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L. Casano, A. Marino, G. Matone, M. Roccella, C. Schaerf and A. Tranquilli: PRODUCTION OF A BEAM OF POLARIZED AND MONOCHROMATIC γ RAYS BY COMPTON SCATTERING OF LASER LIGHT AGAINST HIGH ENERGY ELECTRONS.

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L. Casano[(x)], A. Marino, G. Matone, M. Roccella, C. Schaerf[(x)] and A. Tranquilli[(x)]: PRODUCTION OF A BEAM OF POLARIZED AND MONOCHROMATIC γ RAYS BY COMPTON SCATTERING OF LASER LIGHT AGAINST HIGH ENERGY ELECTRONS.

INTRODUCTION. -

We will discuss here the possibility of obtaining a beam of high energy monochromatic and polarized γ rays by Compton scattering of Argon ion Laser light on the high energy electrons circulating in the Adone storage ring in Frascati.

The high power now available from continuous Argon ion Laser and the large circulating current in a storage ring make feasible a γ ray beam of sufficient high intensity and adequate energy to perform advanced experiments in the field of photo-nuclear reactions.

1. - COMPTON SCATTERING ON HIGH ENERGY ELECTRONS. -

1.1. - Kinematics. -

In the Compton scattering on moving electrons the energy of scattered photon is given by the following formula[(1)]:

$$k_2 = k_1 \frac{1 - \beta \cos \vartheta_1}{1 - \beta \cos \vartheta + \frac{k_1}{E}(1 - \cos \vartheta_2)} . \tag{1}$$

(x) - Istituto di Fisica dell'Università di Roma, and INFN, Sezione di Roma.

Our notation is the following:

$k_1(k_2 = k)$ = energy of incident (scattered) photon;

E, β, m = energy, velocity in unity of c and rest mass of the incident electron;

$\vartheta_1 (\vartheta)$ = angle between the directions of the incident electron and incident (scattered) photon;

ϑ_2 = angle between incident and scattered photon.

The maximum energy k_M of the scattered photon is obtained for $\vartheta = 0$, $\vartheta_1 = \vartheta_2 = \pi$, and in the extreme relativistic limit $(E >> m)$ is given by:

(2) $$k_n = \frac{(1+\beta)^2 k_1 \gamma^2}{1 + \frac{1+\beta}{2} z} = \frac{4 k_1 \gamma^2}{1+z}$$

where:

$$\gamma = \frac{E}{m} \quad , \qquad z = 4 \frac{E k_1}{m^2} \quad .$$

In our case E = 1500 MeV, k_1 = 2.54 ev, z = 0.058, k_M = 83 MeV.

For $\vartheta_1 = \pi$, $\vartheta_2 = \pi$ and if $\vartheta \ll 1$ equation (1) becomes:

$$k = 4 k_1 \gamma^2 \left[1 + z + (\gamma\vartheta)^2 \right]^{-1} \quad .$$

This formula gives the energy dependence of the scattered photon as a function of the scattering angle ϑ for very small values of ϑ. The angle ϑ cannot be defined better than the angular dispersion, $\Delta\vartheta$, of the primary electron beam. Therefore, the energy spread of the photon beam scattered in the forward direction is given by the approximate formulas valid for $(\gamma \Delta\vartheta)_2 < 1$:

(3a) $$\Delta k = \frac{4 k_1 \gamma^2}{(1+z)^2} (\gamma \Delta\vartheta)^2 = \frac{k_M}{1+z} (\gamma \Delta\vartheta)^2$$

and with good approximation:

(3b) $$\frac{\Delta k}{k_M} = (\gamma \Delta\vartheta)^2$$

or:

(3c) $$\Delta k = 0.25 \frac{k_M^2}{k_1} (\Delta \vartheta)^2 .$$

Since the root mean square angular divergence of the electrons circulating in a storage ring is proportional to the electron energy:

$$\Delta \vartheta = \sigma \gamma$$

we can rewrite eqs. (3) as follows:

(4a) $$\frac{\Delta k}{k_M} = \sigma^2 \gamma^4$$

or:

(4b) $$\Delta k = \sigma^2 \frac{k_M^3}{16 k_1^2}$$

where σ is a dimensionless parameter determined by the magnetic structure of the storage ring. The approximate formula (4b) suggests two interesting observations:

a) The energy spread of the gamma ray beam increase with the cube of the energy, therefore such a beam will perform much better at low and intermediate energy where the present interest of nuclear physics lay.

b) The energy spread of a gamma ray beam of given energy decreases with the square of the energy of the laser photons. Therefore, the performance of the beam obtainable with a given storage ring improves drastically with the progress of laser technology. For example, using the ultraviolet lines (351.1 nm) of an Argon ion laser instead of the bleu-green line (488 nm, 2.54 eV) the energy resolution of the gamma ray beam improved by a factor of two.

Some numerical results of eqs. (2) and (4) are indicated in Table I for $k_1 = 2.54$ eV and two different values of σ (2).

TABLE I

E MeV	k_M MeV	σ rad	$\Delta\vartheta$ rad	$\frac{\Delta k}{k}$	Δk MeV
1500	83	5.5×10^{-8}	16×10^{-5}	0.22	18
1500	83	1.0×10^{-8}	3×10^{-5}	0.0074	0.6
750	21	5.5×10^{-8}	8×10^{-5}	0.014	0.3
750	21	1.0×10^{-8}	1.5×10^{-5}	0.0005	0.01

The results of Table I are probably too optimistic since it is hard to believe to-day that an angular resolution much better than 5×10^{-5} can actually be obtained. They however represent the most relevant contribution to the energy spread. In fact the energy resolution of the incident electron is better than 10^{-3} and that of the laser photons of the order of 10^{-5}. This latter contribution is completely neglegible. The former has been calculated and is always smaller than that of the angular dispersion.

1.2. - Cross section. -

The total photon-electron cross section in our energy region is given with good approximation by the classical Thompson formula:

$$\sigma_T = \frac{8}{3} \pi r_o^2 = 0.665 \text{ Barn}$$

where $r_o = 2.81777$ fermi, is the classical electron radius.

The differential cross section in the laboratory system is[1]:

$$\frac{d\sigma}{d\Omega} = r_o^2 \frac{2}{m^2 x_1^2} \left[4\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - 4\left(\frac{1}{x_1} + \frac{1}{x_2}\right) - \left(\frac{x_2}{x_1} + \frac{x_1}{x_2}\right) \right] k_1^2$$

where for $\theta_1 = \pi$, x_1 and x_2 are given by:

$$x_1 = 2k_1(E+p)/m^2 = z\,\frac{1+\beta}{2} = z \ ; \qquad x_2 = 2k_2(E - p\cos\theta)/m^2.$$

The differential photon cross section and the differential photon energy spectrum are given in Fig. 1 for our values of the parameters ($E = 1500$ MeV, $k_1 = 2.54$ eV). In the abscissa of the same figure are indicated also the angles at which photons of a given energy are scattered. Therefore, from this figure we can visualize the energy spectrum of the secondary photon beam as a function of its angular acceptance.

In our case the resulting photon spectrum is indicated by the solid line of Fig. 2 where collimation of the final photon beam has been taken equal to 10^{-4} radiants.

1.3. - Photon polarization. -

For very relativistic electrons their elicity is a good quantum number. Therefore, the electron spin-flip amplitude is negligible and the backward scattered photons retain their original polarization. At other angles the photon polarization can change due to the role of orbital angular momentum. If we use a laser with Brewster windows which produce plane polarized light, the high energy photons produced by Compton scattering will have similar polarization. The polarization has been calculated as a function of the final photon energy and it is given in Appendix. The numerical results, for our values of the kinematical variables, are indicated in Fig. 3. For a scattering angle of 10^{-4} rad this polarization is larger than 0.98.

2. - LASER CAVITY. -

In order to obtain a beam of high intensity and high energy photons we must start with an incident laser beam of high frequency and high intensity.

Our laser is an Ion Argon Laser which produces a 2.54 eV photon beam with a power of 3.7 W, using an output mirror of 0.05 transmittivity. If the interaction region is arranged inside the cavity it is possible to produce a rays beam of adequate intensity for nuclear physics experiments.

To overlap the cavity with the Adone straight section, 6 meters long, it is necessary to realize a Laser cavity of about 15 meters and the stretch containing the interaction region must be kept at the high vacuum of the storage ring; therefore a window must be introduced and this raises the losses of the optical cavity.

We can use a window made out of high purity material with anti-reflection coating and by treating it as a lens we can compensate this disadvantage by optimizing the beam profile in the interaction region.

The linear energy density δ inside the cavity is related to the total losses Γ by the equation[3]

$$\delta = a \left(\frac{\beta}{\beta_c} - 1 \right)$$

where a depends upon the beam geometry in the active medium, β is the pumping rate and β_c the critical pumping rate directly proportional to the total cavity losses Γ.

In our laser, where cavity losses are ≤ 0.1, β / β_c is much greater than 1, and we can assume δ to be inversely proportional to Γ.

Under this approximation, a Γ value of 0.01 is enough to obtain the stored linear energy density inside the cavity approximately one hundred times bigger than the standard output power (3.7 Watt). The effect of the window on the cavity losses has been estimated by comparing the measured output powers of the two cavities of Fig. 4. These are calculated to produce a laser beam with identical geometries in the active medium and perfectly equivalent with respect to the diffraction losses on the mirrors.

Assuming that the total loss in the cavity b) is equal to the transmissivity of the mirror (0.05), the measured external powers for the two cavities, $P_a = 2.0$ Watt and $P_b = 2.3$ Watt respectively, show that the loss introduced by the lens cannot exceed the value of 0.01.

3. - PHOTON YIELD. -

The high energy photon yield is determined by the number of electrons, the number of incident photons and the geometry of the interaction region. It is given by

$$\frac{d\dot{n}}{dk_2} = (1 - \beta \cos \theta) \frac{d\sigma}{dk_2} \; c \int_r n_e \, n_\gamma \, dr$$

where :

$\frac{d\dot{n}}{dk_2}$ indicates the photon yield per MeV and per second ;

c velocity of flight ;

n_e electron density per unit volume ;

n_γ photon density per unit volume ;

$1 - \beta \cos \vartheta \approx 2$ (5) takes into account the motion of the electrons in the laboratory system.

The integral is extended over the entire volume of the interaction region.

In the first approximation we will assume that the electron and photon densities remain constant in the length of the interaction region and are rapidly varying functions of the transversal variables x and y. These latter variations are assumed to have two dimensional gaussian distributions of the form :

$$n_e = n_{oe} \exp - \left(\frac{x^2}{\xi^2} + \frac{y^2}{\eta^2} \right) , \qquad n_\gamma = n_{o\gamma} \exp - \left(\frac{x^2 + y^2}{\rho^2} \right)$$

where ξ, η and ρ are slowly varying functions of the coordinate z in the beam direction and will be assumed constant here. Hence, after the integration we get :

$$(5) \qquad \frac{d\dot{n}}{dk_2} = 2 \frac{d\sigma}{dk_2} \frac{N_e P}{2\pi^2 k_1 e} \frac{1}{R} (\xi^2 + \rho^2)^{-1/2} (\eta^2 + \rho^2)^{-1/2}$$

where :

N_e is the number of electrons circulating in the storage ring;

P is the laser beam power in Watt;

l is the interaction region length in meter;

R is the mean geometrical radius of the electron's orbit in meters;

e is the electron charge: 1.6×10^{-19} Coulomb.

Formula (5) must be corrected for the effect of beam instability in the storage ring. This can be accounted for by introducing a new parameter χ which represents the short term root mean square displacement of the beam from the equilibrium orbit. With this correction our final formula is:

$$\frac{d\dot{n}}{dk_2} = 2\left(\frac{d\sigma}{dk_2}\frac{1}{\sigma_T}\right) N_e P \frac{\sigma_T}{2\pi^2 k_1 e} \frac{l}{R} (\xi^2 + \chi^2 + \rho^2)^{-1/2} (\eta^2 + \chi^2 + \rho^2)^{-1/2}$$

where $\sigma_T = 0.665$ barn is the Thompson total cross section. For the storage ring Adone we may assume[4]:

$$N_e = 2 \times 10^{11}, \quad R = 16.7\ \mathrm{m}, \quad l = 6.0\ \mathrm{m}, \quad \xi = 1.5\ \mathrm{mm},$$

$$\eta = 0.5\ \mathrm{mm}, \quad \chi = 0.5\ \mathrm{mm}.$$

For our calculated laser cavity we have

$$P = 250\ \mathrm{W}, \quad k_1 = 2.54\ \mathrm{eV}, \quad \rho = 1\ \mathrm{mm}$$

which gives

$$\frac{d\dot{n}}{dk_2} = 1.3 \times 10^8 \left(\frac{d\sigma}{dk_2}\frac{1}{\sigma_T}\right).$$

The number of photons per unit in a given energy interval Δk is given by:

$$\dot{n} = 1.3 \times 10^8 \left(\frac{d\sigma}{dy} y\right) \frac{1}{\sigma_T} \frac{\Delta k_2}{k_2}, \qquad y = \frac{k_2}{k_{2max}}.$$

From Fig. 1 and for an energy interval of 8%:

$$\frac{d\sigma}{dy} y = 0.84 \text{ barn} ; \quad \frac{\Delta k}{k} = 0.08 ; \quad \dot{n} = 1.4 \times 10^7 \text{ photon sec}^{-1} .$$

4. - BACKGROUND PHOTONS. -

The bremsstrahlung of the circulating electrons against the nuclei of the residual gas in the storage ring vacuum chamber is the only appreciable source of background photons. The total power radiated in this way is given by:

$$W_B = t \frac{N_e c}{2 \pi R} E a$$

where many symbols have been previously defined and:

a is the fraction of gamma rays accepted by our solid angle. In our case we can assume: $\Delta\Omega = \pi\vartheta^2 \approx 3 \times 10^{-8}$ ster and from Fig. 4 of ref. (6) we obtain:

$$a = 0.08 ;$$

t is the thickness of gas traversed by the electrons in a straight section measured in units of its radiation length:

$$t = \frac{l}{X_o} \frac{p}{p_a} = 2.6 \times 10^{-14} ;$$

$l = 6$ m is the length of the straight section;

$p = 10^{-9}$ torr is the pressure in the vacuum chamber;

$X_o = 3029$ m is the radiation length of the residual gas assumed to be air;

$p_a = 765$ torr is the atmospheric pressure.

The final result is:

$$W_B = 1.8 \times 10^6 \text{ MeV/sec}$$

at an electron energy of 1500 MeV.

This number must be compared with the total power in the beam of Compton scattered photons:

$$W_C = \dot{n}k = 1.2 \times 10^9 \text{ MeV/sec} .$$

Therefore the ratio of the background power to the power of the signal is given by:

$$r_w = \frac{W_B}{W_C} = 0.15 \times 10^{-2} .$$

We can also calculate the number of background photons present in an energy interval equal to that of the signal. This is given by:

$$\dot{n}_B = \frac{\Delta k}{k} \frac{W_B}{E}$$

for an energy interval of 8 % the result is:

$$\dot{n}_B = 96 \text{ sec}^{-1} .$$

The ratio of the background photons to the useful ones in the same energy interval is given by:

$$r_n = \frac{\dot{n}_B}{n} = 6.8 \times 10^{-6} .$$

APPENDIX. - Photon polarization.

An extremely important property of photons scattered from moving electron is that they hold their original polarization.

To see this we shall introduce the Stokes parameters in the form of a four vector $\overleftrightarrow{p} = (I, P_1, P_2, P_3) = (I, \vec{P})$ where I is the beam intensity, $\vec{P} = I\vec{\xi}$ and the ξ_i are the usual Stokes parameters given by the equations:

$$W_i = \frac{1}{2}(1 + \xi_i) \qquad (i = 1, 2, 3)$$

where:

W_1 is the probability to find a linear polarization along the x-axis;
W_2 a linear polarization of $\pi/4$ in the x, y plane; and
W_3 a circular polarization in the same plane.

The number

$$\xi = \frac{|\vec{P}|}{I} = |\xi|$$

measures the beam polarization.

Being k_1 and k_2 the momenta of the incoming and scattered photon we shall choose as reference planes

$$\text{(A. 1)} \qquad \vec{\chi}_1^{(1)} = \frac{\vec{k}_1 \wedge \vec{k}_2}{|\vec{k}_1 \wedge \vec{k}_2|} \qquad \vec{\chi}_2^{(1)} = \frac{\vec{k}_1 \wedge \vec{\chi}_1^{(1)}}{|\vec{k}_1|}$$

$$\vec{\chi}_1^{(2)} = \frac{\vec{k}_1 \wedge \vec{k}_2}{|\vec{k}_1 \wedge \vec{k}_2|} \qquad \vec{\chi}_2^{(2)} = \frac{\vec{k}_2 \wedge \vec{\chi}_1^{(2)}}{|\vec{k}_2|}$$

for the Stokes vectors of the incident and scattered photon respectively.

If we deal only with plane polarization, only the first two components must be retained. In this case the Stokes vector of the scattered photon is related to the incoming vector $(1, \vec{\xi}\,')$ assumed of unitary intensity by

(A. 2) $$\left(\frac{I'}{P'}\right) = T\left(\frac{1}{\xi'}\right)$$

where T is a 3 x 3 matrix given by

(A. 3) $$T = \frac{1}{2} T_o^2 \left(\frac{k_2'}{k_1'}\right)^2 \begin{vmatrix} \frac{k_2'}{k_1'} + \frac{k_1'}{k_2'} - \mathrm{sen}^2\,\theta' & \mathrm{sen}^2\,\theta' & 0 \\ \mathrm{sen}^2\,\theta' & 1 + \cos\theta' & 0 \\ 0 & 0 & 2\cos\theta' \end{vmatrix}$$

The two vectors are referred to the two different reference frames attached to the plane of scattering previous defined.

k_1', k_2', θ' are respectively the energies of incoming and scattered photons and the angle between them in a reference frame where the electron is at rest. It can be shown that if the photon and the electron were moving at angle $\theta = 0$ or $\theta = \pi$ before the collision; the equation (A. 2) and (A. 3) are still valid with respect to references frames (A. 1) attached to the laboratory, provided that k_2', k_1', θ' are expressed in the system where the electron is at rest.

Then these quantities must be related to those of the laboratory by the equations

$$\cos\theta' = \frac{k_1\left(1 - \frac{k_2}{E}\right) - \frac{1}{1+\beta}\left(\frac{k_1}{E}\right)k_2}{k_1\left(\beta - \frac{k_2}{E}\right)}$$

$$k_1' = k_1 \frac{1+\beta}{\sqrt{1 = \beta^2}} \qquad k_2' = \frac{k_1'}{1 + \frac{k_1'}{m}(1 - \cos\theta')}$$

If we are interested only to very small angle of scattering in the forward ($\theta \sim 0$) and backward ($\theta = \pi$) directions, can assume a common fixed reference frame for the incoming and scattered photons, such that the reference frames attached to the scattering plane for the incoming and outgoing photons are rotated approximatively of the same angle φ with respect to the fixed reference frame. Then, if $(\frac{1}{\xi} p)$ and $(\frac{I}{P} p)$ express the two Stokes vectors in this frame we will have

(A. 4) $$\left(\frac{I}{P} p\right) = S_{+(-)} \left(\frac{1}{\xi} p\right)$$

where

(A. 5) $$S_{+(-)} = M_{\varphi} T_{+(-)} M_{-\varphi}$$
$$\dot{T} = T_{+}(\theta) = T_{-}(\theta + \pi)$$

the sign +(-) must be assumed in the forward (backward) hemisphere and

(A. 6) $$M_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\varphi & \text{sen } 2\varphi \\ 0 & -\text{sen } \varphi & \cos 2\varphi \end{pmatrix}$$

is the matrix that operates on Stokes vectors when the reference frame is ritated by φ (7).

By sostituting (A. 5) and (A. 6) in equation (A. 4) and averaging over φ that cannot be practically experimentally distinguisched at $\theta \sim 0$, $\theta \sim \pi$ we obtain

$$\langle \xi_1^{(2)} \rangle = \frac{\overline{P}_1}{I} = \frac{(1 + |\cos \theta'|)^2}{2\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \text{sen}^2 \theta'\right)} \xi_1^{(1)}$$

$$\langle \xi_2^{(2)} \rangle = \frac{\overline{P}_2}{I} = \frac{(1 + |\cos \theta'|)^2}{2\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \text{sen}^2 \theta'\right)} \xi_2^{(1)} .$$

For the polarization we have

$$\langle \xi^2 \rangle = \left(\frac{|P|}{I} \right)^2 = \frac{0.25\,(1+|\cos\theta'|)^4\,(\xi^{(1)})^2}{\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \mathrm{sen}^2\,\theta' \right)^2} \qquad \text{(A. 7)}$$

The mean value $\langle \xi \rangle$ has been graphed in Fig. 3 by using equation (A. 7).

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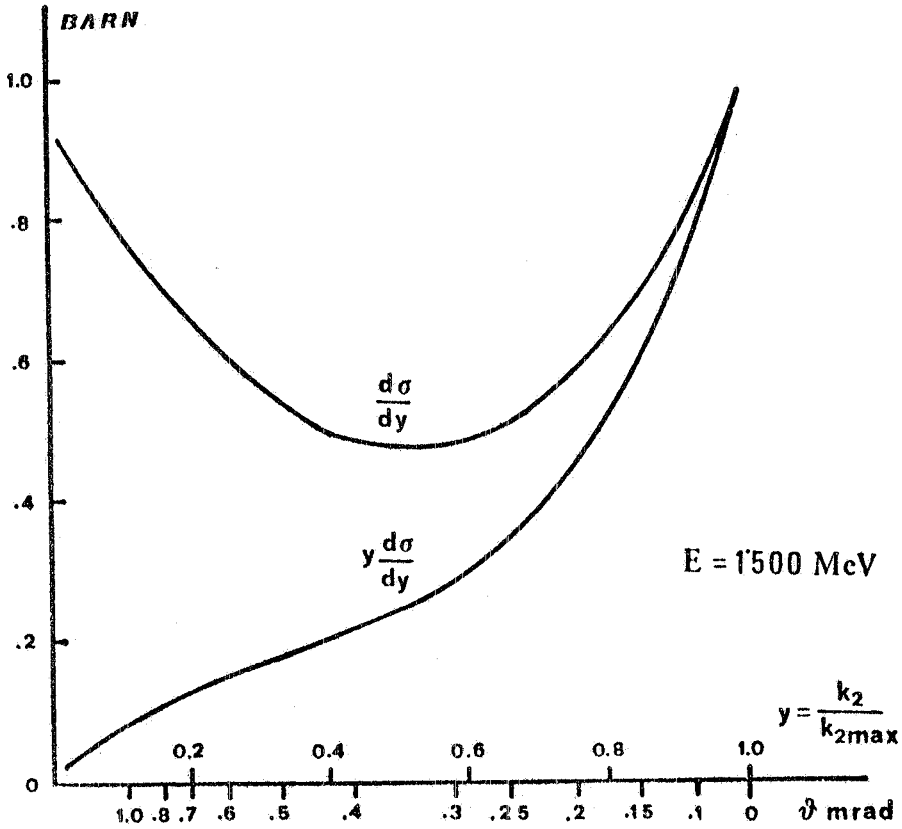
BARN
1.0
.8
.6
.4
.2
0
dσ/dy
y dσ/dy
E = 1500 MeV
y = k2/k2max
0.2
0.4
0.6
0.8
1.0
1.0 .8 .7 .6 .5 .4 .3 .25 .2 .15 .1 0
ϑ mrad

FIG. 1

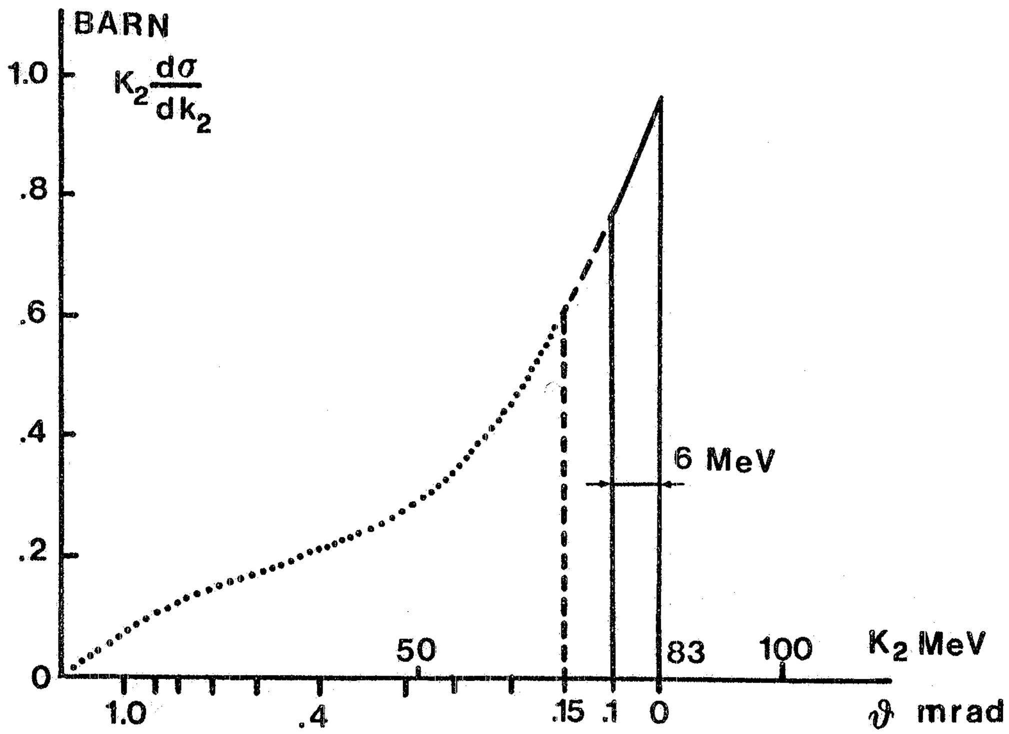
BARN
K2 dσ/dk2
1.0
.8
.6
.4
.2
0
6 MeV
50
83
100
K2 MeV
1.0
.4
.15 .1 0
ϑ mrad

FIG. 2

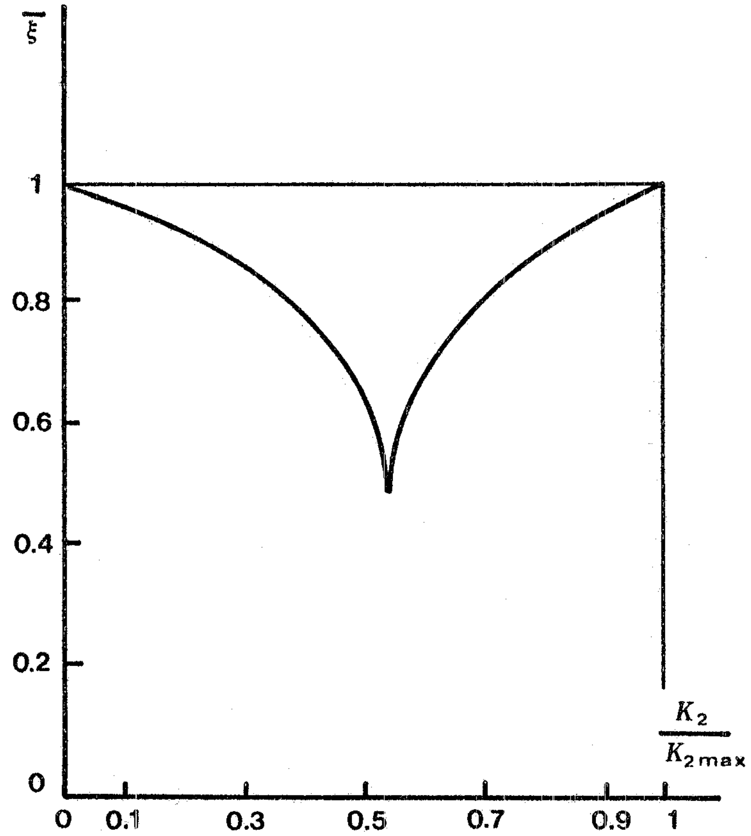

FIG. 3

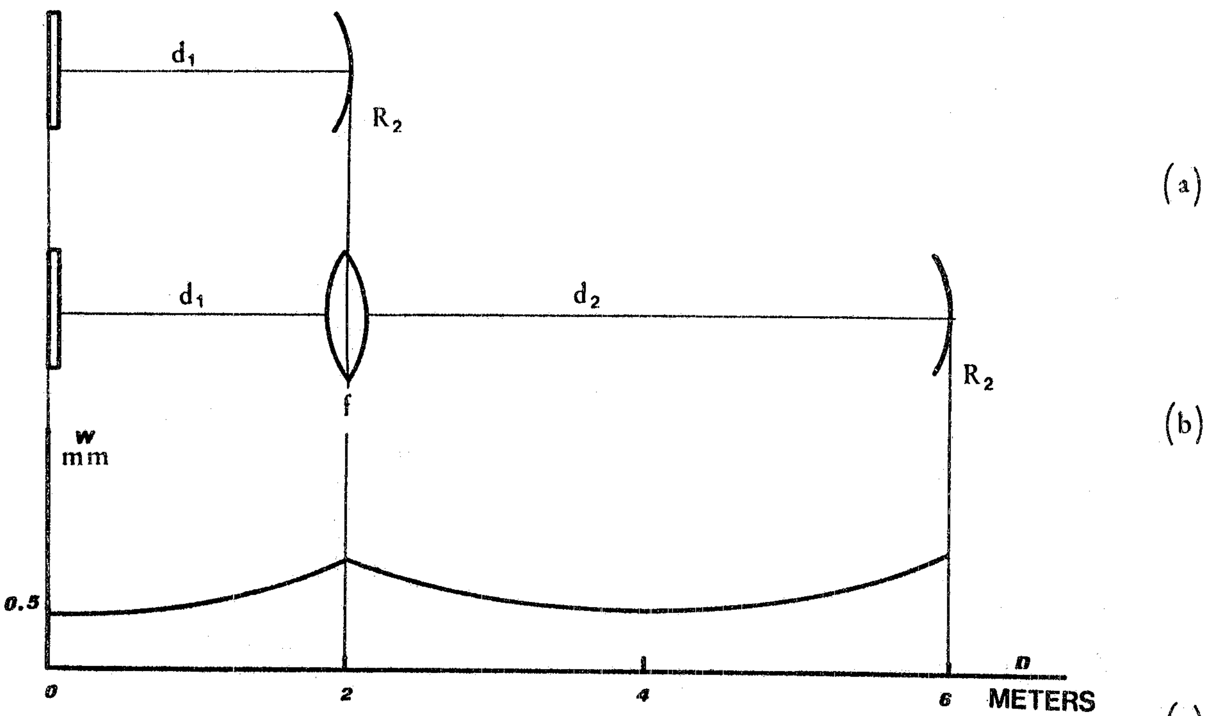

FIG. 4 - Sketch of the cavity with and without lens and beam profile inside them.

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PRODUCTION OF A BEAM OF POLARIZED AND MONOCHROMATIC γ RAYS BY A COMPTON SCATTERING OF LASER LIGHT AGAINST HIGH ENERGY ELECTRONS

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INTRODUCTION

We will discuss here the possibility of obtaining a beam of high energy monochromatic and polarized γ rays by Compton scattering of Argon ion Laser light on the high energy electrons circulating in the Adone storage ring in Frascati.

The high power now available from continuous Argon ion Laser and the large circulating current in a storage ring make feasible a γ ray beam of sufficient high intensity and adequate energy to perform advanced experiments in the field of photo-nuclear reactions.

1. - COMPTON SCATTERING ON HIGH ENERGY ELECTRONS.

1.1 - Kinematics.

In the Compton scattering on moving electrons the energy of scattered photon is given by the following formula[1]:

(1)
$$k_2 = k_1 \frac{1 - \beta \cos \vartheta_1}{1 - \beta \cos \vartheta + \frac{k_1}{E}(1 - \cos \vartheta_2)}$$

Our notation is the following:

$k_1 (k_2' = k)$ = energy of incident (scattered) photon;

E, β, m = energy, velocity in unity of c and rest mass of the incident electron;

$\vartheta_1 (\vartheta)$ = angle between the directions of the incident electron and incident (scattered) photon;

ϑ_2 = angle between incident and scattered photon.

The maximum energy k_M of the scattered photon is obtained for $\vartheta = 0$, $\vartheta_1 = \vartheta_2 = \pi$, and in the extreme realativistic limit ($E >> m$) is given by:

(2)
$$k_n = \frac{(1+\beta)^2 k_1 \gamma^2}{1 + \frac{1+\beta}{2} z} = \frac{4 k_1 \gamma^2}{1+z}$$

where:
$$\gamma = \frac{E}{m} , \qquad z = 4 \frac{E k_1}{m^2}$$

In our case E = 1500 MeV, k_1 = 2.54 ev, z = 0.058, k_M = 83 MeV.

For $\vartheta_1 = \pi$, $\vartheta_2 = \pi$ and if $\vartheta \ll 1$ equation (1) becomes:

$$k = 4 k_1 \gamma^2 \left[1 + z + (\gamma \vartheta)^2 \right]^{-1} .$$

This formula gives the energy dependence of the scattered photon as a function of the scattering angle ϑ for very small values of ϑ. The angle ϑ cannot be defined better than the angular dispersion, $\Delta\vartheta$, of the primary electron beam. Therefore, the energy spread of the photon beam scattered in the forward direction is given by the approximate formulas valid for $(\gamma \Delta \vartheta)^2 < 1$:

(3a)
$$\Delta k = \frac{4 k_1 \gamma^2}{(1+z)^2} (\gamma \Delta \vartheta)^2 = \frac{k_M}{1+z} (\gamma \Delta \vartheta)^2$$

and with good approximation:

(3b)
$$\frac{\Delta k}{k_M} = (\gamma \Delta \vartheta)^2$$

or:

(3c)
$$\Delta k = 0.25 \frac{k_M^2}{k_1} (\Delta \vartheta)^2 .$$

Since the root mean square angular divergence of the electrons circulating in in a storage ring is proportional to the electron energy:

$$\Delta\vartheta = \sigma\gamma$$

we can rewrite eqs. (3) as follows:

(4a)
$$\frac{\Delta k}{k_M} = \sigma^2 \gamma^4$$

or:

(4b)
$$\Delta k = \sigma^2 \frac{k_M^3}{16\, k_1^2}$$

where σ is a dimensionless parameter determined by the magnetic structure of the storage ring. The approximate formula (4b) suggests two interesting observations:

a) The energy spread of the gamma ray beam increase with the cube of the energy, therefore such a beam will perform much better at low and intermediate energy where the present interest of nuclear physics lav.

b) The energy spread of a gamma ray beam of given energy decreases with the square of of the energy of the laser photons. Therefore, the performance of the beam obtainable with a given storage ring improves drastically with the progress of laser technology. For example, using the ultraviolet lines (351.1 nm) of an Argon ion laser instead of the bleu-green line (488 nm, 2.54 eV) the energy resolution of the gamma ray beam improved by a factor of two.

Some numerical results of eqs. (2) and (4) are indicated in Table I for k_1 = = 2.54 eV and two different values of σ [(2)].

TABLE I

E MeV	k_M MeV	σ rad	$\Delta\vartheta$ rad	$\frac{\Delta k}{k}$	Δk MeV
1500	83	5.5×10^{-8}	$16\ \times10^{-5}$	0.22	18
1500	83	1.0×10^{-8}	$3\ \times10^{-5}$	0.0074	0.6
750	21	5.5×10^{-8}	$8\ \times10^{-5}$	0.014	0.3
750	21	1.0×10^{-8}	1.5×10^{-5}	0.0005	0.01

The results of Table I are probably too optimistic since it is hard to beleive to-day that an angular resolution much better than 5×10^{-5} can actually be obtained. They however represent the most relevant contribution to the energy spread. In fact the energy resolution of the incident electron is better than 10^{-3} and that of the laser photons of the order of 10^{-5}. This latter contribution is completely neglegible. The former has been calculated and is always smaller than that of the angular dispersion.

1.2 - Cross section. -

The total photon-electron cross section in our energy region is given with good approximation by the classical Thompson formula:

$$\sigma_T = \frac{8}{3} \pi r_o^2 = 0.665 \text{ Barn}$$

where $r_o = 2.81777$ fermi, is the classical electron radius.

The differential cross section in the laboratory system is[1]:

$$\frac{d\sigma}{d\Omega} = r_o^2 \frac{2}{m^2 x_1^2} \left[4\left(\frac{1}{x_1} + \frac{1}{x_2}\right)^2 - 4\left(\frac{1}{x_1} + \frac{1}{x_2}\right) - \left(\frac{x_2}{x_1} + \frac{x_1}{x_2}\right) \right] k_1^2$$

where for $\theta_1 = \pi$, x_1 and x_2 are given by:

$$x_1 = 2k_1 (E+p)/m^2 = z \frac{1+\beta}{2} = z \; ; \qquad x_2 = 2k_2(E-p\cos\theta)/m^2 .$$

The differential photon cross section and the differential photon energy spectrum are given in Fig. 1 for our values of the parameters ($E = 1500$ MeV, $k_1 = 2.54$ eV). In the abscissa of the same figure are indicated also the angles at which photons of a given energy are scattered. Therefore, from this figure we can visualize the energy spectrum of the secondary photon beam as a function of its angular acceptance.

In our case the resulting photon spectrum is indicated by the solid line of Fig. 2 where collimation of the final photon beam has been taken equal to 10^{-4} radiants.

1.3. - Photon polarization. -

For very relativistic electrons their elicity is a good quantum number. Therefore, the electron spin-flip amplitude is negligible and the backward scattered photons retain their original polarization. At other angles the photon polarization can change due to the role of orbital angular momentum. If we use a laser with Brewster windows which produce plane polarized light, the high energy photons produced by Compton scattering will have similar polarization. The polarization has been calculated as a function of the final photon energy and it is given in Appendix. The numerical results, for our values of the kinematical variables, are indicated in Fig. 3. For a scattering angle of 10^{-4} rad this polarization is larger than 0.98.

2. - LASER CAVITY. -

In order to obtain a beam of high intensity and high energy photons we must start with an incident laser beam of high frequency and high intensity.

Our laser is an Ion Argon Laser which produces a 2.54 eV photon beam with a power of 3.7 W, using an output mirror of 0.05 transmittivity. If the interaction region is arranged inside the cavity it is possible to produce a rays beam of adequate intensity for nuclear physics experiments.

To overlap the cavity with the Adone straight section, 6 meters long, it is necessary to realize a Laser cavity of about 15 meters and the stretch containing the interaction region must be kept at the high vacuum of the storage ring: therefore a window must be introduced and this raises the losses of the optical cavity.

We can use a window made out of high purity material with antireflection coating and by treating it as a lens we can compensate this disadvantage by optimizing the beam profile in the interaction region.

The linear energy density δ inside the cavity is related to the total losses Γ by the equation[3]

$$\delta = a \left(\frac{\beta}{\beta_c} - 1 \right)$$

where a depends upon the beam geometry in the active medium, β is the pumping rate and β_c the critical pumping rate directly proportional to the total cavity losses Γ.

In our laser, where cavity losses are ≤ 0.1, β / β_c is much greater than 1, and we can assume δ to be inversely proportional to Γ.

Under this approximation, a Γ value of 0.01 is enough to obtain the stored linear energy density inside the cavity approximately one hundred times bigger than the standard output power (3.7 Watt). The effect of the window on the cavity losses has been estimated by comparing the measured output powers of the two cavities of Fig. 4. These are calculated to produce a laser beam with identical geometries in the active medium and perfectly equivalent with respect to the diffraction losses on the mirrors.

Assuming that the total loss in the cavity b) is equal to the transmissivity of the mirror (0.05), the measured external powers for the two cavities, $P_a = 2.0$ Watt and $P_b = 2.3$ Watt respectively, show that the loss introduced by the lens cannot exceed the value of 0.01.

3. - PHOTON YIELD. -

The high energy photon yield is determined by the number of electrons, the number of incident photons and the geometry of the interaction region. It is given by

$$\frac{d\dot{n}}{dk_2} = (1 - \beta\cos\theta)\frac{d\sigma}{dk_2}\, c \int_r n_e n_\gamma \, dr$$

where:

$\frac{d\dot{n}}{dk_2}$ indicates the photon yield per MeV and per second;

c velocity of flight;

n_e electron density per unit volume;

n_γ photon density per unit volume;

$1 - \beta\cos\vartheta \approx 2$ (5) takes into account the motion of the electrons in the laboratory system.

The integral is extended over the entire volume of the interaction region.

In the first approximation we will assume that the electron and photon densities remain constant in the length of the interaction region and are rapidly varying functions of the transversal variables x and y. These latter variations are assumed to have two dimensional gaussian distributions of the form:

$$n_e = n_{oe} \exp - \left(\frac{x^2}{\xi^2} + \frac{y^2}{\eta^2}\right), \qquad n_\gamma = n_{o\gamma} \exp - \left(\frac{x^2 + y^2}{\rho^2}\right)$$

where ξ, η and ρ are slowly varying functions of the coordinate z in the beam direction and will be assumed constant here. Hence, after the integration we get:

$$(5) \qquad \frac{d\dot{n}}{dk_2} = 2\frac{d\sigma}{dk_2}\frac{N_e P}{2\pi^2 k_1 e}\frac{l}{R}(\xi^2 + \rho^2)^{-1/2}(\eta^2 + \rho^2)^{-1/2}$$

where:

N_e is the number of electrons circulating in the storage ring;

P is the laser beam power in Watt;

l is the interaction region length in meter;

R is the mean geometrical radius of the electron's orbit in meters;

e is the electron charge: 1.6×10^{-19} Coulomb.

Formula (5) must be corrected for the effect of beam instability in the storage ring. This can be accounted for by introducing a new parameter χ which represents the short term root mean square displacement of the beam from the equilibrium orbit. With this correction our final formula is:

$$\frac{d\dot{n}}{dk_2} = 2\left(\frac{d\sigma}{dk_2}\,\frac{1}{\sigma_T}\right) N_e P \frac{\sigma_T}{2\pi^2 k_1 e}\,\frac{1}{R}\,(\xi^2+\chi^2+\rho^2)^{-1/2}(\eta^2+\chi^2+\rho^2)^{-1/2}$$

where $\sigma_T = 0.665$ barn is the Thompson total cross section. For the storage ring Adone we may assume[(4)]:

$N_e = 2 \times 10^{11}$, $R = 16.7$ m, $1 = 6.0$ m, $\xi = 1.5$ mm,

$\eta = 0.5$ mm, $\chi = 0.5$ mm.

For our calculated laser cavity we have

$P = 250$ W, $k_1 = 2.54$ eV, $\varrho = 1$ mm

which gives

$$\frac{d\dot{n}}{dk_2} = 1,3 \times 10^8 \left(\frac{d\sigma}{dk_2}\,\frac{1}{\sigma_T}\right).$$

The number of photons per unit in a given energy interval Δk is given by:

$$\dot{n} = 1,3 \times 10^8 \left(\frac{d\sigma}{dy}\,y\right)\frac{1}{\sigma_T}\,\frac{\Delta k_2}{k_2}, \qquad y = \frac{k_2}{k_{2max}}.$$

From Fig. 1 and for an energy interval of 8%:

$$\frac{d\sigma}{dy}\,y = 0,84 \text{ barn}; \quad \frac{\Delta k}{k} = 0,08; \quad \dot{n} = 1,4 \times 10^7 \text{ photon sec}^{-1}.$$

4. - BACKGROUND PHOTONS. -

The bremsstrahlung of the circulating electrons against the nuclei of the residual gas in the storage ring vacuum chamber is the only appreciable source of background photons. The total power radiated in this way is given by:

$$W_B = t\,\frac{N_e c}{2\pi R}\,E a$$

where many symbols have been previously defined and:

a is the fraction of gamma rays accepted by our solid angle. In our case we can assume: $\Delta\Omega = \pi\vartheta^2 \approx 3 \times 10^{-8}$ ster and from Fig. 4 of ref. (6) we obtain:

$$a = 0.08;$$

t is the thickness of gas traversed by the electrons in a straight section measured in units of its radiation length:

$$t = \frac{1}{X_o} \frac{p}{p_a} = 2.6 \times 10^{-14};$$

1 = 6 m is the length of the straight section;

p = 10^{-9} torr is the pressure in the vacuum chamber;

X_o = 3029 m is the radiation length of the residual gas assumed to be air;

p_a = 765 torr is the atmosphere pressure.

The final result is:

$$W_B = 1.8 \times 10^6 \text{ MeV/sec}$$

at an electron energy of 1500 MeV.

This number must be compared with the total power in the beam of Compton scattered photons:

$$W_C = \dot{n}k = 1.2 \times 10^9 \text{ MeV/sec.}$$

Therefore the ratio of the background power to the power of the signal is given by:

$$r_w = \frac{W_B}{W_C} = 0.15 \times 10^{-2}$$

We can also calculate the number of background photons present in an energy interval equal to that of the signal. This is given by:

$$\dot{n}_B = \frac{\Delta k}{k} \frac{W_B}{E}$$

for an energy interval of 8% the result is:

$$\dot{n}_B = 96 \text{ sec}^{-1}$$

The ratio of the background photons to the useful ones in the same energy interval is given by:

$$r_n = \frac{\dot{n}_B}{\dot{n}} = 6.8 \times 10^{-6}$$

APPENDIX. - Photon polarization.

An extremely important property of photons scattered from moving electron is that they hold their original polarization.

To see this we shall introduce the Stokes parameters in the form of four vector $\overleftrightarrow{p} = (I, P_1, P_2, P_3) = (I, \vec{P})$ where I is the beam intensity, $\vec{P} = I\,\vec{\xi}$ and the ξ_i are the usual Stokes parameters given by the equations:

$$W_i = \frac{1}{2}(1 + \xi_i) \qquad (i = 1, 2, 3)$$

where:

W_1 is the probability to find a linear polarization along the x-axis;

W_2 a linear polarization of $\pi/4$ in the x, y plane; and

W_3 a circular polarization in the same plane.

The number

$$\xi = \frac{|\vec{P}|}{I} = |\xi|$$

measures the beam polarization.

Being k_1 and k_2 the momenta of the incoming and scattered photon we shall choose as reference planes

$$\text{(A.1)} \qquad \vec{\chi}_1^{(1)} = \frac{\vec{k}_1 \wedge \vec{k}_2}{|\vec{k}_1 \wedge \vec{k}_2|} \qquad \vec{\chi}_2^{(1)} = \frac{\vec{k}_1 \wedge \vec{\chi}_1^{(1)}}{|\vec{k}_1|}$$

$$\vec{\chi}_1^{(2)} = \frac{\vec{k}_1 \wedge \vec{k}_2}{|\vec{k}_1 \wedge \vec{k}_2|} \qquad \vec{\chi}_2^{(2)} = \frac{\vec{k}_2 \wedge \vec{\chi}_1^{(2)}}{|\vec{k}_2|}$$

for the Stokes vectors of the incident and scattered photon respectively.

If we deal only with plane polarization, only the first two components must be retained. In this case the Stokes vector of the scattered photon is related to the incoming vector $(1, \vec{\xi}\,')$ assumed of unitary intensity by

$$\text{(A.2)} \qquad \left(\frac{I'}{P'}\right) = T\left(\frac{1}{\xi'}\right)$$

where T is a 3 x 3 matrix given by

$$(A.3) \qquad T = \frac{1}{2} T_o^2 \left(\frac{k'_2}{k'_1}\right)^2 \begin{vmatrix} \frac{k'_2}{k'_1} + \frac{k'_1}{k'_2} - \mathrm{sen}^2\,\theta' & \mathrm{sen}^2\,\theta' & 0 \\ \mathrm{sen}^2\,\theta' & 1+\cos\theta' & 0 \\ 0 & 0 & 2\cos\theta' \end{vmatrix}$$

The two vectors are refered to the two different reference frames attached to the plane of scattering previous defined. k'_1, k'_2, θ' are respectively the energies of incoming and scattered photons and the angle between them in references frame where the electron is at rest. It can be shown that if the photon and the elect... were moving at angle $\theta = 0$ or $\theta = \pi$ before the collision; the equation (A.2) and (A.3) are still valid with respect to references frames (A.1) attached to the laboratory, provided that k'_2, k'_1, θ' are expressed in the system where the electron is at rest.

Then these quantities must be related to those of the laboratory by the equations

$$\cos\theta' = \frac{k_1\left(1 - \frac{k_2}{E}\right) - \frac{1}{1+\beta}\left(\frac{k_1}{E}\right)k_2}{k_1\left(\beta - \frac{k_2}{E}\right)}$$

$$k'_1 = k_1 \frac{1+\beta}{\sqrt{1-\beta^2}} \qquad\qquad k'_2 = \frac{k'_1}{1 + \frac{k'_1}{m}(1-\cos\theta')}$$

If we are interested only to very small angle of scattering in the forward ($\theta \sim 0$) and backward ($\theta = \pi$) directions, can assume a common fixed reference frame for the incoming and scattered photons, such that the reference frames attached to the scattering plane for the incoming and outgoing photons are rotated approximatively of the same angle φ with respect to the fixed reference frame. Then, if $(\frac{1}{\xi}p)$ and $(\frac{1}{P}p)$ express the two Stokes vectors in this frame we will have

(A.4) $$\left(\frac{I}{P}p\right) = S_{+(-)} \left(\frac{1}{\xi}p\right)$$

where $$S_{+(-)} = M_{\varphi} T_{+(-)} M_{-\varphi}$$

(A.5) $$T = T_{+}(\theta) = T_{-}(\theta + \pi)$$

the sign +(-) must be assumed in the forward (backward) hemisphere and

(A.6) $$M_{\varphi} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos 2\varphi & \text{sen } 2\varphi \\ 0 & -\text{sen } \varphi & \cos 2\varphi \end{pmatrix}$$

is the matrix that operates on Stokes vectors when the reference frame is ritated by φ (7).

By sostituting (A.5) and (A.6) in equation (A.4) and averaging over φ that cannot be practically experimentally distinguished at $\theta \sim 0$, $\theta \sim \pi$ we obtain

$$\langle \xi_1^{(2)} \rangle = \frac{\bar{P}_1}{I} = \frac{(1 + |\cos \theta'|)^2}{2\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \text{sen}^2 \theta'\right)} \xi_1^{(1)}$$

$$\langle \xi_2^{(2)} \rangle = \frac{\bar{P}_2}{I} = \frac{(1 + |\cos \theta'|)^2}{2\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \text{sen}^2 \theta'\right)} \xi_2^{(1)}$$

For the polarization we have

(A.7) $$\langle \xi^2 \rangle = \left(\frac{|P|}{I}\right)^2 = \frac{0,25\,(1 + |\cos \theta'|)^4 \,(\xi^{(1)})^2}{\left(\frac{k_1'}{k_2'} + \frac{k_2'}{k_1'} - \text{sen}^2 \theta'\right)^2} .$$

The mean value $\langle \xi \rangle$ has been graphed in Fig. 3 by using equation (A.7).

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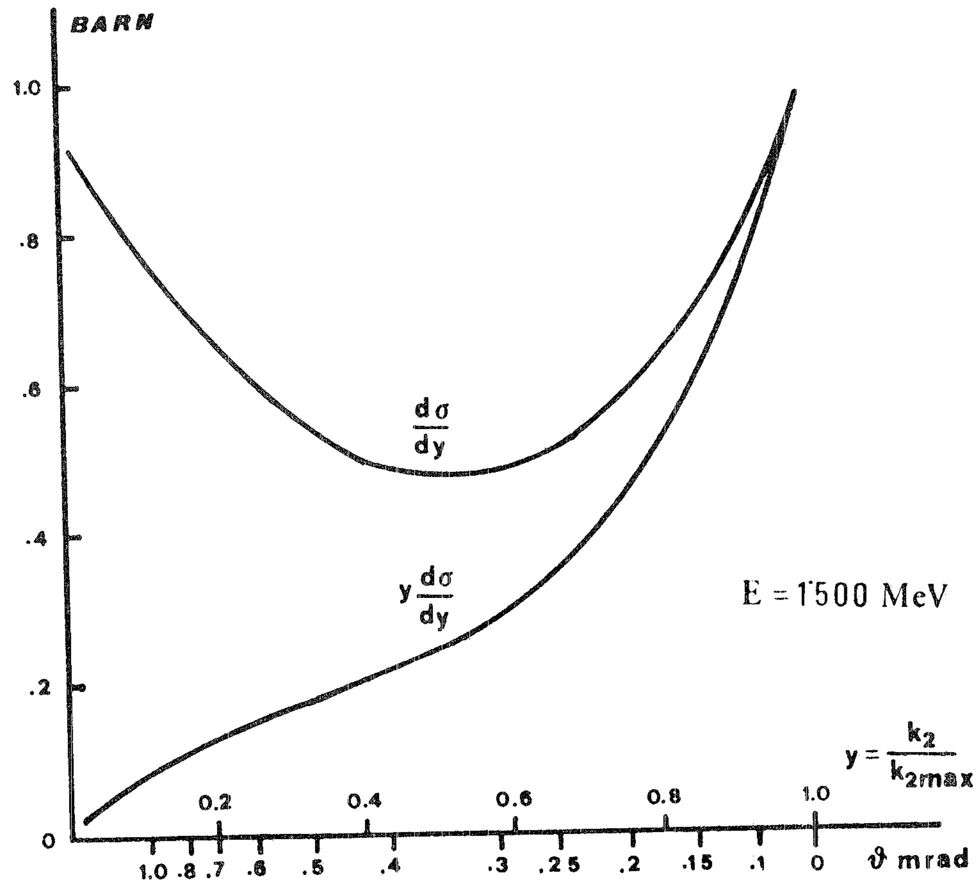

FIG. 1

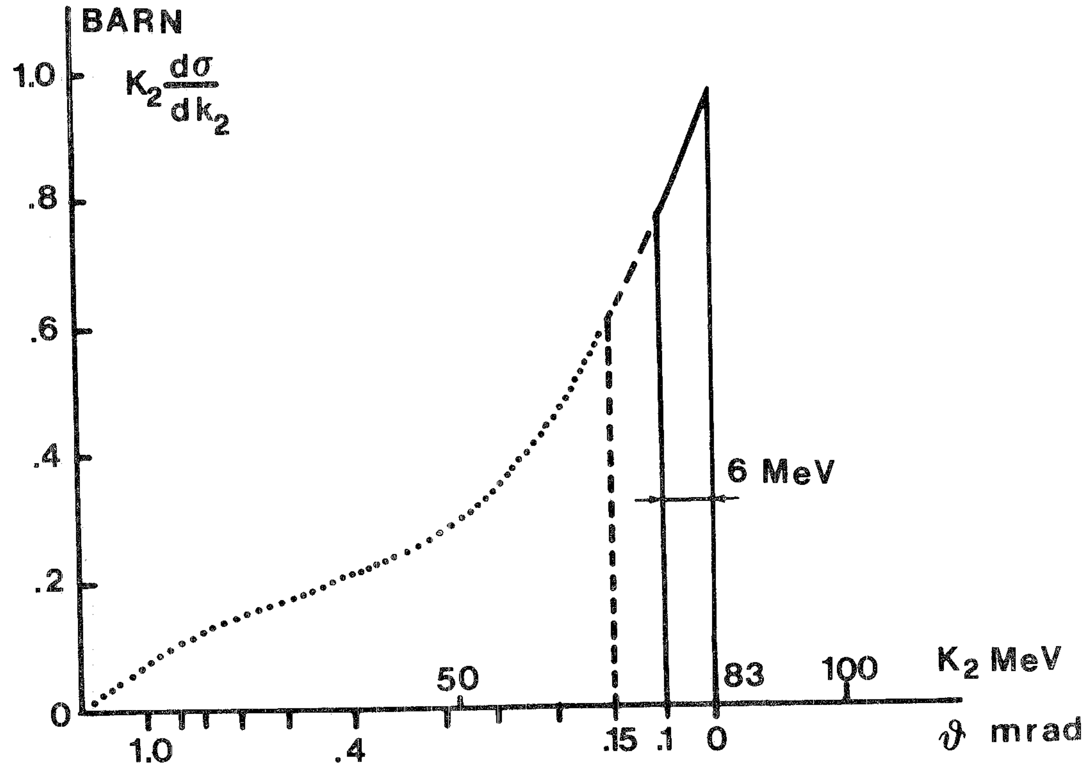
BARN
$K_2 \frac{d\sigma}{dk_2}$
1.0
.8
.6
.4
.2
0
6 MeV
50
83
100
K_2 MeV
1.0
.4
.15
.1
0
ϑ mrad

FIG. 2

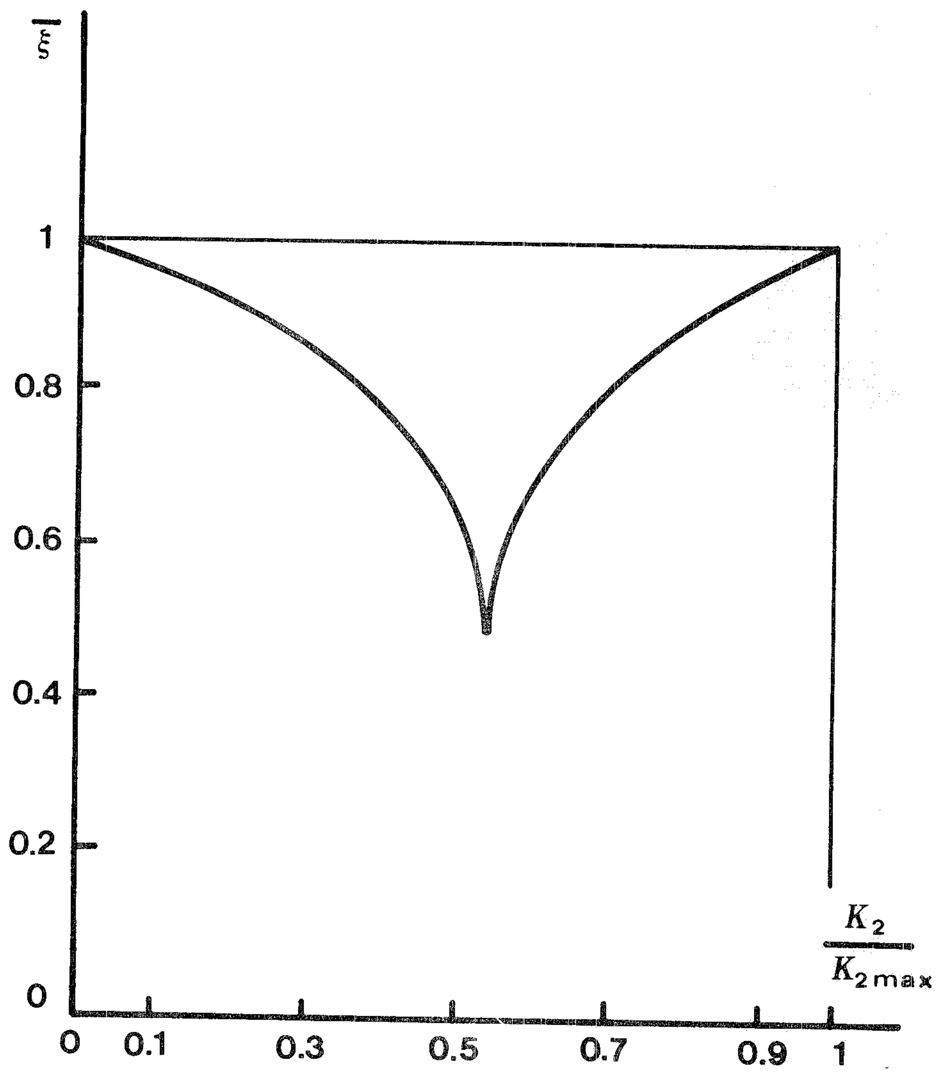

FIG. 3

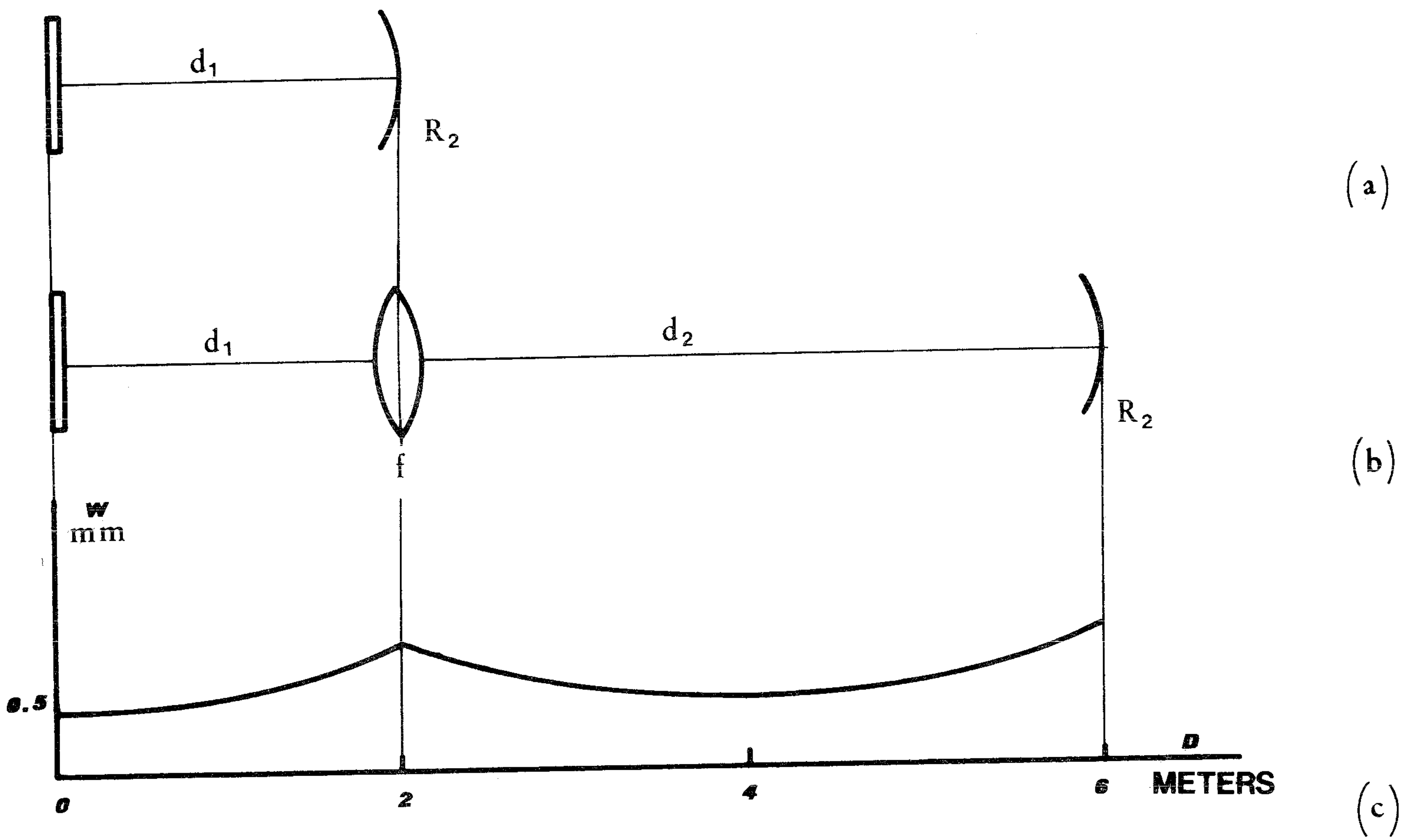

FIG. 4

Sketch of the cavity with and without lens and beam profile inside them.

ON THE RANGE OF VIDEO TRANSMISSION IN WATER BY LASER BEAM

Translation of an article by G.I. Vlasov and I. M. Levin; Problems of Information Transmission, Russian, pp 650-655.

The range of video transmission in water by laser is discussed. A Communications system consisting of an optical laser emitting a narrow light beam and a receiver consisting of lens and photoreceiver is examined.

The use of laser emission as a carrier of information is promising fields where ordinary wide-band communication lines cannot be employed. Such fields include, in particular, wide-band underwater communication, where there is strong attenuation of radio frequencies.

The chief characteristic of communication lines, determining their capabilities, is range of communications, which in the case of water is limited by the absorption and scattering of radiation in the sea.

Two factors act independently here: weakening of the amplitude of the transmitted signal, caused by absorption and one-time scattering, and distortion of the shape of the transmitted signal, attributed to multiple scattering. Obviously distortion of a signal of arbitrary form can be calculated if the response is known of a linear system representing a diffusing layer of given thickness, located between a laser and an optical receiver, to action in delta-function form (infinitely short light pulse). This response (instrument function or scatter function[x]) is determined by multiple scattering. L. S. Dolin proved that the width of the scatter function up to optical thickness not less than 50[xx], does not exceed a few hundreds of

---

x) The scatter function is considered as the dependence of the intensity of emission on time, in contrast to the often-used space scatter function, an argument of which are linear or angular coordinates.

xx) The optical thickness τ_H is defined as the dimensionless product of range H and the index of total attenuation of light in a medium ε_m (with the base ten logarithm).