q}_\perp^{\,2} + \lambda^2}\, e^{-i\vec{\rho}\vec{q}_\perp} = -\alpha K_o(\rho\lambda) \tag{79}$$

where $K_o(\rho\lambda)$ is the modified Bessel function and when $\Delta \gg 1$ the amplitude $f_c(E,\vec{\Delta})$ is equal to

$$f_c(E,\vec{\Delta}) = ik\int \rho\, d\rho\; I_o(\Delta\rho)\,(1 - e^{2i\delta^{c}(\rho)}) = ik\left\{\left[(1 - e^{-2i\alpha K_o(\rho\lambda)})\,\frac{1}{\Delta}\,\rho\, I_1(\Delta\rho)\right]_0^\infty - \right.$$

$$\left. - 2i\alpha\int_0^\infty \frac{1}{\Delta}\,\rho\, d\rho\; I_1(\Delta\rho)\, e^{-2i\alpha K_o(\rho\lambda)}\, K_o'(\rho\lambda)\, d\rho\right\} .$$

In this expression the first term is equal to zero because $K_o(x) \to \sqrt{\frac{\pi}{2\pi}}\, e^{-x}$ for $x \to \infty$. The integration of the second term may easily be performed for $\Delta \gg \lambda$, when the main contribution to the integral gives the region $\rho \sim \Delta^{-1}$ and $\rho\lambda \ll 1$. For small value of $\rho\lambda$ we have $K_o(\rho\lambda) = -\ln\rho\lambda\gamma$ (where $\ln\gamma = e = 0.58$ is the Euler's constant) and

$$f_c(E,\vec{\Delta}) = -2k\alpha\frac{1}{\Delta}\int_0^\infty (\rho\lambda\gamma)^{2i\alpha}\, I_1(\Delta\rho)\, d\rho = -\alpha\,\frac{2k}{\Delta^2}\left(\frac{2\lambda\gamma}{\Delta}\right)^{2i\alpha}\frac{\Gamma(1+i\alpha)}{\Gamma(1-i\alpha)} . \tag{80}$$

Exp. (80) may be written in the form

$$f_c(E,\vec{\Delta}) = -\alpha\,\frac{2k}{\Delta^2}\, e^{2i(\delta_c + \delta_o)} \tag{81}$$

where

$$\delta_c = -\alpha\ln\frac{\Delta}{2k} + \arg\frac{\Gamma(1+i\alpha)}{\Gamma(1-i\alpha)} \tag{82}$$

and

$$\delta_o = \alpha\ln\frac{\lambda\gamma}{k} . \tag{83}$$

From standard expressions which can be found in text books (see, for example ref. (34)) Exp. (80) differs by the phase factor $e^{2i\delta_o}$. This phase factor is important when we have to imply the optical theorem.

Bethe proposed the following expression for the amplitude $f_s^{(c)}(E,\vec{\Delta})$ [35]

$$f_s^{(c)}(E,\vec{\Delta}) = f_c(E,\vec{\Delta})\, e^{2i(\delta_B + \delta_o)} \tag{84}$$

where

$$\delta_B = \frac{\int_0^\infty \rho\, d\rho\left[\delta_c(\rho) - \delta_o\right]\delta^{P}(\rho)}{\int_0^\infty \rho\, d\rho\; \delta^{P}(\rho)} . \tag{85}$$

The difference between the Bethe's and the Coulomb phase shifts is approximately equal to

$$\delta_c - \delta_B = \alpha\ln\frac{1}{\Delta a} \tag{86}$$

where a is the radius of the strong interactions.

The differential cross-section of the elastic π-p scattering with the Coulomb-nuclear interference term may be written in the form

$$\frac{d\sigma}{d\Omega} = \left| -\alpha \frac{2k}{\Delta^2} e^{2i(\delta_c - \delta_B)} + f_s(E,\vec{\Delta}) \right|^2 . \tag{87}$$

Exp. (87) is used for determination of the ratio $\mathrm{Re}\, f_s(0)/\mathrm{Im}\, f_s(0)$ from data on the π-p elastic scattering at small angles. This formula may be used in the region $\Delta a \ll 1$ where it is possible to distinguish between the long range coulomb interaction and the short range strong interaction.

The Bethe formula (87) does not take into account the contribution of hadronic excitation spectrum in intermediate states. This contribution was analyzed in Refs. (36-37) where it was shown that for elastic scattering it is not big. So the real part of the π-p scattering amplitude determined with the help of Exp. (87) may be considered as pure strong up to the terms of the order of $\alpha\, \mathrm{Im}\, f_{\pi p}$.

6. 3. - Elimination of the infrared divergency. -

The Coulomb phase shift (69) contains the photon mass λ. All the electric charges of nuclei in our world are screened by electrons, and λ^{-1} is equal in the order of magnitude to the atomic radius. Obviously in experiments with beams of high energy particles the atomic energy scale can not be involved. So in the final expressions the value of λ must be changed on the experimental resolution over energy or momentum transfer, or the final expression must not be dependent on λ.

The differential cross-section as we can see from Exp. (85) is not dependent on λ, because the λ-dependence of the amplitude may be extracted in the form of the phase factor $e^{2i\delta_o}$ which has no influence on a value of differential cross-seetion.

The different situation appears when we want to calculate the imaginary part of the forward scattering amplitude which is related through the optical theorem to the total cross-section. In this case all the phase factors in the amplitude are essential, and in the final answer the value of λ must be changed on the experimental resolution over the momentum transfer. For example, if in an experiment the minimal value of the momentum transfer squared is equal to $\vec{\Delta}^2_{min} = -t_{min}$, it means that the value

$$\int_{t_{max}}^{t_{min}} \frac{d\sigma_{ab}}{dt}\, dt \tag{88}$$

is measured.

The optical theorem now may be represented in the form

$$\mathrm{Im}\, f_{ab}(0) - \frac{k}{4\pi} \int_{t_{min}}^{0} \frac{d\sigma_{ab}}{dt}\, dt = \frac{k}{4\pi} \int_{t_{max}}^{t_{min}} \frac{d\sigma_{ab}}{dt}\, dt . \tag{89}$$

If the integral (88) has a singularity at $t_{min} \to 0$, the same singularity is also in $\mathrm{Im}\, f_{ab}(0)$. At fixed value of t_{min} the wright and left hand part of Exp. (89) have not singularity and for $-t_{min} \gg \lambda^2$ the photon mass λ may be put equal to 0.

6. 4. - Coulomb-nuclear interference in π-nucleus scattering. -

The differences of the total and differential cross-sections for the scattering of π^- and π^+ mesons on nuclei with zero isospin are dominated by the contribution of the Coulomb-nuclear interference. Because the strong interactions are charge independent these differences are equal to zero when the electromagnetic interaction is neglected.

The $\pi^{\mp}$-difference for total cross-sections was analyzed in paper (38). The difference may be written in the form

$$r_A = 2\, \frac{\sigma^{tot}_{\pi^- A} - \sigma^{tot}_{\pi^+ A}}{\sigma^{tot}_{\pi^- A} + \sigma^{tot}_{\pi^+ A}} = 2\alpha z \eta \left[C_1 \ln \frac{4}{R^2 \vec{\Delta}^2_{min}} + C_2 \ln \frac{2}{b \vec{\Delta}^2_{min}} + C_3 \right] \tag{90}$$

where $\eta = \mathrm{Re}(f_{\pi^+p}(0) + f_{\pi^-p}(0)/\mathrm{Im}(f_{\pi^+p}(0) + f_{\pi^-p}(0))$, R is the radius of the nucleus, $C_1 \sim 1$, $C_2/C_1 \sim A^{-2/3}$, $C_3/C_1 \lessapprox 0.1$.

The main interesting feature of Exp. (90) is the energy dependence of r_A. Because the coefficients C_1, C_2, C_3 are determined by nuclear parameters, this dependence is mainly connected with the energy dependence of η.

The contribution of the Coulomb-nuclear interference to the differential cross-sections was considered in Refs. (22, 39, 40). As it was shown in paper (40) this contribution for $\Delta R \gtrsim 1$ is equivalent to the addition ~ 0.1 to the ratio $\mathrm{Re}\, f_{ap}/\mathrm{Im}\, f_{ap}$.

The general properties of the Coulomb-nuclear interference in hadron-nucleus scattering are following:

a) The correction to the imaginary part of the elastic scattering amplitude is proportional to z and is small at high energy where η is small;
b) A more appreciable contribution from the electromagnetic corrections is to the real part of the amplitude. For $\Delta R \gtrsim 1$ the relative contribution is of the order of $z\alpha/\eta$;
c) In differential cross section $d\sigma/d\Omega = |\mathrm{Re}F_{\pi A}|^2 + |\mathrm{Im}F_{\pi A}|^2$ the contribution of the Coulomb correction is mostly important in the regions of the diffraction minima, where $\mathrm{Im}F_{\pi A}$ is close to 0.

7. - PECULIAR POLARIZATION PHENOMENA FOR KNOCK-OUT REACTIONS (p, 2p) AND (p, p) ON NUCLEI. -

7.1. - Target fragments in the backward hemisphere in the lab. system. -

In previous sections we discussed the processes in which the fragments of nuclei were not detected. Detection of nuclear fragments may also give interesting information about nuclear structure and reaction mechanism. As examples, there may be the well known reactions (e, e'p) and (p, 2p) on nuclei. Detection of two particles on the final state leads to a very low cross-section. Interesting information about nuclear structure may also be received from the reactions (p, p) on nuclei. It is of great interest the spectrum and polarization of the final protons emitted into the backward hemisphere in the lab. system. This emission of protons into the backward hemisphere is related to the Fermi motion of nucleus inside a nucleus and can not be realyzed for the scattering of protons on free protons. The ITEP group[41] when analyzed the backward proton spectra in reaction

$$p_0 + {}^{12}C \rightarrow p_2 + \text{anything} \tag{91}$$

has found the presence of high-momentum component in the spectrum and a rather big polarization of the final protons. Both this phenomena - the significant deviation of spectrum from the prediction of the indipendent particle model and big polarization - were qualitatively explained in paper (42) in the framework of the DWBA.

Following to the results of Ref. (42) let us consider final-protons p_2 in reaction (91) which are emitted into sector 2 of the backward hemisphere (see Fig. 9a). The momentum of the incident

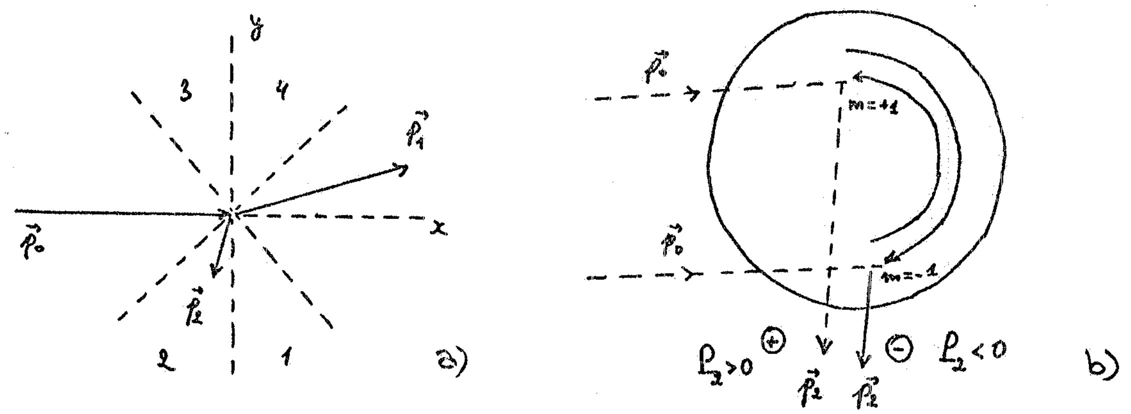


FIG. 9 - Illustration of kinematics for the reaction (p, 2p) on ^{12}C.

and final protons are p_o and p_2. The polarization of outgoing protons p_2 appears as a result (a) orbital motion of nucleons inside a nucleus and presence of the spin-orbit interaction; (b) absorption of outgoing protons in nuclear matter. Protons emitted into the backward hemisphere before the collision were moving against the incoming proton. On the other hand due to absorption, the protons coming from the nuclear side faced the detector give the dominant contribution to the cross-section (see Fig. 9b). It means that the p-shell protons which come to sector 2 Fig. 9a were moving clockwise before the collision, i.e. thay had the definite projection of angular momentum (on Fig. 9a, the z-axis is directed on a reader and the projection is equal to $m = -1$). Due to the spin-orbit coupling the nucleon with definite projection of angular momentum has the definite polarization. For the $j=3/2$ state and $m = -1$ this polarization is equal to $P_2 = 0.5$.

At high energy the spin dependent part of the N-N scattering matrix is small. So after collision the outgoing proton has practically the same polarization which it had before the collision. The contribution of the s-shell protons and protons coming from the opposite side of nucleus is small and may decrease the polarization only on 10 - 20 %.

The ITEP group(41) found a large left-wright assymetry $\sim 0.3 - 0.4$ in the secondary scattering of protons emitted from ^{12}C on angle 137^o for energy of incident protons 2 and 3.66 GeV. The outgoing protons had the momenta 550 and 645 MeV/c and their polarization was estimated to be not less than 0.5. The effective polarization of protons inside a nucleus for the reaction (p, 2p) was also discussed in Ref. (45), where for detection of this effect it was proposed to measure the left-wright assymetry in the quasi elastic scattering of polarized protons with selection of definite quantum numbers of residual nuclei. This asymetry would be absent for the scattering of unpolarized protons. The JINR group(44) found the assymetry of this type in the quasi elastic scattering of polarized 653 MeV protons on ^6Li.

Note that for bad resolution on the excitation energy of the recoil nucleus it is more preferable for the detection of this effect to use nuclei with not complete p-shell. For the complete $P_{3/2}$ and $P_{1/2}$ shells, as for the ^{16}O nucleus which was discussed in Ref. (43), the effective polarizations of $P_{3/2}$- and $P_{1/2}$- protons are compensated.

7.2. - The reaction (p, 2p) in the framework of the DWBA. -

Let us discuss the quasi elastic p-p scattering on protons in nuclei

$$p_o + {}^{12}C \rightarrow p_1 + p_2 + \text{anything} \tag{92}$$

in the lab. system. The momentum transfer is $\vec{\Delta} = \vec{p}_o - \vec{p}_1$, $|\vec{\Delta}| \ll |p_o|$. We shall imply the condition

$$\vec{\Delta}^2 \gg R^{-2} \tag{93}$$

when it is possible to distinguish between the know-out protons and evaporating protons which have the momenta of the order of R^{-1}. In this case it is also reasonable to use the DWBA for the description of the absorption of outgoing protons. For estimationswe choose $|\vec{p}_o| = 20$ GeV/c, $|\vec{p}_2| = 540$ MeV/c ($T_2 = 155$ MeV) and the simplest shell model for the ground state of ^{12}C with completely occupied $S_{1/2}$ and $P_{3/2}$ shells.

In the DWBA the differential cross-section of reaction (92) and the normal to the scattering plane polarization of the knock-out protons p_2 are given by the expressions

$$\frac{d^5\sigma}{d^2\vec{\Delta}\, d^3\vec{p}_2} = \frac{d^2\sigma_{pp}^{el}}{d^2\vec{\Delta}} G_o((\vec{\Delta} - \vec{p}_2)^2), \tag{94}$$

$$P_2 = G_o^{-1}(\vec{p}_3^{\,2}) \frac{2}{3} (|g_p^{+1}(\vec{p}_3)|^2 - |g_p^{-1}(\vec{p}_3)|^2), \tag{95}$$

where

$$G_o(\vec{p}_3) = 2|g_s(\vec{p}_3)|^2 + \frac{4}{3}(|g_p^{+1}(\vec{p}_3)|^2 + |g_p^{-1}(\vec{p}_3)|^2 + |g_p^o(\vec{p}_3)|^2), \tag{96}$$

$\vec{p}_3 = \vec{\Delta} - \vec{p}_2$. Due to the condition (93) we have $\vec{p}_3^{\,2} \ll \vec{\Delta}^2$. The functions $g_e^m(\vec{p}_3)$ can be expressed through the single particle wave functions

$$g_e^m(\vec{p}_3) = \frac{1}{(2\pi)^{3/2}} \int e^{i\vec{p}_3\vec{r}} D_o(x,y,z)\, \psi_{e,m}(\vec{r})\, d^3\vec{r} \tag{97}$$

where the absorption factor is

$$D_o(x,y,z) = \exp\left\{-\frac{\sigma_o(1-i\eta_o)}{2}\int_{-\infty}^{\infty}\rho(x_1^2+y^2+z^2)\,dx_1 - \frac{\sigma_2(1-i\eta_2)}{2}\int_{-\infty}^{\infty}\rho(x^2+y_1^2+z^2)\,dy_1\right\} \tag{98}$$

Here the x-axis is directed along $\vec{p}_o$, the y-axis along the momentum transfer $\vec{\Delta}$, which is considered to be normal to $\vec{p}_o$.

For the description of the nucleon absorption in (98) the optical model is used. The factors $\exp\left\{-\frac{\sigma_o(1-i\eta_o)}{2}\int_{-\infty}^{\infty}\rho(x_1,y,z)dx_1\right\}$ and $\exp\left\{-\frac{\sigma_2(1-i\eta_2)}{2}\int_{-\infty}^{y}\rho(x,y_1,z)dy_1\right\}$ takes into account the absorption (and reflection) of the fast and slow protons respectively. For $\vec{\Delta}^2 \gg \vec{p}_3^{\,2}$ the trajectory of the slow proton is approximately parallel to the y-axis.

The nuclear density is given by

$$\rho(\vec{r}) = \frac{4}{\pi^{3/2}a_o^3}(1+\delta\, r^2/a_o^2)\, e^{-r^2/a_o^2}, \tag{99}$$

where $\delta = (A-4)/6$. For ^{12}C $\delta = 4/3$, $a_o = 1.6$ fm. The optical model with nuclear density (99) describes very well the elastic and inelastic $p-^{12}C$ scattering. For incident protons with energy 20 GeV $\sigma_o = \frac{1}{2}(\sigma_o^{pp} + \sigma_o^{pn}) = 39$ mbarn, $\eta_o \approx 0$. For outgoing protons with energy 155 MeV $\sigma_2 = \frac{1}{2}(\sigma_2^{pp} + \sigma_2^{pn}) = 35$ mbarn, $\eta_2 \approx 0.7 - 0.8$. The wave functions of s- and p-states have the forms:

$$\psi_s(\vec{r}) = \pi^{-3/4} a_o^{-3/2} e^{-r^2/2a_o^2},$$

$$\psi_{1m}(r) = \pi^{-3/4} a_o^{-5/2} \left(\frac{8\pi}{3}\right)^{1/2} r\, I_{1m}(\vartheta,\varphi)\, e^{-r^2/2a_o^2}. \tag{100}$$

The dependences of $|g_e^m(\vec{p}_3)|^2$ and P_2 on p_3 are presented in Fig. 10 for $p_{3z} = 0$, $p_{3y} = 0$ (for $p_{3z} = 0$, $g_e^o = 0$). The region $p_3 a_o \sim 1$ is of main interest. At high energy the longitudinal component of the momentum transfer Δ is small, and we may put $p_{2x} \approx p_{3x}$. Really, $\Delta_x \sim \Delta_\perp^2/2p_o$ and for $p_o \approx 20$ GeV/c, $\vec{\Delta} \approx 0.5$ GeV/c we have $\Delta x \approx 6$ MeV/c. If we choose the interval 60 MeV/c $\le p_3 \le$ 200 MeV/c, then for overtaking (or contrary-moving) kinematics will correspond the position of p_3 in sectors 1 and 4 (or 2 and 3) in Fig. 9a. Positive values of p_3 in Fig. 10 correspond to overtaking kinematics, negative ones - to contrary - moving. Therefore in corrispondence with Fig. 10 protons in Sectors 1 and 3 have positive polarization, and in Sectors 2 and 4 negative one (let us remember, that the z-axis is directed on the reader).

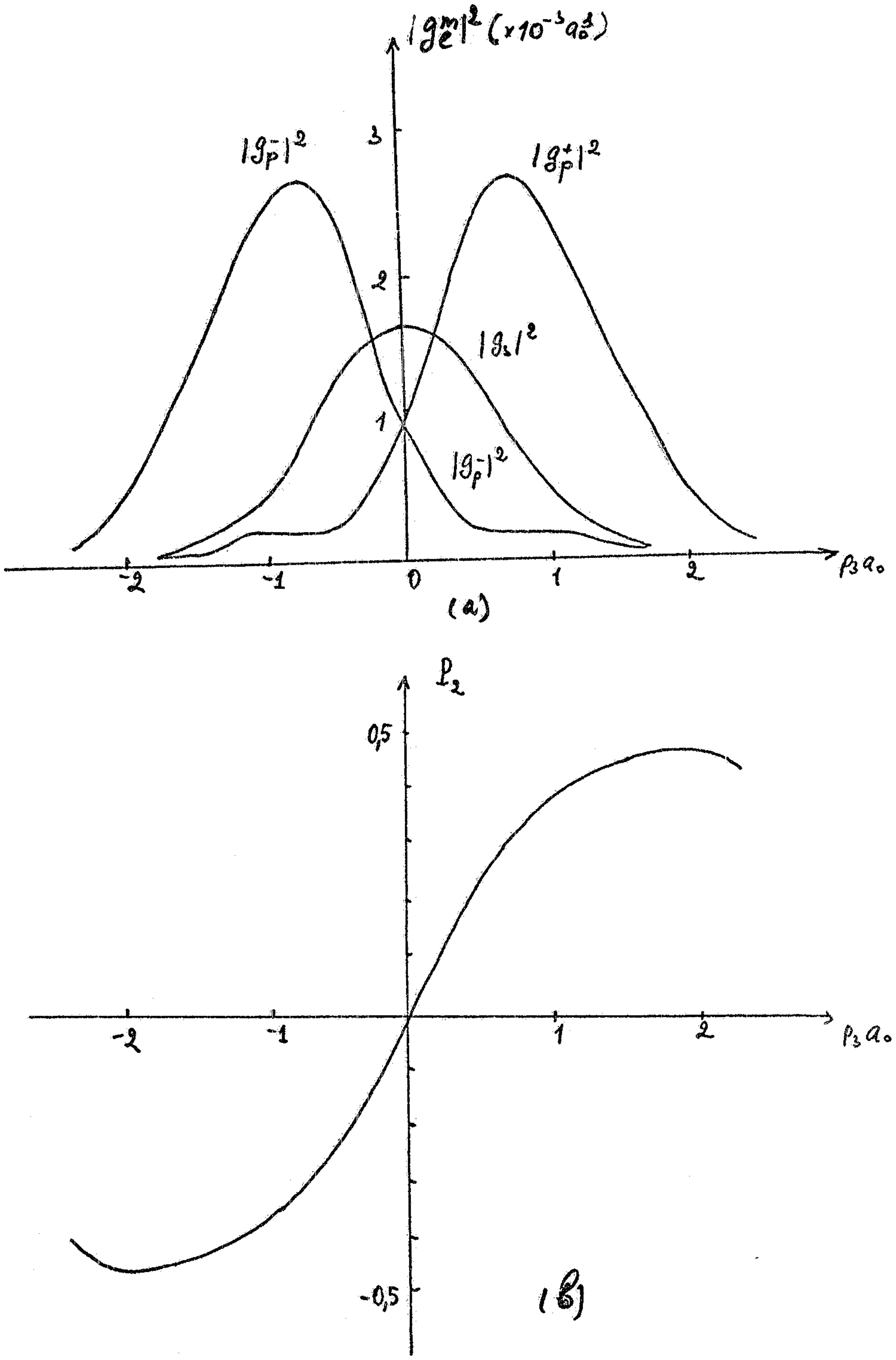


FIG. 10 - Longitudinal spectra (a) and polarization (b) of recoil protons in the reaction (p, 2p) on ^{12}C.

7.3. - The reaction (p, p) with proton fragments in the backward hemisphere. -

If the fast proton p_1 in reaction (92) is not detected and only the proton p_2 going into the backward hemisphere is measured (reaction (91)), then in Exps. (94)-(98) it is necessary to exclude the absorption of the fast proton in the final state. This is equivalent to change of the absorption factor

$$D_o(x, y, z) \to D(x, y, z) ,$$

where

$$D(x, y, z) = \exp\left\{- \frac{\sigma_o(1-i\eta_o)}{2} \int_{-\infty}^{x} \rho(x_1, y, z)dx_1 - \frac{\sigma_2(1-i\eta_2)}{2} \int_{-\infty}^{y} \rho(x, y_1, z)dy_1 \right\} . \qquad (101)$$

Now the cross-section of the reaction (91) may be written in the form

$$\frac{d^3\sigma^{(o)}}{d^3\vec{p}_2} = \int d^2\vec{\Delta} \, \frac{d^2\sigma_{pp}^{el}}{d^2\vec{\Delta}} \, G((\vec{\Delta} - \vec{p}_2)^2) , \qquad (102)$$

where in the region $0 \lesssim p_3 \lesssim 200$ MeV/c the function G can be parametrized as

$$G(\vec{p}_3^{\,2}) = G(0)\, e^{-a\vec{p}_3^{\,2}} \qquad (103)$$

$G(0) \simeq 100\ \mathrm{GeV}^{-3}$, $a \approx 20\ \mathrm{GeV}^{-2}$.

After substitution of the p-p elastic cross-section $\frac{d\sigma^{el}}{d\vec{\Delta}^2} = A e^{-b\vec{\Delta}^2}$ into Exp. (102) we get

$$\frac{d^3\sigma^{(o)}}{d^3\vec{p}_2} = \frac{A\,G(0)}{a+b}\, e^{-b'p_{2\perp}^2 - a p_{2\parallel}^2} \qquad (104)$$

where $b' = \frac{ab}{a+b}$, $p_{2\perp}$ and $p_{2\parallel}$ are the perpendicular and parallel to $\vec{p}_o$ components of the momentum $\vec{p}_2$. The polarization of outgoing protons P_2 is not changed practically after the exchange $D_o(x, y, z) \to D(x, y, z)$ in the region 60 MeV/c $\lesssim p_3 \lesssim$ 200 MeV/c. At the same time the differential cross-section increases by 3 times.

Note two interesting features of the cross-section (14): i) different dependence on the $p_{2\perp}$ and $p_{2\parallel}$; the $p_{2\perp}$-dependence is mainly determined by the slope of the elastic p-p scattering $b' = b\,(1 + \frac{b}{a+b})$, because the ratio $\frac{b}{a+b}$ is small; the $p_{2\parallel}$- dependence is determined by the parameters of nucleus; ii) the slope of the $p_{2\parallel}$-dependence is approximately 3 times less than it would be in the independent particle model without taking into account the absorption of the outgoing proton.

In the cross-section (14) is not taken into account the contribution from the inelastic elementary reaction

$$p_o + "p_3" \to X_o + p_2 . \qquad (105)$$

It is possible to distinguish two mechanisms of the inclusive reaction (105): diffraction dissotiation and multiperipheral. In the former the cross-section is dominated at energies $\lesssim$ 100 GeV by small masses $M_o \lesssim 2$ GeV. The contribution of the diffraction dissotiations is very similar to the contribution of the elastic scattering. The main difference is the appearence in the kinematical relations of the additional longitudinal component of the momentum transfer $\Delta x^{(M)} = (M_o^2 - m^2)/2p_o$. But in the energy region $E_o \gg m^2 a_o$ this term in Δx may be neglected. In result we may expect the increasing of proton yield on 15-20% due to the contribution of the diffraction dissociation.

The multiperipheral mechanism gives a flat distribution on the variable M_o^2/ρ, where $\rho = 2mE_o + m^2$. For $M_o^2/2p_o \gg a_o^{-2}$ all the recoil protons will go due to the longitudinal momentum transfer $\Delta x^{(M)} \sim M_o^2/2p_o$ into the forward hemisphere. Into the backward hemisphere there will be contribution from the mass region

$$\frac{M_o^2 - m^2}{2p_o} \lesssim a^{-1/2} \tag{106}$$

where a is determined by Exp. (103). Due to a flat distribution of inelastic cross-section over the variable M_o^2/s, the inclusion of the events satisfying to the condition (106) will increase the yield of protons into the backward hemisphere by the ratio

$$R = \frac{(1/ma^{1/2})\,\sigma_{inel} + \sigma_{el}}{\sigma_{el}} \approx 2 . \tag{107}$$

Therefore, the cross-section of reaction (91) for backward protons with the inclusion of the inelastic p-p collisions is equal

$$\frac{d^3\sigma(p_{2\parallel} < 0)}{d^3\vec{p}_2} \approx 2\,\sigma_{el}\,\frac{b}{a+b}\,G(0)\,e^{-b'\vec{p}_{2\perp}^{\,2} - a\vec{p}_{2\parallel}^{\,2}} \qquad (\vec{p}_{2\perp}^{\,2} >> a_o^{-2}) \tag{108}$$

Here it was assumed that the $\vec{\Delta}$-dependence of the inelastic cross-section in the mass region $M_o^2 \lesssim 2p_o a^{-1/2}$ is the same as for the elastic scattering.

It may also be expected that the spin-flip in inelastic collisions (105) will not be big and the inclusion of the inelastic contribution will not change appreciably the polarization. The cross-section (108) is in a qualitative agreement with the results of the ITEP group(41).

7. 4. - Sensitivity to nuclear models. -

The presented calculations of the cross-section and the final proton polarization in reaction (91) are leaned essentially against two points: 1) the simplest shell-model for the nuclear wave function; 2) the DWBA which was used for the calculation of the probability of the proton to be emitted from nucleus with the given magnitude of momentum. The polarization is very sensitive to model of nuclear wave function. If, for example, the wave function of the ^{12}C ground state model contain (apart from s-shell) the admixture of the $p_{3/2}$ and $p_{1/2}$ shells, then the polarization could be appreciably less in the considered here experimental conditions when the final state of nucleus is not identified. This is because the contribution of the $p_{1/2}$ shell gives the polarization with the opposite sign, than the contribution of the $p_{3/2}$ shell. As it was mentioned above the complete cancellation of polarization can take place for ^{16}O, which has the completely occupied $p_{3/2}$ and $p_{1/2}$ shells.

Concerning the point about the use of the DWBA we may note that outside of this approximation is the contribution of elastic (noncoherent) scattering which transfer the nucleons produced initially in different momentum interval into the given interval. This contribution may slightly change the cross-section. But it is very unlikely that the polarization may be changed very much, because the depolarization is mainly connected with the contribution of protons coming from the nuclear side opposite to the detector. This contribution is not big because for medium nuclei the mean free path of nucleon on nuclear matter is at least two times less than the nuclear diameter.

The depolarization connected with the presence of spin-slip in the scattering matrix when the nucleons is passing through nuclear matter can not be big because the spin dependent part of the optical potential for residual nucleus is small $\sim 1/A$.

8. - NEW THEOREM FOR FINAL STATE INTERACTION IN DEUTRON BREAK UP. -

Deutron target is very often used for determination of neutron cross-sections. Let us discuss the π^o-photoproduction on deutron

$$\gamma(\vec{p}_o) + d \to \pi^o(\vec{k}) + n(\vec{p}_1) + p(\vec{p}_2) . \tag{109}$$

This reaction is used for determination of the $\gamma n \to \pi^o n$ amplitude (see for example Ref. (45)). The problem is how to take into account the final state interaction of pions. Let us now neglect for a moment by charge-exchange and consider only the elastic π-N rescattering.

Using the method developed in Set I it is easily to write the following expression for the amplitude of the reaction (109) when apart from π^o the proton-spectator is detected

$$F^{(p)} = f_1 + f_2 \tag{110}$$

where f_1 is the single scattering term

$$f_1(\vec{p}_o; \vec{p}_2, \vec{k}) = F_n(\vec{p}_o, \vec{k})\, \varphi(\vec{p}_2)\,, \tag{111}$$

where F_n is the $\gamma n \to \pi^o n$ amplitude and $\varphi(\vec{p}_2)$ is the deutron wave function in the momentum space. The term f_2 which describes the final state interaction may be represented in the form

$$f_2 = - F_n(\vec{p}_o, \vec{k}) \int d^3\vec{r}\, e^{-i\vec{p}_2\vec{r}}\, \varphi(\vec{r})\, \Gamma(\vec{b})\, \theta(z)\,, \tag{112}$$

where the z-axis is directed along the π^o momentum $\vec{k}$, $\Gamma(\vec{b})$ can be expressed through the π^o-p elastic scattering amplitude

$$\Gamma(\vec{b}) = \frac{1}{2\pi i k} \int d^2\vec{q}\, e^{-i\vec{q}\vec{b}}\, f_{\pi^o n}(\vec{q})\,, \tag{113}$$

θ function $\theta(z) = \begin{cases} 1 & z \geq 0 \\ 0 & z < 0 \end{cases}$ takes into account the ordering along z-axis of neutron and proton inside the deutron, i.e. the pion must be produced before rescattering.

In Exp. (112) it is taken into account that the dominant contribution to the final state interaction is due to the small angle π^o-n rescattering. It is also neglected by the radius of π^o-n interaction as compared with the radius of deutron. The protonmay be identified as spectator if $|\vec{p}_o - \vec{k}| \gg \vec{p}_2$.

The sum of two terms (111) and (112) may be written as

$$f_1 + f_2 = F_n(\vec{p}_o, \vec{k}) \int d^3\vec{r}\, e^{-i\vec{p}_2\vec{r}}\, \varphi(\vec{r}) \left[1 - \theta(z)\, \Gamma(\vec{b})\right]. \tag{114}$$

The differential cross-section of the reaction (109) with proton spectator is given by

$$\frac{d^5 \sigma_d^{(p)}}{d\Omega_\pi\, d^3\vec{p}_2} = \left| f_1 + f_2 \right|^2 . \tag{115}$$

It may be seen from Exps. (114)-(115) that the final state interaction leads to the change of the spectator spectrum. Let us perform the integration over the spectator momenta p_2 we get

$$\int \left| f_1 + f_2 \right|^2 d^3\vec{p}_2 = \int d^3\vec{r} \left| \varphi(\vec{r}) \right|^2 \left| F_n(\vec{p}_o, \vec{k}) \right|^2 \cdot \left\{ 1 - \theta(z)(\Gamma(\vec{b}) + \Gamma^*(\vec{b})) + \theta^2(z)\, \Gamma(\vec{b})\, \Gamma^*(\vec{b}) \right\}. \tag{116}$$

Due to the two-body unitarity condition

$$\Gamma(\vec{b}) + \Gamma^*(\vec{b}) = \Gamma(\vec{b})\, \Gamma^*(\vec{b}) \tag{117}$$

the two last terms in curly brackets of (116) are compensated because $\theta^2(z) = \theta(z)$. We have

$$\int \left| f_1 + f_2 \right|^2 d^3\vec{p}_2 = \int \left| f_1 \right|^2 d^3\vec{p}_2 = \left| F_n(\vec{p}_o, \vec{k}) \right|^2 = \frac{d\sigma_n^{free}}{d\Omega} \tag{118}$$

where $d\sigma_n^{free}/d\Omega$ is the cross-section of the reaction $\gamma n \to \pi^o n$. Exp. (118) means that the elastic rescattering does not change the value of the cross-section

$$\int \frac{d^5\sigma_d^{(p)}}{d\Omega_\pi d^3\vec{p}_2} d^3\vec{p}_2 = \frac{d\sigma_n^{free}}{d\Omega_\pi} . \tag{119}$$

The relation (119) has the simple physical meaning: the total number of the produced π^o-mesons can not be changed by elastic rescattering.

When we include charge-exchange of pions the proton spectator can be transformed to the neutron spectator and vice versa. Now it can be formulated the following theorem:

The sum of the cross-sections of the reaction (109) with the proton-spectator and the neutron--spectator is equal to the sum of the cross-sections for π^o-photoproduction on free neutron target and free proton-target.

$$\frac{d\sigma_d^{(p)}(\vec{p}_o,\vec{k})}{d\Omega_\pi} + \frac{d\sigma_d^{(n)}(\vec{p}_o,\vec{k})}{d\Omega_\pi} = \frac{d\sigma_n^{free}(\vec{p}_o,\vec{k})}{d\Omega_\pi} + \frac{d\sigma_p^{free}(\vec{p}_o,\vec{k})}{d\Omega_\pi} . \tag{120}$$

Therefore contrary to the usual belief that in determination of $d\sigma_n^{free}$ near the first resonance through the ratio

$$d\sigma_n^{free} = \frac{d\sigma_d^{(p)}}{d\sigma_d^{(n)}} d\sigma_p^{free} \tag{121}$$

the final state interaction correction is compensated, we see that it is much better to determine $d\sigma_n^{free}$ from the difference

$$d\sigma_n^{free} = (d\sigma_d^{(p)} + d\sigma_d^{(n)}) - d\sigma_p^{free} \tag{122}$$

which is not influenced by rescattering. The term $(d\sigma_d^{(p)} + d\sigma_d^{(n)})$ determines the total counting rate of π^o-mesons on deutron target.

Introducing isospin indexes we may write the amplitude of reaction (109) in the form

$$\begin{aligned} \langle \pi_\alpha, N_\nu | f | \gamma_\tau, d \rangle &= \langle \pi_\alpha, N_\nu | f_1 + f_2 | \gamma_\tau, d \rangle = D_{\mu\nu} \langle \pi_\alpha, N_\mu | F(\vec{p}_o,\vec{k}) | \gamma_\tau, N_\mu \rangle \varphi(\vec{p}_2) - \\ &- \sum_{N',\nu',\beta} D_{\mu'\nu'} \int d^3\vec{r}\, \varphi(\vec{r}) e^{-i\vec{p}_2\vec{r}} \theta(z) \langle \pi_\alpha, N_\nu | \Gamma(\vec{b})\, \pi_\beta, N_{\nu'} \rangle \langle \pi_\beta, N_\mu | F(\vec{p}_o,\vec{k}) | \gamma_\tau, N_{\mu'} \rangle \end{aligned} \tag{123}$$

where $N_{+1/2} = p$, $N_{-1/2} = n$, $\pi_{+1} = \pi^+$, $\pi_{-1} = \pi^-$, $\pi_o = \pi^o$, $D_{\mu\nu}$ describes the vertex $d \to N_\mu N_\nu$, and has the properties

$$D_{\mu\nu} = -D_{\nu\mu} , \qquad \sum_\nu D_{\mu'\nu} D^*_{\mu''\nu} = \delta_{\mu'\mu''} . \tag{124}$$

The sum of the cross-sections with free proton and neutron target is

$$\frac{d\sigma_p^{free}}{d\Omega_\pi} + \frac{d\sigma_n^{free}}{d\Omega_\pi} = \sum_\mu \left| \langle \pi_o, N_\mu | F(\vec{p}_o,\vec{k}) | \tau_\gamma, N_\mu \rangle \right|^2 . \tag{125}$$

The unitarity condition has the form

$$\langle \pi_\alpha, N_\nu | \Gamma(\vec{b}) | \pi_\beta, N_{\nu'} \rangle + \left| \langle \pi_\alpha, N_\nu | \Gamma(\vec{b}) | \pi_\beta, N_{\nu'} \rangle \right|^* = \tag{126}$$
$$= \sum_{\beta', \nu''} \langle \pi_\alpha, N_\nu | \Gamma(\vec{b}) | \pi_{\beta'}, N_{\nu''} \rangle \left| \langle \pi_\beta, N_{\nu'} | \Gamma(\vec{b}) | \pi_{\beta'}, N_{\nu''} \rangle \right|^* .$$

Now with the help of the unitarity condition (126) it may be easily verified that

$$\int \sum_{v} \left| \langle \pi_\alpha, N_\nu | f | \gamma_\tau, d \rangle \right|^2 d^3\vec{p}_2 = \int \sum_{v} \left| \langle \pi_\alpha, N_\nu | f_1 | \gamma_\tau, d \rangle \right|^2 d^3\vec{p}_2 . \tag{127}$$

From Exp. (127) and (125) it follows the theorem (120).

Similar theorems may also be derived for different processes with deutron break up. Among of the most interesting are the processes of $\bar{p}d$-annihilations (see, for example, the experimental data of Ref. (46))

$$\bar{p}d \rightarrow p + \text{pions} ,$$
$$\bar{p}d \rightarrow n + \text{pions} , \tag{128}$$

when the $\bar{p}$ momentum is equal to a few hundred MeV/c the pions in the final state have the momenta peaked around 300 MeV/c. So in the rescattering of pions it is necessary to take into account the elastic scattering and charge-exchange. It is also necessary to take into account the rescattering of $\bar{p}$ in the initial state which may be done by the expression similar to Exp. (112). So the corresponding theorem for the processes (128) can be formulated in a following way

$$\boxed{\sigma^{(p)}_{\bar{p}d} + \sigma^{(n)}_{\bar{p}d} = \sigma_{\bar{p}p} + \sigma_{\bar{p}n} - \delta} \tag{129}$$

where the shadow correction δ is given by

$$\delta = \frac{\sigma^{inel}_{\bar{p}p} \, \sigma^{inel}_{\bar{p}n}}{4\pi a} , \tag{130}$$

$\sigma^{inel}_{\bar{p}N}$ being the $\bar{p}N$ annihilation cross-sections and a corresponds to the choice of the deutron formfactor in the form $S_d(\vec{q}) = e^{-aq^2}$. If we use a more suitable parametrization of the deutron formfactor[47]

$$S(\vec{q}) = C_1 e^{-\gamma_1 \vec{q}^2} + C_2 e^{-\gamma_2 \vec{q}^2} \tag{131}$$

where $C_1 = 0.4$, $C_2 = 0.6$, $\gamma_1 = 4.6 \text{ fm}^{-2}$, $\gamma_2 = 0.88 \text{ fm}^{-2}$, then $\frac{1}{a} = \frac{C_1}{\gamma_1} + \frac{C_2}{\gamma_2}$. With these parameters the cross-sections $\sigma_{\bar{p}p}$, $\sigma_{\bar{p}n}$ and the shadow correction at 60 MeV are equal to

$$\sigma^{tot}_{\bar{p}p} = 240 \text{ mbarn}, \qquad \sigma^{tot}_{\bar{p}n} = 197 \text{ mbarn} ,$$

$\sigma^{inel}_{\bar{p}p} = 133$ mbarn, $\sigma^{inel}_{\bar{p}n} = 106$ mbarn, $\delta = 88$ mbarn. Here the $\bar{p}$-p and $\bar{p}$-d cross-sections are taken from Refs. (46, 48-50), the cross-section $\sigma^{tot}_{\bar{p}n}$ is calculated using the Glauber formula

$$\sigma^{tot}_{\bar{p}d} = \sigma^{tot}_{\bar{p}p} + \sigma^{tot}_{\bar{p}n} - \frac{\sigma^{tot}_{\bar{p}n} \, \sigma^{tot}_{\bar{p}p}}{8\pi a} \tag{132}$$

and it was assumed that $\sigma^{inel}_{\bar{p}n} / \sigma^{inel}_{\bar{p}p} = \sigma^{tot}_{\bar{p}n} / \sigma^{tot}_{\bar{p}p}$.

9. - THE REACTION $\pi^- d \to p\Delta^-$ FOR FORWARD PROTONS AND THE PROBLEM OF AN -ADMIXTURE IN DEUTRON. -

As we could see in Sect. 4 the scattering of the fast particles on nuclei at small angles may very well be described by the Glauber approximation. It is much less understood the scattering on large angles when to the target nucleon it may be transfer a rather big momentum. Here it may be appeared the alternative : (a) the dominance of a few step mechanism with essential contribution of high momentum component of nuclear wave function, i. e. with behaviour of nuclear wave functions at small separations of nucleons ; (b) the dominance of multistep mechanism when the most important contribution is given by normal separations of nucleons inside a nucleus and nucleon resonances are produced as a result of final state interaction.

The mechanism (a) is directly connected with the admixture of the nucleon resonances in the nuclear wave functions. As it was shown in Ref. (51) the N^*-admixture on nuclear wave functions may be of a rather appreciable magnitude $\sim 10\%$ (the probability of about 1-2 %). In Refs. (51-53) it was demonstrated that the contributions of N^*-admixture lead to a better theoretical fit of the magnetic moments of d, ^3H, ^3He ; the e-d scattering data for large momentum transfer and p-d backward scattering data.

For verification of the hypothesis about a big magnitude of the $\Delta\Delta$-admixture in the deuteron wave function it was proposed in Ref. (54) to measure the cross-section of the reaction

$$\pi^- d \to p\Delta^- \tag{133}$$

for forward protons.

If the n-channel Δ-exchange were dominated the cross-section of this reaction then it would be possible to determine the $d \to \Delta^{++}\Delta^-$ vertex using this cross-section. The estimates of Ref. (54) gave the cross-section of 10-100 microbarns at energy of 1 GeV and the probability of $\Delta\Delta$-admixture of about 1 %.

In Ref. (55) the cross-section of reaction (133) was described in the framework of mechanism (b), when the initial π-meson is scattered on proton on large angle and after produced the Δ-isobar through the final state interaction with neutron. In this model the differential cross-section of reaction (133) is given by

$$\frac{d\sigma_{\pi^- d \to p\Delta^-}(S, n)}{dn} = F(n) \frac{d\sigma_{\pi^- p \to p\pi^-}(S_1, n)}{dn} \tag{134}$$

where $\frac{d\sigma_{\pi^- p \to p\pi^-}(S_1, n)}{dn}$ is the π^--p backward scattering cross-section, $S = (p_d + p_\pi)^2$, $S_1 = (\frac{p_d}{2} + p_\pi)^2$, $n = (p_\pi - p_p)^2$. The function is dependent on a model of φd and is present in Fig. 11 for three different models :

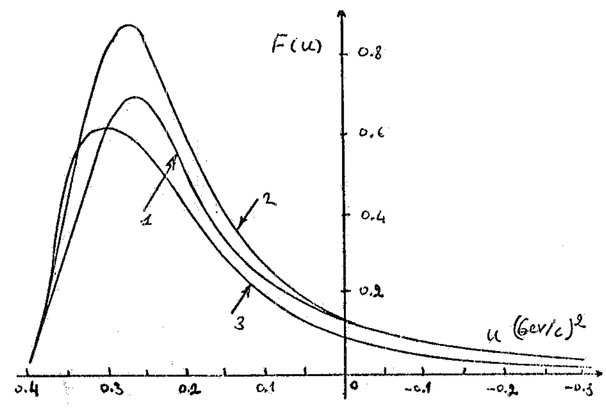


FIG. 11 - The function F(u) for three models of the deuteron wave function :
1 - Hülthen ;
2 - Moravcsik ;
3 - Gaussian .

$$1 - \text{Hülthen} \quad : \quad \varphi_d^H(r) = \frac{N_1}{r}\,(e^{-0.232r} - e^{-1.202r})\,; \tag{135}$$

$$2 - \text{Moravcsik} \quad : \quad \varphi_d^M(r) = \frac{N_2}{r}\,(1 - e^{-1.59r})(e^{-0.232r} - e^{-1.59r})\,; \tag{136}$$

$$3 - \text{Gaussian} \quad : \quad \varphi_d^G(r) = N_3\, e^{-r^2/2R^2}\,, \qquad R = 2.55\ \text{fm}\ . \tag{137}$$

In Fig. 12 there are given the differential cross-sections (134) for different incident momenta p_π = = 1.03; 1.50; 2.38 and 3.0 GeV/c (The curves 1-4 respectively). For the π^--p backward scattering cross-sections the data of Refs. (56-58) were used. For comparison the cross-section of Ref. (54) at p_π = 1 GeV/c is given by dashed line.

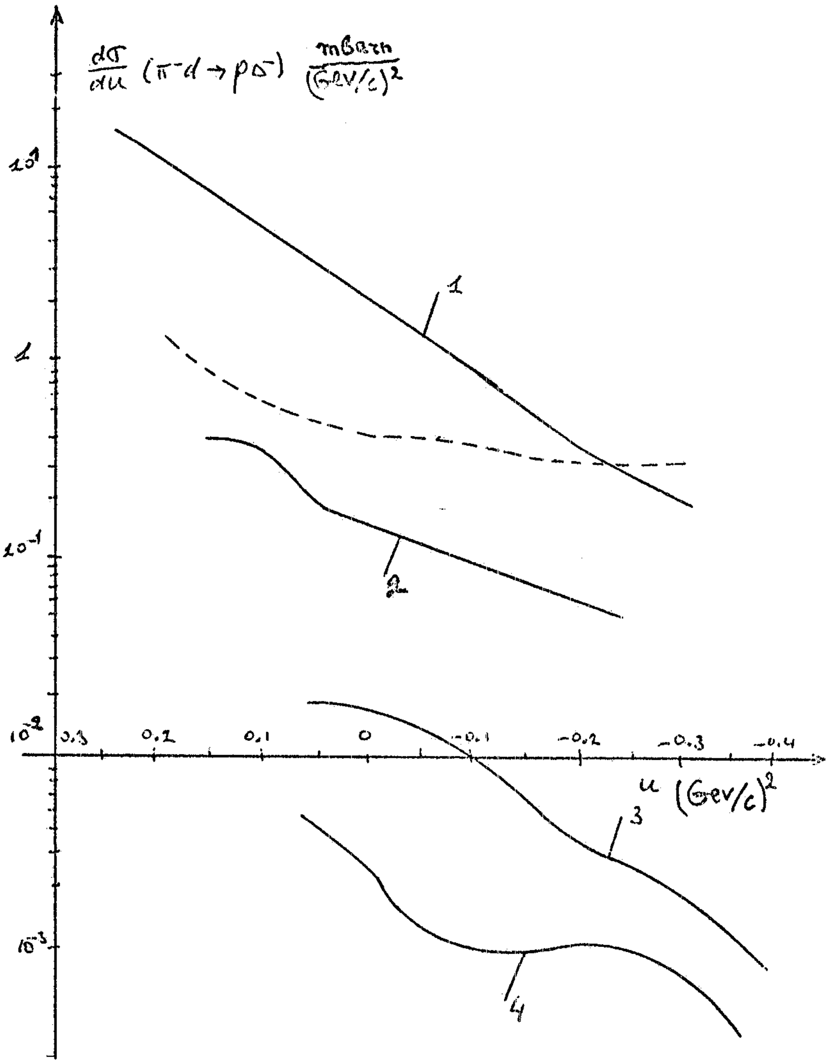


FIG. 12 - The differential cross-section of the reaction $\pi^-d \rightarrow p\Delta^-$ for p_π = 1.03 GeV/c (1); 1.50 GeV/c (2); 2.38 GeV/c (3); 3.0 GeV/c (4)(55). The dashed line is the prediction of Ref. (54) for 1 GeV/c.

The main difference between two models is the energy dependence of the cross-section. In the model of Ref. (54) the cross-section of reaction (133) does not decrease with energy (moreover it increase). In the model of Ref. (55) the energy dependence of the cross-section is determined by the energy dependence of the π^--p backward scattering cross-section. The experimental data of the ITEP group[59] at energy 1.7 GeV are in agreement with the model of Ref. (55). It means that in reaction (133) the dominant role is playing by not small n-p separaions in deutron.

Note that the cross-section of the reaction $\pi^- p \to p\pi^-$ can not be described by the Δ-isobar exchange in n-channel.

This exchange may be written as

$$M^{(\Delta)}(n) = \frac{\lambda^2}{\mu^2} \frac{k'_\mu \bar{U}(p') \Pi_{\mu\nu}(q) U(p) K_\nu}{q^2 - m_\Delta^2} \qquad (138)$$

where μ is the pion mass, λ is the coupling constant $\Delta \to \pi N$,

$$\Pi_{\mu\nu}(q) = (\hat{q}+m_\Delta)\left\{-\delta_{\mu\nu} + \frac{1}{3}\gamma_\mu\gamma_\nu - \frac{1}{3m_\Delta}(q_\mu\gamma_\nu - \gamma_\mu q_\nu) + \frac{2}{3m_\Delta^2} q_\mu q_\nu\right\} \qquad (139)$$

is the Δ-propagator. For $u=0$ the corresponding differential cross-section is equal to

$$\frac{d\sigma^{(\Delta)}_{\pi^- p \to p\pi^-}}{du} = 180\, E \, \frac{\text{mbarn}}{(\text{GeV}/c)^2} \qquad (140)$$

where E in GeV. This cross-section is in a contradiction with experimental data. At E = 1 GeV $(d\sigma_{\pi^- p \to p\pi^-}/du)_{exp} \approx 20$ mbarn/$(\text{GeV}/c)^2$ and decreases fastly with energy. This fact demonstrates impossibility to extract the vertex $d \to \Delta\Delta$ from the data on reaction (133).

It is also necessary to discuss the problem with off-shell effects. If even the Δ-exchange were dominated in the $\pi^- p$ backward cross-section, all the same it would be difficult to compare the magnitude of the $d \to \Delta^{++}\Delta^-$ vertex determined from the reaction $\pi^- p \to p\Delta^-$ with the magnitude of the $\Delta\Delta$-admixture in nuclear wave function which gives the correction to the magnetic moment or to the e-d scattering. As a matter of fact the vertex $d \to \Delta\Delta$ is strongly dependent on the departure of Δ-isobar from the mass-shell $n = m_\Delta^2$. In reaction (133) the difference between u and m_Δ^2 is large $n - m_\Delta^2 \approx m_\Delta^2$. But for the isobar bound inside deuteron this difference is much smaller

$$m_\Delta^2 - (m_\Delta^{bound})^2 \sim m_\Delta^2 - m_r^2 \ .$$

The dependence of the $d \to \Delta\Delta$ vertex squared on u may be described by the u-dependence of F(u) (see Fig. 11). For example, if u changes from 0 to 0.2 $(\text{GeV}/c)^2$ the function F(u) changes on the order of magnitude. Thos means that un ambigous extrapolation of the $d \to \Delta^{++}\Delta^-$ vertex from $u = 0$ to $u \sim m_N^2$ would be strongly dependent on the model of $\Delta\Delta \to NN$ interaction. For the determination of thos vertex the reaction $\pi^- d \to \pi^- \Delta^{++}\Delta^-$ is a more suitable.

But experimental investigation of reaction $\pi^- d \to p\Delta^-$ is very interesting for understanding the mechanism of large angle scattering of fast particles on nuclei. The backward hadron-hadron scattering is satisfactorily described by the reggeon exchanges in u-channel. In the reactions $\pi^- p \to p\pi^-$ and $\pi^- d \to p\Delta^-$ the dominant contribution is given by the Δ-reggeon exchange. For the model of Ref. (55) the coupling $d \to \Delta^- + \Delta_k^{++}$ (where Δ_k^{++} is reggeon) is given by

$$g^2_{d \to \Delta_k^{++}\Delta^-} = F(u)\, g^2_{\pi^- \Delta_k^{++} \to p} \qquad (141)$$

where $g^2_{\pi^- \Delta_k^{++} \to \Delta}$ is the residue of Δ-trajectory for the reaction $\pi^- p \to p\pi^-$.

Exps. (134) (141) may be useful for predictions of double-charge exchange cross-sections on deuteron targets. For example, similar expressions may be written for reactions

$$k^- p \to \Sigma^+\pi^- \qquad \text{and} \qquad k^- d \to \Sigma^+\Delta^-, \ \text{etc.}$$

ACKNOWLEDGEMENT. -

I am grateful to the Director of the Frascati National Laboratories, Prof. G. Bellettini, administration and physicists of CNEN and Rome Section of INFN for hospitality during my staying in Italy in october-december of 1974. I am very grateful to Prof. A. Reale whose encouraging and very stimulating discussions were very important during the writing of these lectures. Discussions with Prof. R. Bizzarri on the problems of p-d annihilations at low energy were decisive for formulation of the theorem on final state interaction for inelastic interactions with deutron. I am also very grateful to S. Stipcich for his great care in the printing of this work.

REFERENCES. -

(1) - M. L. Golberger and K. M. Watson, Collision theory (Wiley, New York, 1964).
(2) - L. A. Kondratyuk, in "Nuclear reactions at high energies", Lectures given in Moscow Ingeneering Physical Institute (Moscow, 1972).
(3) - R. R. Lewis Jr., Phys. Rev. 102, 537, 544 (1956).
(4) - L. Van Hove, Phys. Rev. 95, 249 (1954).
(5) - R. J. Glauber, in "Lectures on theoretical physics", ed. by W. E. Britten (Interscience Publ., New York), Vol. 1, p. 315.
(6) - L. A. Kondratyuk and Yu. A. Simonov, JETP Letters 17, 619 (1973).
(7) - K. M. Watson, Phys. Rev. 89, 575 (1953).
(8) - V. M. Kolybasov and L. A. Kondratyuk, Phys. Letters 39B, 439 (1972); Soviet Jnl. Nuclear Phys. 18, 316 (1973); Proc. 1972 Rome Symp. on Present Status and Novel Development in Nuclear Many Body Problem (1974), p. 347.
(9) - L. S. Kisslinger, Phys. Letters 29B, 211 (1969).
(10) - V. M. Kolybasov and L. A. Kondratyuk, in "Some topics of nuclear structure and methods of its investigation", Lectures given in Moscow Ingeneering Physical Institute (Moscow, 1973).
(11) - K. Göttfried, Ann. Phys. 66, 868 (1971).
(12) - E. A. Abers, H. Burkhardt, V. L. Teplitz and C. Wilkin, Nuovo Cimento 42, 365 (1966).
(13) - J. Pumplin and M. Ross, Phys. Rev. Letters 21, 1778 (1968).
(14) - V. N. Gribov, Soviet Phys.-JETP 56, 892 (1969).
(15) - A. B. Kaydalov and L. A. Kondratyuk, (a) Nuclear Phys. B56, 90 (1973); (b) JETP Letters 15, 170 (1972).
(16) - V. A. Karmanov and L. A. Kondratyuk, JETP Letters 18, 451 (1973).
(17) - W. Czyz and L. C. Maxymon, Ann. Phys. 52, 59 (1969).
(18) - A. Dar and Z. Kirzon, Phys. Letters 37B, 166 (1971).
(19) - H. Kanada, K. Sakai and M. Yasuno, Progr. Theor. Phys. 46, 1071 (1971).
(20) - A. Tekon, Nuclear Phys. B46, 141 (1972).
(21) - J.W. Wilson, Phys. Letters 52B, 149 (1974).
(22) - R. J. Glauber and E. G. Matthiae, Nuclear Phys. B21, 135 (1970).
(23) - G. Bellettini et al., Nuclear Phys. 79, 609 (1966).
(24) - V. V. Jurkin, V. P. Kanavets, L. A. Kondratyuk et al., Preprint ITEP-103 (Moscow, 1973, in russian).
(25) - G. D. Alehazov et al., Preprint ZIYaP no. 48 (Leningrad, 1973, in russian).
(26) - V. E. Starodubsky and O. A. Domchenkov, Phys. Letters 42B, 319 (1972).
(27) - H. Palevsky et al., Phys. Rev. Letters 18, 1200 (1967); Nuclear Phys. A104, 294 (1967).
(28) - G. D. Alehasov et al., Phys. Letters 42B, 121 (1972).
(29) - J. C. Hilbert and G. T. Garvey, Phys. Rev. 135B, 346 (1964).
(30) - G. Alberi, L. Bertocchi and M. A. Gregorio, Nuovo Cimento 10A, 37 (1972).
(31) - E. T. Boschitz, in Proc. Topical Seminar on Interactions of Elementary Particles with Nuclei, Trieste, Sept. 1970 (INFN, Trieste).
(32) - M. G. Albrow et al., Phys. Letters 35B, 247 (1971).
(33) - A. B. Kaydalov, Yu. F. Pirogov, V. A. Khoze and N. L. Ter-Isaakian, JETP Letters 17, 626 (1973).
(34) - N. Mott and H. Massey, The theory of atomic collisions (Clarendon Press, Oxford, 1965).
(35) - H. A. Bethe et al., Ann. Phys. 3, 190 (1958).
(36) - L. A. Kondratyuk, V. B. Kopeliovich and V. V. Soloviev, Soviet Jnl. Nuclear Phys. 14, 1237 (1971).
(37) - L. A. Kondratyuk and V. B. Kopeliovich, JETP Letters 16, 201 (1972).

(38) - L. A. Kondratyuk and V. B. Kopeliovich, JETP Letters 13, 176 (1972).
(39) - W. Czyz, L. Lesniak and M. Wolek, Nuclear Phys. B19, 125 (1970).
(40) - M. Lesniak and L. Lesniak, Preprint CERN - TH 1368 (1971).
(41) - Yu. D. Bayukov et al., Soviet Jnl. Nuclear Phys. 5, 337 (1967).
(42) - L. A. Kondratyuk and I. I. Levintov, Beam of high energy nuclei as a source of polarized nucleons, Preprint ITEP (1974).
(43) - G. Jacob, Th. A. J. Maris, C. Schneider and M. R. Theodoro, Phys. Letters 45B, 181 (1973).
(44) - V. S. Nadejdin, N. I. Petrov and V. I. Satarov, Preprint JINR, E1-7579 (1973).
(45) - E. DiCapua, E. Fiorentino, F. Palmonari, V. Pocci, A. Reale, L. Satta, M. Severi, L. Tau and G. Ubaldini, Preprint Frascati LNF-73/29 (1973).
(46) - R. Bizzarri, P. Guidoni, F. Marcelja, F. Marzano, E. Castelli and M. Sessa, Nuovo Cimento 22A, 225 (1974).
(47) - V. M. Kolybasov and M. S. Marinov, Uspekhi Fizicheskich Nauk (Soviet Phys.-Uspekhi), 109, 137 (1973).
(48) - U. Amaldi Jr., B. Confronto, G. Fidecaro, H. Steiner, G. Baroni, R. Bizzarri, P. Guidoni, V. Rossi, G. Brautti, E. Castelli, M. Ceschia, L. Chersovani and M. Sessa, Nuovo Cimento 46A, 171 (1966).
(49) - B. Confronto, G. Fidecaro, H. Steiner, R. Bizzarri, P. Guidoni, F. Marcelja, G. Brautti, E. Castelli, M. Ceschia and M. Sessa, Nuovo Cimento 54A, 441 (1968).
(50) - R. Bizzarri, B. Confronto, G. C. Gialanella, P. Guidoni, F. Marcelja, E. Castelli, M. Ceschia and M. Sessa, Nuovo Cimento 54A, 456 (1968).
(51) - A. K. Kerman and L. S. Kisslinger, Phys. Rev. 180, 1483 (1969).
(52) - L. S. Kisslinger, Phys. Letters 29B, 211 (1969).
(53) - H. Williams, H. Arenhövel and H. Miller, Phys. Letters 36B, 278 (1971).
(54) - R. H. Math, M. J. Weber and P. K. Kabiz, Phys. Rev. Letters 26, 1404 (1971).
(55) - V. A. Karmanov and L. A. Kondratyuk, Talk at USSR All-union Conference on Nuclear Reactions at High Energy, June 1972; JETP Letters, to be published (1974).
(56) - P. J. Duke et al., Phys. Rev. 149, 1077 (1966).
(57) - P. S. Aplin et al., Nuclear Phys. B32, 253 (1971).
(58) - R. R. Cristensen et al., Phys. Rev. D1, 3050 (1970).
(59) - V. Abramov, I. Dukhovskoy, I. Radkevich et al., Paper presented at Intern. Conf. on High Energy Physics, London, June 1974; to be published.