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M. Greco and Y.N. Srivastava: INCLUSIVE PION DISTRIBUTIONS IN e^+e^- ANNIHILATION FROM SUM RULES AND DUALITY. -

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M. Greco and Y.N. Srivastava[(x)(+)]: INCLUSIVE PION DISTRIBUTIONS IN e^+e^- ANNIHILATION FROM SUM RULES AND DUALITY.

ABSTRACT -

The threshold behaviour of pion production presented in our earlier work is successfully compared with the new SPEAR data.

By using duality and sum rules we derive

$$F_T^{(\pi^+)}(x) \simeq F_L^{(\pi^+)}(x) \simeq F_T^{(\pi^0)}(x) \gg F_L^{(\pi^0)}(x)$$

for x near 1.

An accompanying result is

$$\sigma_{\pi A_2}(s) \simeq 2\,\sigma_{\pi\omega}(s) \simeq 4\sigma_{\pi\pi}(s) \simeq 9(m_\rho^2/s)^3\sigma_{\mu\bar{\mu}} \qquad \text{for large s.}$$

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The recent data[1] on e^+e^- annihilation into hadrons have indicated that the inclusive distribution behaves quite differently in the x region explored. In addition to the large violation of scaling observed for $x = 2E/\sqrt{q^2} \lesssim 0.5$, which reflects itself in the rise of the ratio R ($R \equiv \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$), the data are quite consistent with $q^2\, d\sigma/dx$ scaling for $x \gtrsim 0.5$, giving also evidence for an essential isotropy of the angular distributions. Furthermore, the pion structure function near x = 1 is more than an order of magnitude larger than the proton one, as measured in deep inelastic electron-proton scattering.

This striking result is however in excellent agreement with the prediction we made some time ago[2] (hereafter referred to as I), based on the use of Bloom-Gilman[3] type FESR together with an EVMD (extended vector dominance) model for the pion form factor $F_\pi(s)$.

In this letter we extend the analysis of I, by investigating the threshold behaviour of both transverse and longitudinal structure functions of charged pions and the transverse one for neutral pions. These are connected, by duality to the main processes $e^+e^- \to \pi^\pm A_2^\mp$, $e^+e^- \to \pi^+\pi^-$ and $e^+e^- \to \pi^0\omega$ respectively, the corresponding form factors being altogether related by modified Harari-Pagels type sum rule[4,5]. We are then able to predict

$$F_L^{(\pi^+)}(x) \simeq F_T^{(\pi^+)}(x) \simeq F_T^{(\pi^0)}(x),$$

for $x \simeq 1$, in agreement with data. We also predict near x = 1 a purely transverse distribution for neutral pion production.

Let us begin by summarizing the results of I. The single inclusive differential cross section may be written in terms of the usual structure functions $\bar{F}_1$ and $\bar{F}_2$ as

$$\frac{d\sigma}{dxdz} = \frac{\pi\alpha^2}{2s}\sqrt{x^2 - \frac{4\mu^2}{s}}\left\{2\bar{F}_1(x,s) + \frac{1}{2x}\left(x^2 - \frac{4\mu^2}{s}\right) \times (1-z^2)\,\bar{F}_2(x,s)\right\}, \tag{1}$$

where $q^2 = s$, μ is the pion mass and z is the cosine of the angle made by the pion with respect to the e^+e^- beam. For later purposes, it is more convenient to introduce longitudinal and transverse structure functions as follows:

$$\frac{d\sigma}{dxdz} = \frac{\pi\alpha^2}{2s}\sqrt{x^2 - 4\mu^2/s}\left\{F_T(x,s)(1+z^2) + F_L(x,s)(1-z^2)\right\}. \tag{2}$$

The cross section for pion pairs production is given by:

$$(3)\qquad \frac{d\sigma(e^+e^- \to \pi^+\pi^-)}{dz} = \frac{\pi\alpha^2}{4s}\left(1-\frac{4\mu^2}{s}\right)^{3/2}\left|F_\pi(s)\right|^2(1-z^2).$$

For large s, assuming scaling and a threshold behaviour $F_L(x) \sim c_\pi(1-x)^2$, which are both consistent with the present data, Bloom-Gilman FESR gives:

$$(4)\qquad \frac{1}{s}\int_{\mu^2}^{\sim m_\rho^2} F_L(x,s)\,dM_x^2 \approx \int_{1-\frac{1}{s}(m_\rho^2-\mu^2)}^{1} F_L(x)\,dx,$$

which implies

$$(5)\qquad \left|F_\pi(s)\right|^2 \underset{\text{large}\,s}{\sim} \frac{2}{3}c_\pi\left(\frac{m_\rho^2}{s}\right)^3.$$

A model for $F_\pi(s)$, with such an asymptotic behaviour was explicitly constructed using the scaling model of e.m. interactions in which the photon is coupled to a Veneziano-type spectrum of hadronic states. We found:

$$(6)\qquad \left|F_\pi(s)\right| \underset{\text{large}\,s}{\sim} 3\left(\frac{m_\rho^2}{s}\right)^{3/2},$$

which together with (5) gives $c_\pi \simeq 27/2$. By using the experimental information that

$$F_L^{(\pi^+)}(x) \simeq F_T^{(\pi^+)}(x),$$

we finally predict:

$$(7)\qquad \frac{d\sigma(e^+e^- \; \pi^+ + x)}{dx} \underset{x\approx 1}{\sim} 27\,\frac{\pi\alpha^2}{s}(1-x)^2,$$

which is in excellent agreement with the data, for $x \gtrsim 0.5$. This result shows that the observed scaling behaviour of the pion structure function, for large values of x, agrees with our expectations based on simple duality arguments. This is also supported by the following analysis of the transverse structure functions.

The main contributions to the transverse structure function for π^0 and π^+ production near threshold come from the exclusive processes $e^+e^- \to \pi^0\omega$ and $e^+e^- \to \pi^+A_2^-$ respectively. The corresponding cross sections are given by:

$$\frac{d\sigma(e^+e^- \to \pi^0\omega)}{dxdz} = \left(\frac{\pi\alpha^2}{16}\right)\left(x^2 - \frac{4\mu^2}{s}\right)^{3/2} \left|G_{\omega\pi\gamma}(s)\right|^2 (1+z^2) \quad \times \tag{8}$$

$$\times\, \delta\left(1 - \frac{m_\omega^2 - \mu^2}{s} - x\right),$$

$$\frac{d\sigma(e^+e^- \to \pi^+A_2^-)}{dxdz} = \left(\frac{\pi\alpha^2}{16}\right)\left(x^2 - \frac{4\mu^2}{s}\right)^{5/2} \left|G_{A_2\pi\gamma}(s)\right|^2 \quad \times \tag{9}$$

$$\times\, \frac{s^2}{8m_{A_2}^2}(1+z^2)\, \delta\left(1 - \frac{m_{A_2}^2 - \mu^2}{s} - x\right)$$

where $G_{\omega\pi\gamma}(s)$ and $G_{A_2\pi\gamma}(s)$ are the transition form factors which are normalized at $q^2 = s = 0$ to the corresponding radiative decay widths:

$$\Gamma_{\omega\to\pi^0\gamma} = \frac{\left|G^2_{\omega\pi\gamma}(o)\right|^2}{4\pi}\,\frac{P_\pi^3}{3}, \quad \Gamma_{A_2\to\pi\gamma} = \frac{\left|G^2_{A_2\pi\gamma}(o)\right|^2}{4\pi} \times$$

$$\times\, \frac{P_\pi^5}{20}.$$

In principle also processes like $e^+e^- \to \pi\rho$, πA_1, ... would contribute to $F_T(x,s)$. The corresponding transition form factors however are espected to be smaller than those considered above. In particular $/G_{\rho^0\pi^0\gamma}(s)/^2 \simeq 1/9\, /G_{\omega\pi^0\gamma}(s)/^2$ and $/G_{A_1\pi\gamma}(s)/^2$ is also smaller than both $/G_{\omega\pi\gamma}(s)/^2$ and $/G_{A_2\pi\gamma}(s)/^2$ at $s = 0$, as shown below. Hence, we assume as a working hypothesis that this would be true also for large s. Consequences of possible violations of this assumption will be discussed later.

In order to exploit the duality sum rules as in (4), one needs to know the asymptotic behaviour of the form factors which appear in eqs. (8) and (9). To get this information we shall use a modified Harari-Pagels sumrules which we apply first at $q^2 = 0$ following a recent analysis by Pancheri-Srivastava and Srivastava(5) for pion Compton scattering.

In reference (5) a self-consistent saturation of three sum rules for real pion-Compton scattering has been obtained, on the basis of a

J = 0 fixed pole in the non-sense channel and a Kronecker-delta singularity in the sense channel. Clearly one has no predictive power if these non-Regge terms have unknown parameters. So, an economical and attractive hypothesis was made in ref. (5) , that the non-Regge terms are "dual" to the pion Born terms. Remarkably enough, if one saturates the 3-sum rules with ω, A_1 and A_2 mesons, one ends up with the rather pleasing result for the radiative decays:

$\Gamma(\omega \rightarrow \pi\gamma) \simeq 0.95$ MeV (Experimentally and through VMD $\approx$ 0.87 MeV)

$\Gamma(A_2 \rightarrow \pi\gamma) \simeq 0.92$ MeV (VMD $\approx$ 0.85 MeV)

$\Gamma(A_1 \rightarrow \pi\gamma) \simeq 0.11$ MeV.

The relevance of the above discussion for this work is the following. One of the above sum rules can be generalized to virtual ($q^2 \neq 0$) Compton scattering. Since the saturation scheme for this sum rule is in agreement with experiment and VMD etc. for $q^2 = 0$, we may hope that such is the case for $q^2 \neq 0$ as well. For $\pi(p)+\gamma(q) \rightarrow \pi(p)+\gamma(q)$ we write as usual:

(10) $$T_{\mu\nu} = Ap_\mu p_\nu + B(p_\mu q_\nu + p_\nu q_\mu) + Cq_\mu q_\nu + Dg_{\mu\nu} .$$

Consider the amplitude $A(\nu, Q^2)$ for the difference of $\pi^+\gamma$ - $\pi^o\gamma$ elastic scatterings, such that it is proportional to the isospin $I_t = 2$ in the t-channel. ($\nu \equiv p\cdot q$, $\omega = 2\,p\cdot q/Q^2$ and $Q^2 = -q^2 > 0$). Writing a dispersion relation in ν (for fixed Q^2) and assuming that the leading behaviour is governed by the J = 0 fixed pole, we obtain the sum rule

(11) $$R(Q^2) = F_\pi^2(Q^2) + \int_{\omega_o}^{\infty} d\omega \left[F_2^{(\pi^+)}(\omega, Q^2) - F_2^{(\pi^o)}(\omega, Q^2) \right] ,$$

where $R(Q^2)$ is the residue of the fixed pole and $\omega_o = 1 + 3\mu^2/Q^2$.

As discussed above, at $Q^2 = 0$, the fixed pole cancelled the pion term and the sum rule was satisfied with the ω term being approximately equal to the A_2 term (with a small, $\lesssim$ 10% contribution from the A_1 term, which we neglect from now on). We also remember that in this sum rule (for $Q^2 = 0$), the pion term numerically equals the ω term (and hence also the A_2 term). Thus, assuming that the fixed pole is "dual" to the pion term also for $Q^2 \neq 0$, the saturation of the sum rule with ω and A_2 mesons leads us to the relation

$$(12) \qquad \left| \frac{G_{\omega\pi\gamma}(Q^2)}{G_{A_2\pi\gamma}(Q^2)} \right|^2 \underset{Q^2 \text{ large}}{\sim} \frac{(Q^2)^2}{8\, m^2_{A_2}}$$

If the pion term equals the ω and A_2 contributions as at $Q^2 = 0$ we also have

$$(13) \qquad \left| \frac{G_{\omega\pi\gamma}(s)}{F_\pi(s)} \right|^2 \underset{Q^2 \text{ large}}{\sim} \frac{4}{Q^2} .$$

This last result (13) can also be obtained by EVMD arguments as follows. In our earlier work(2) $F_\pi(Q^2)$ was obtained by summing an infinite series of vector mesons. We try a similar model for the vertex function $G_{\omega\pi\gamma}(Q^2)$, with the known normalization at $Q^2 = 0$:

$$G^2_{\omega\pi\gamma}(0) = \frac{4}{m^2_\rho} F^2_\pi(0).$$

The higher vector mesons, in the narrow width approximation, simply replace the denominator m^2_ρ at $Q^2 = 0$ to Q^2, for Q^2 large, to give us approximately the result(13).

Thus, we have found that under reasonable assumptions - which have been successfully tested at $Q^2 = 0$ - the pion form factor $F_\pi(Q^2)$ is related to the vertex functions $G_{\omega\pi\gamma}(s)$ and $G_{A_2\pi\gamma}(s)$ as in eqs. (12) and (13).

With all that in mind, we an finally able to estimate the transverse structure functions near threshold, via duality sum rules. We obtain near $x = 1$,

$$(14a) \qquad \frac{1}{s}\int F_T^{(\pi^0)}(x,s)\,dM_x^2 \simeq \frac{1}{8}\int s\left| G_{\omega\pi\gamma}(s)\right|^2 \delta\left(1 - \frac{m^2_\omega - \mu^2}{s} - x\right) \simeq \int F_T^{(\pi^0)}(x)\,dx ,$$

$$(14b) \qquad \frac{1}{s}\int F_T^{(\pi^+)}(x,s)\,dM_x^2 \simeq \frac{1}{8}\int \frac{s^3}{8m^2_{A_2}}\left| G_{A_2\pi\gamma}(s)\right|^2 \delta\left(1 - \frac{m^2_{A_2} - \mu^2}{s} - x\right) \simeq \int F_T^{(\pi^+)}(x)\,dx ,$$

(14c) $$\frac{1}{s}\int F_L^{(\pi^+)}(x,s)dM_x^2 = \frac{1}{2}\int \left| F_\pi(s)\right|^2 \delta(1-x) \simeq \int F_L^{(\pi^+)}(x)\,dx ,$$

and through eqs. (12) and (13),

(15) $$F_L^{(\pi^+)}(x) \simeq F_T^{(\pi^+)}(x) \simeq F_T^{(\pi^o)}(x) \simeq c_\pi (1-x)^2 .$$

We therefore "predict" an almost isotropy in the π^+ inclusive angular distribution, the possible deviations being attributed to the contributions from $(e^+e^- \to \pi^+\rho^-) \simeq (1/9)(e^+e^- \to \pi^+A_2^-)$ and eventually from $e^+e^- \to \pi^+A_1^-$. Thus, assuming a small πA_1 component ($\approx 10\%$ as at $q^2 = 0$), a distribution of the type $1 + \alpha \cos^2\theta$, with $\alpha \approx 0.1$-0.2, is obtained in excellent agreement with experiment.

As far as π^o production is concerned, due to the absence of the pion pole, as well as the absence of important exclusive channels contributing to $F_L^{(\pi^o)}(x)$ near $x = 1$, we predict at threshold essentially a transverse component. Experimental checks on this point will be very important for duality ideas.

The total cross section for the exclusive channels considered above are also straightforward to compute. We have, for large s :

(16) $$\sigma_{\pi A_2}(s) \underset{\rho}{\simeq} 2\sigma_{\pi\omega}(s) \simeq 4\sigma_{\pi\pi}(s) = \sigma_{\mu\bar{\mu}} \left| F_\pi(s)\right|^2 \simeq 9\sigma_{\mu\mu}(m_\rho^2/s)^3 .$$

Next, we briefly discuss the angular dependence of n-particle inclusive cross sections. The approximate z-independence of the single inclusive cross-section for large Q^2[(1)] gives $F_T(x,s) \simeq F_L(x,s)$, i.e. the "photon" behaves as if it were a "scalar" particle. This property is maintained for all inclusive processes. It can be verified, for example, by considering the sum rules eqs. (2.5a-2.5e) given in I, which connect the single and double inclusive structure functions. By recursion, similar results follow for any n. Thus, the n-particle inclusive matrix element[(2)]

$$H_{\mu\nu}^{(n)} = \frac{1}{2^n}\frac{1}{(2\pi)^{3n}} \sum_m \frac{1}{m!} (2\pi)^4 \delta^4(q-k_1-k_2-\ldots-k_n-P_m) \times$$

$$\times (d\mathcal{P}_m) \langle 0 | J_\mu | k_1k_2\ldots k_n;m\rangle \langle k_1k_2\ldots k_n;m | J_\nu | 0\rangle$$

for large s reduces to

(17) $$H_{\mu\nu}^{(n)}(q;k_1k_2\ldots k_n) \simeq \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right) G^{(n)} ,$$

where the only surviving invariant $G^{(n)}$ depends upon the kinematic invariants formed out of the available momenta. This allows for a considerable simplification. In terms of $G^{(n)}$ the n-particle inclusive cross section can be written compactly as

$$f^{(n)}_{e^+e^-} = \frac{d\sigma^{(n)}}{\prod_{i=1}^{n}\left(\frac{d^3k_i}{E_i}\right)} \simeq \frac{2(2\pi)^2\alpha^2}{s^2}\, G^{(n)} . \tag{18}$$

What can we say about the structure function $G^{(n)}$? One rather general qualitative result about the distributions in x_i ($x_i \equiv = 2k_i q/s$, $i = 1,..,n$) follows from the present SPEAR data on single particle spectra. These, through the sum rules of the type (2.5a-2.5e) of I, imply that in $G^{(n)}$ whenever any of the x_i's are $\gtrsim 1/2$ the structure function "scales", i.e.

$$\hat{G}^{(n)}(x_1x_2..x_n;s) \xrightarrow[\text{any } x_i \gtrsim 1/2]{} \frac{1}{s^{n-1}}\,\hat{g}^{(n)}(x_1x_2..x_n) , \tag{19}$$

where $\hat{G}^{(n)}$ is defined by integrating $G^{(n)}$ over all the angular variables leaving only the x_i, and $g^{(n)}$ is independent of s. Conversely, all the non-scaling behaviour is confined to the kinematic region with every x_i small.

To conclude, we summarize our basic results :

(i) The data on the scaling part of the single π^+ distributions seem to be well reproduced by the results of our earlier work I. A further check of the model in I would be provided by the measurement of the pion form factor at high q^2.

(ii) Duality and our sum rules lead us to predict $F_T^{(\pi^+)}(x) \simeq F_L^{(\pi^+)}(x) \simeq \simeq F_T^{(\pi^0)}(x) >> F_L^{(\pi^0)}(x)$ for large x. In particular they imply for the exclusive channels eqs. (16). Similar results are obtained for a resonance of mass $M_R \ll \sqrt{s}$ lying on the ω-A_2 trajectory.

(iii) On the basis of energy-momentum sum rules and the z independence of the single inclusive cross section, restrictions are obtained for the n-particle inclusive distributions.

REFERENCES. -

(1) - B. Richter, Proc. of Trieste Conf. on Colliding Beam Interactions, IAEA Trieste, June 1974 (unpublished).
(2) - M. Greco and Y. Srivastava, Nuovo Cimento 18A, 601 (1973).
(3) - E. Bloom and F. Gilman, Phys. Rev. Letters 25, 1140 (1970).
(4) - H. Pagels, Phys. Rev. Letters 18, 316 (1967); H. Harari, Phys. Rev. Letters 18, 319 (1967); H. Abarbanel and M. Goldberger, Phys. Rev. 165, 1594 (1968).
(5) - G. Pancheri-Srivastava and Y. Srivastava, Frascati preprint LNF-74/42(P) (1974).