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ABSTRACT

It is shown that the most important contribution to the energy gap of nuclear matter comes from the tensor potential. When this is taken into account and the exact gap equations are solved (rather than their BCS approximations), the energy gap results of the order of 1 Mev at normal density, corresponding to $k_F = 1.4 \ fm^{-1}$.

The suggestion that nuclear matter should be super-fluid [1] has originated from the analysis of the spectra of heavy deformed even-even nuclei. The first intrinsic excited states of these nuclei lie at about 1 Mev above the ground state, while the spacing between single-nucleon excited states is of about 1/4 Mev. The resulting energy gap is of the order of 1 Mev.

After this observation the energy gap has been estimated by many authors [2, 3, 4] for infinite nuclear matter for different nucleon-nucleon pair states, and has been found to be of the order of 0.1 Mev or smaller at " normal density " corresponding to $k_F = 1.4$ fm^{-1}, with an effective mass M^* of the order of 0.7.

The connection between these results and the spectra of finite nuclei is not clear, because of the finite spacing of single-nucleon levels and surface effects. Emery and Sessler [3] suggested that one should average the energy gap as obtained in infinite nuclear matter over an appropriate range of densities, before a comparison with the spectra of finite nuclei could be done (the energy gap is very sensitive to the density and increases when k_F ranges from 1.4 fm^{-1} to 0.8 fm^{-1}). It has been observed, however, that is not obvious that such an average is meaningful, unless the correlation length turns out not to exceed the diameter of finite nuclei in the range of densities one is averaging over, which is not the case if the energy gap is too small.

In any case we consider the above results unsatisfactory

from a theoretical point of view, for the reason we are going
to explain.

The nucleon-nucleon interaction is able to bind the
proton-neutron system, and this makes divergent the perturbative
expansion for the energy of any N-body system $^{(6)}$. In an
infinite system, moreover, singularities rimain in the G-matrix$^{(7)}$
after the Brueckner-Goldstone theory is applied. These
singularities may escape notice either because numerical
computations are not accurate enough $^{(8)}$, or because of the
large energy gap introduced in the single particle spectrum
by partial summations of the perturbative series. The
singularity disappears if enough proton-neutron attraction
is taken into account in a non perturbative way, for instance
by pairing the proton to the neutron. So this pairing should
play an essential role both in the theory of finite nuclei
and infinite nuclear matter, and should be expected to give
rise to an energy gap of the order of the deuteron binding
energy. This leads to study the pairing in what we shall
call the quasideuteron state, which is an eigenstate of the
spin S and isospin T with eigenvalues resp. 1 and 0, but is
not an eigenstate of the total angular momentum J. In this
state coupled partial waves are allowed, so that the tensor
potential contributes to the energy gap. The theory for
pairing with coupled partial waves has been developed by
Takatsuka $^{(9)}$ for neutron matter in connexion with the study
of superfluidity in neutron stars, and the extension
to neutron-proton matter is very simple. One starts with the

trial wave function

$$\Psi = e^{iS}\, \Phi,$$

where Φ is the Fermi gas ground state and e^{iS} a generalized Bogoliubov transformation with

$$iS = \frac{1}{2}\sum_{\underline{k},\sigma_1,\tau_1,\sigma_2,\tau_2}\left[\Theta(\underline{k},\sigma_1,\tau_1,\sigma_2,\tau_2)\, C^{*}_{\underline{k},\sigma_1,\tau_1}\cdot C^{*}_{-\underline{k},\sigma_2,\tau_2}\right.$$

$$\left. -\,\Theta^{*}(\underline{k},\sigma_1,\tau_1,\sigma_2,\tau_2)\, C_{-\underline{k},\sigma_2,\tau_2}\, C_{\underline{k},\sigma_1,\tau_1}\right], \tag{1}$$

where

$$\Theta(\underline{k},\sigma_1,\tau_1,\sigma_2,\tau_2) = \sqrt{2}\sum_{\ell J_z}\varphi_{\ell J_z}(\kappa)\, Y_{\ell}^{J_z-\sigma_1-\sigma_2}(\hat{\kappa}) .$$

$$\cdot \sqrt{2}\,\langle \tfrac{1}{2}\tau_1\, \tfrac{1}{2}\tau_2 \mid \tfrac{1}{2}\,\tfrac{1}{2}\,00\rangle\langle \tfrac{1}{2}\sigma_1\, \tfrac{1}{2}\sigma_2 \mid \tfrac{1}{2}\,\tfrac{1}{2}\, S\,(\sigma_1+\sigma_2)\rangle .$$

$$\cdot \langle S\,(\sigma_1+\sigma_2)\,\ell\,(J_z-\sigma_1-\sigma_2)\mid S\,\ell\,J\,J_z\rangle . \tag{2}$$

In these formulae the $\varphi_{\ell J_z}(\kappa)$'s are arbitrary functions and the other symbols should be selfexplanatory. Application of the Bogoliubov condition, or, equivalently, variation of the trial wave function Ψ with respect to the $\varphi_{\ell J_z}(\kappa)$'s, gives rise to a system of coupled equations for the $\varphi_{\ell J_z}(\kappa)$'s, which is more conveniently written in terms of gap functions

$\Delta_{\ell J_z}(\kappa)$. The details of the theory can be deduced from the paper by Takatsuka [9], with the simple generalization introduced by the presence of the factor $\sqrt{2} \langle \frac{1}{2} c_1 \frac{1}{2} c_2 | \frac{1}{2} \frac{1}{2} 00 \rangle$ in eq. (2).

In order to have a first estimate of the importance of the inclusion of the tensor potential, we have taken only $\varphi_{00}(\kappa)$ and $\varphi_{20}(\kappa)$ different from zero in eq. (2). Moreover we shall confine to interaction models which in the ^3S-^3D channel only have J = 1. In this case the system of equations for the gap functions becomes

$$\Delta_{\ell 0}(\kappa) = -4 \int_0^\infty d\kappa' \kappa'^2 \int_0^1 d\mu \left[e_\ell(\kappa, \kappa') + \mu^2 h_\ell(\kappa, \kappa') \right] \cdot$$

$$\cdot \left[\hat{\xi}^2(\kappa', \mu) + D^2(\kappa', \mu) \right]^{-\frac{1}{2}}, \quad \ell = 0, 2, \tag{3}$$

corresponding to eqs. (2.25) of Takatsuka [9]. In eqs. (3)

$$e_0(\kappa, \kappa') = \frac{\sqrt{2}}{8\pi} \left[\sqrt{2} \langle \kappa' | \hat{v}_{00} | \kappa \rangle - \langle \kappa' | \hat{v}_{20} | \kappa \rangle \right] \cdot$$

$$\cdot \left[\Delta_{00}(\kappa') + \frac{1}{\sqrt{2}} \Delta_{20}(\kappa') \right],$$

$$e_2(k,k') = \frac{\sqrt{2}}{8\pi}\left[\langle k'| \sigma_{22}|k\rangle - \sqrt{2}\langle k'|\sigma_{20}|k\rangle\right] \cdot$$

$$\cdot\left[\Delta_{00}(k') + \frac{1}{\sqrt{2}}\Delta_{20}(k')\right],$$

$$h_0(k,k') = \frac{3}{8\pi}\left[-\sqrt{2}\langle k'|\sigma_{00}|k\rangle \Delta_{20}(k') + \sqrt{2}\langle k'|\sigma_{20}|k\rangle \Delta_{00}(k')\right.$$

$$\left. -\langle k'|\sigma_{20}|k\rangle \Delta_{20}(k')\right],$$

$$h_2(k,k') = \frac{3}{8\pi}\left[-\sqrt{2}\langle k'|\sigma_{22}|k\rangle \Delta_{00}(k') + \langle k'|\sigma_{22}|k\rangle \Delta_{20}(k')\right.$$

$$\left. +\sqrt{2}\langle k'|\sigma_{20}|k\rangle \Delta_{20}(k')\right],$$

$$\langle k'|\sigma_{\ell\ell'}|k\rangle = \int_0^\infty dr\, r^2\, j_\ell(k'r)\, \sigma_{\ell\ell'}(r)\, j_\ell(kr),$$

$\sigma_{\ell\ell'}$ being the interaction in the state $T = 0$, $S = 1$, $J = 1$, between the l-th , l'-th partial waves. Moreover $D(k,\mu)$ is

the anisotropic energy gap, which depends on k as well as the cosine of the angle between $\underline{k}$ and the axis of spin quantization μ

$$D^2(K,\mu) = \delta_0^2(K) + s\,\delta_2^2(K)\,\mu^2 \,, \quad s = \pm 1 \,,$$

with

$$\delta_0^2(K) = \frac{1}{8\pi}\left[\Delta_{00}(K) + \frac{1}{\sqrt{2}}\Delta_{20}(K)\right]^2 \,,$$

$$\delta_2^2(K) = \frac{3}{16\pi}\left|\Delta_{20}(K)\left[\Delta_{20}(K) - 2\sqrt{2}\,\Delta_{00}(K)\right]\right| \,.$$

We have only found solutions to eqs. (3) with $s = 1$. Note that for $s = 1$ $\delta_0(K)$ is the minimum of $D(k,\mu)$ with respect to μ, so that $\delta_0(K_F)$ is the minimum excitation energy of the system.

Finally $\tilde{\varepsilon}(k,\mu)$ is the Hartree-Fock single-particle potential minus the chemical potential. In this paper we always use for it the effective mass approximation

$$\tilde{\varepsilon}(k,\mu) = \frac{\hbar^2 k^2}{2m M^*} - \varepsilon_F \,, \quad \varepsilon_F = \frac{\hbar^2 K_F^2}{2m M^*} \,.$$

Eqs. (3) can easily be integrated with respect to μ yielding, for $s = 1$

$$\Delta_{\ell 0}(K) = -4 \int_0^\infty dK' K'^2 \left\{ \left[\frac{e_\ell(K,K')}{\delta_2(K')} - \frac{1}{2} \frac{h_\ell(K,K') E^2(K')}{\delta_2^3(K')} \right] \right.$$

$$\cdot \ln \frac{\delta_2(K') + \sqrt{\delta_2^2(K') + E^2(K')}}{E(K')} + \frac{1}{2} \frac{h_\ell(K,K') E(K')}{\delta_2^2(K')}$$

$$\left. \cdot \left[1 + \frac{\delta_2^2(K')}{E^2(K')} \right]^{\frac{1}{2}} \right\}, \quad \ell = 0, 2, \qquad\qquad (4)$$

where

$$E(K) = \left[\left(\frac{\hbar^2 K^2}{2m M^*} - \frac{\hbar^2 K_F^2}{2m M^*} \right)^2 + \delta_0^2(K) \right]^{\frac{1}{2}}.$$

We have also considered the BCS approximations to eqs. (4), obtained by observing that everything in the integrand is a slowly varying function of k with respect to $\hbar^2(K^2-K_F^2)/(2mM^*)$, and the greatest contribution to the integral comes from $k = k_F$. Then putting $\langle K_F | U_{\ell\ell'} | K_F \rangle = W_{\ell\ell'}/K_F^3$, $\Delta_{\ell 0}(K_F) = \bar{\Delta}_{\ell 0}$, $\delta_0(K_F) = \bar{\delta}_0$, $\delta_2(K_F) = \bar{\delta}_2$, $e_\ell(K_F, K_F) = \bar{e}_\ell$, $h_\ell(K_F, K_F) = \bar{h}_\ell$, and replacing $\int_0^\infty dK K^2$ by $K_F^3/(2\varepsilon_F) \int_{-\varepsilon_F}^{\varepsilon_F} d\varepsilon$

we obtain, for s = 1

$$\bar{\Delta}_{\ell 0} = \frac{2 \kappa_F^3}{\varepsilon_F} \left\{ (\bar{e}_\ell + \frac{1}{3} \bar{h}_\ell) \ln \frac{\bar{\delta}_0^2 + \bar{\delta}_2^2}{4 \varepsilon_F^2} - \frac{2}{9} \bar{h}_\ell + \right.$$

$$\left. -2 \left[1 - \frac{\bar{\delta}_0}{\bar{\delta}_2} \text{arctg} \frac{\bar{\delta}_2}{\bar{\delta}_0} \right] \left[\bar{e}_\ell - \frac{1}{3} \left(\frac{\bar{\delta}_0}{\bar{\delta}_2} \right)^2 \bar{h}_\ell \right] \right\}, \quad \ell = 0, 2 .$$

(5)

We want to investigate the effect on the energy gap of the strength of the tensor potential, when this is varied within its experimental range of uncertainty. For this reason we have applied our equations to the separable potentials of Afnan et al.[10], which have a D-state probability PD ranging between 0.01 and 0.09.

In order to exhibit the effect of the tensor potential, we want to compare the minimum energy gap $\delta_0(\kappa_F)$ which obtains in the quasideuteron pairing, with the energy gap $\delta(k_F)$ in the T = 0, S = 1, l = 0 pairing, where the tensor potential does not contribute. The equations for the gap in the T = 0, S = 1, l = 0 pairing obtain from eqs. (4) and (5) by putting $\bar{J}_{20} = \bar{J}_{22} = \Delta_{20} = 0$. The results are reported in Tab. I.

TABLE I

The energy gap $\delta(k_F)$, and its BCS approximation $\bar{\delta}$, evaluated at $k_F = 1.4$ fm^{-1} with $M^* = 0.75$, for the potentials of Afnan et al.[10] classified according to their D-state probability PD.

PD	$\delta(k_F)$ (MeV)	$\bar{\delta}$ (MeV)
0.03	0.29	0.14
0.04	0.12	0.0054
0.055	0	0
0.07	0	0

The gap in this state has already been estimated to be $\delta(k_F) = 0.16$ Mev by Brueckner et al.[4], in BCS approximation, with $M^* = 0.73$ and $k_F = 1.5$ fm^{-1}. This result is greater than our results in the same approximation with close values of $M^* = 0.75$ and $k_F = 1.4$ fm^{-1}. This ensures that the potentials we are using are not too attractive in the $T = 0$, $S = 1$, $l = 0$ state, as also implied by the value of the energy per particle of nuclear matter yielded to first order in the G-matrix[10].

In particular the potentials with PD > 0.04 are repulsive in this state and do not give rise to superfluidity.

In Tab. II we report the values of $\delta_0(k_F)$ and $\delta_2(k_F)$

obtained for the quasideuteron pairing.

TABLE II

The functions $\delta_0(k_F)$ and $\delta_2(k_F)$, and their BCS approximations $\bar{\delta}_0$ and $\bar{\delta}_2$, evaluated at $k_F = 1.4$ fm^{-1} with $M^* = 0.75$, for the potentials of Afnan et al.[10] classified according to their D-state probability PD.

PD	$\delta_0(k_F)$ (MeV)	$\delta_2(k_F)$ (MeV)	$\bar{\delta}_0$ (MeV)	$\bar{\delta}_2$ (MeV)
0.03	0.94	2.1	0.60	1.4
0.04	0.87	2.3	0.45	1.3
0.055	0.71	2.4	0.22	1.0
0.07	0.50	2.2	0.065	0.57

They show that the tensor potential considerably enhances the the minimum energy gap $\delta_0(k_F)$ for PD ≤ 0.04, and gives rise to superfluidity also for PD > 0.04. Moreover $\delta_0(k_F)$ turns out of the expected order of magnitude. The BCS approximation becomes worse and worse when PD increases, and is generally unreliable. Finally in Tab. III we have reported the results obtained with $M^* = 1$ in order to show the sensitivity of the energy gap to effective mass.

TABLE III

The functions $\delta_0(k_F)$ and $\delta_2(k_F)$ evaluated at $k_F = 1.4$ fm^{-1} with $M^* = 0.75$, for the potentials of Afnan et al.[10], classified according to their D-state probability PD.

PD	$\delta_0(k_F)$ (MeV)	$\delta_2(k_F)$ (MeV)
0.03	2.3	5.1
0.04	2.1	5.6
0.055	1.8	5.8
0.07	1.5	5.8

Summarizing, the most important contribution to the energy gap of nuclear matter, comes from the tensor potential, and when this is taken into account by the quasideuteron pairing, and the exact gap equations are solved (rather than their BCS approximations), the minimum energy gap turns out of the expected order of magnitude. This result has been admittedly obtained with a naive potential model, but the displayed mechanism is quite general.

The quasideuteron pairing generates an energy gap which is an anisotropic function of the momentum.

This point should be investigated further to see

whether it is a true feature of infinite nuclear matter, or there are other states favoured in energy, obtained for example by repairing the quasideuterons in a $T = 0$, $J = 0$ state (quasialpha).

The existence of an energy gap has been clearly observed in the spectra of deformed nuclei, the spherical nuclei being more difficult to analyze in this respect. (1) Our intoductory considerations, however, along with our numerical results, support the conjecture that quasideuterons, possibly repaired to quasialphas, should be present in all finite nuclei. If so, standard perturbation theory for the effective interaction should be modified to include pairing.

We conclude by addressing the interested reader to the literature (11), for the discussion of some phenomenological implications of the pairing in $T = 0$ states.

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