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G. Pancheri-Srivastava and Y.N. Srivastava[o]: EVIDENCE FOR A J = 0 FIXED POLE AND KRONECKER-DELTA SINGULATIRY IN PION COMPTON SCATTERING[x]. -

In this paper we wish to present some theoretical reasons and phenomenological evidence (albeit indirect) for the presence of non-Regge singularities (fixed poles and Kronecker-delta terms) in pion Compton scattering. Fixed poles at J = 1 and J = 0 have been considered in nuclear Compton scattering by many authors, to restore, in one case, the Pomeron coupling in the total cross-section[(1)] and to obtain a good fit to the data in the other[(2)]. With regard to pion Compton scattering, suspicions about a J = 0 fixed pole have been advanced[(3)] but no clear cut evidence has been forwarded. Our result is based on the internal consistency of a set of sum rules, which is possible only with the aid of non-Regge terms at J = 0. Our result may be of particular relevance for the forthcoming measurements on photon-photon processes (e.g. $\gamma\gamma \to \pi^+\pi^-$) via $e^\pm e^-$ colliding beam facilities. We comment on this briefly towards the end of this work.

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For a study of possible fixed $J = 0$ singularities (pole, if non--sense, and kronecker delta if sense) $\pi\gamma$ offers an excellent choice. The reason is pion's one unit of isospin and hence the existence of an "exotic" reaction (2 units of isopin in the t-channel) obtained by considering the amplitude for the difference $\pi^{+}\gamma - \pi^{o}\gamma$. The high energy behaviour of the (real part of the) above amplitude is thus controlled by the fixed $J = 0$ singularities. Hence, any sum rule (s) of the super convergent type based on exoticity, if fail, show a direct evidence for the existance of non-Regge terms.

For real photons on pions, there are two independent scattering amplitudes which we take to be

$$A(s,t,u) = \frac{T^{s}_{1,0;1,0}(s,t,u)}{2(\mu^{4}-su)} = \frac{T^{t}_{0,0;1,-1}(s,t,u)}{2(\mu^{4}-su)}$$

and

$$B(s,t,u) = \frac{T^{s}_{1,0;-1,0}(s,t,u)}{-t/2} = \frac{T^{t}_{0,0;1,1}(s,t,u)}{-t/2}$$

where $T^{(s(t)}$ are the s-channel (t-channel) helicity amplitudes and the (kinematic) factors below them are chosen so as to render A and B free of kinematic singularities and thus fit for dispersion. Both A and B are even under $\nu \leftrightarrow -\nu$ where $\nu = (s-u)/2$. For the reasons stated above, it will be understood that we are considering only $I = 2$ in the t-channel so as to take advantage of the fact that the for leading Regge trajectory (in the t-channel) $\alpha_{I=2}(t)$ 0.

We can thus write an unsubtracted dispersion relation for A and B at fixed t:

$$A(\nu,t) = \frac{e^{2}}{(\frac{t}{2})^{2} - \nu^{2}} + \frac{2}{\pi}\int_{\nu_{o}}^{\infty} \frac{\nu'\, d\nu'}{\nu'^{2} - \nu^{2}}\, \mathrm{Im}\, A(\nu',t) \qquad (1a)$$

(1b) $$B(\nu, t) = \frac{4\mu^2 e^2}{(\frac{t}{2})^2 - \nu^2} + \frac{2}{\pi}\int_{\nu_o}^{\infty} \frac{\nu' d\nu'}{\nu'^2 - \nu^2} \quad m\, B(\nu', t)$$

The sum rule due to Pagels and Harari[4] (hereafter referred to as HP sum rule) is obtained from (1a) by demanding that $A(\nu, t=0) \underset{\nu \to}{\sim} \nu^{\alpha_{I=2}(0)-2}$ and hence, with $\alpha_{I=2}(0) < 0$, that A be superconvergent:

(2) $$0 = e^2 + \frac{2}{\pi}\int_{\nu_o}^{\infty} \nu' d\nu' \quad m\, A(\nu', t=0)$$

If we saturate (2) with ω, A_1 and A_2 resonances we obtain (neglecting the pion mass)

(3) $$\frac{\alpha}{6} = \frac{\Gamma_{\omega \to \pi^o}}{m_\omega} - \frac{\Gamma_{A_1 \to \pi\gamma}}{m_{A_1}} - \frac{5}{3}\,\frac{\Gamma_{A_2 \to \pi\gamma}}{m_{A_2}}$$

We now wish to test this equation by comparison with a sum rule due to Abarbanel and Goldberger[5] (here after referred to as AG sum rule). This sum rule is obtained by writing a fixed s-dispersion relation for the B amplitude. It is assumed that, at s = 0, $B(s,t,u) \underset{s=0}{\overset{t \to \infty}{\sim}} t^{\alpha_\pi(0)-1}$. We have another superconvergence relation since $\alpha_\pi(0) < 0$:

$$e^2 = \frac{1}{4\pi}\int_{t_o}^{\infty} dt' \quad m\, B(s=0, t') - \frac{1}{4\pi}\int_{u_o}^{\infty} du' \quad m\, B(s=0, u')$$

Since the t-channel is "exotic", we keep only the ω, A, and A_2 contributions from the u-channel, to obtain the AG sum rule (again we neglect the pion mass)

(4) $$\frac{\alpha}{3} = \frac{\Gamma_{\omega \to \pi^o\gamma}}{m_\omega} - \frac{\Gamma_{A_1 \to \pi\gamma}}{m_{A_1}} + \frac{5}{3}\,\frac{\Gamma_{A_2 \to \pi\gamma}}{m_{A_2}}$$

Eqs. (3) and (4) can be made consistent only with the rather displeasing choice

$$(5) \qquad \frac{\Gamma_{\omega \to \pi^o \gamma}}{m_\omega} \quad \frac{\alpha}{4} \quad \text{and} \quad \frac{\Gamma_{A_2 \to \pi\gamma}}{m_{A_2}} \simeq \frac{\alpha}{20}$$

and ignoring the A_1 term alltogether (keeping it would only worsen the estimate for $\Gamma_{\omega \to \pi^o\gamma}$). The experimental value for $\Gamma_{\omega \to \pi^o\gamma}/m_\omega$ is very close to $\alpha/6$. We can estimate $\Gamma_{A_2 \to \gamma}$ through VMD using the experimental value $\Gamma_{A_2 \to \pi\varrho} = 71$ MeV. We find $\Gamma_{A_2 \to \pi\gamma}/m_{A_2} \simeq \frac{\alpha}{10}$. Thus on both experimental and theoretical accounts the estimate (5) seems incorrect. For later discussion, let us record here that the estimates $\alpha/6$ for both $\Gamma_{\omega \to \pi^o\gamma}/m_\omega$ and $5/3\,(\Gamma_{A_2 \to \pi\gamma}/m_{A_2})$ are in excellent agreement with eq. (4), i.e. with AG sum rule (and eventually a tiny contribution from the A_1). Thus the culprit seems to be eq. (3). The fault is due to the neglect of a $J = 0$ fixed pole in the A amplitude. We make the hypothesis in complete analogy with the nucleon case[2], that the residue of this fixed pole is related to the Born term (i.e. the pion pole). More precisely we claim that

$$A(\nu, t) \underset{\substack{\nu \to \infty \\ t=0}}{\longrightarrow} -\frac{e^2}{\nu^2} + O(\nu^{\alpha_{I=2}(0)-2})$$

which changes the Harari and Pagels result, eq. (3), to read

$$(6) \qquad 0 \quad \frac{\Gamma_{\omega \to \pi^o \gamma}}{m_\omega} - \frac{\Gamma_{A_1 \to \pi\gamma}}{m_{A_1}} - \frac{5}{3}\,\frac{\Gamma_{A_2 \to \pi\gamma}}{m_{A_2}}$$

Now, the discrepancy disappears: eqs. (4) and (6) are self-consistent and in agreement with VMD and experiment when available.

Now we derive a new sum rule and show that its satisfaction demands again a fixed $J = 0$ singularity. This time, however, it is in the

B amplitude, which is non-spin flip in the t-channel, and hence it is of Kronecker delta type in the J-plane.

Let us first consider a heuristic argument to motivate our sum rule. Near the Adler point, $s = u = 0$ and $t = \mu^2$, we can make a small s, t, u expansion to write

$$B(s,t,u) \simeq b_1(t-\mu^2) + b_2(s+u)$$

But we are considering an "exotic" I_t=2 channel and hence we expect $b_1 = 0$. This implies that for <u>physical</u> pion Compton scattering

$$B^{I_t=2}(s,t,u) \sim t - 2\mu^2$$

near threshold. A more elegant argument based on unitarity leads us to a more precise requirement:

$$B^{I_t=2}(J=0,\ t \simeq 2\mu^2) = 0$$

which is to say that the (I_t=2) t-channel J=0 partial wave amplitude inherits the Adler zero which is present near $t \simeq 2\mu^2$ in (I_t=2) $\pi\pi$ amplitude. The argument runs as follows.

Consider the elastic unitarity condition for the t-channel partial waves for the three processes $\pi\pi \to \pi\pi$, $\gamma\gamma \to \pi\pi$ and $\gamma\gamma \to \gamma\gamma$. We write for $t \geq 4\mu^2$

(t) $\pi\pi \to \pi\pi$: $A^{I_t}(J,t)$; $\qquad \mathrm{Im}\, A^{I_t}(J,t) = \varrho(t) |A(J,t)|^2$

(t) $\gamma\gamma \to \pi\pi$: $B^{I_t}_{\lambda_1\lambda_2}(J,t)$; $\qquad \mathrm{Im}\, B^{I_t}_{\lambda_1\lambda_2}(J,t) = \varrho(t)\, B^{I_t}_{\lambda_1\lambda_2}(J,t)\, A^{I_t *}(J,t)$

(t) $\gamma\gamma \longrightarrow \pi\pi$: $C_{\lambda_3\lambda_4;\lambda_1\lambda_2}(J,t)$;

$$\mathrm{Im}\, C_{\lambda_3\lambda_4;\lambda_1\lambda_2}(J,t) = \varrho(t) \sum_{I_t=0,2} B^{I_t}_{\lambda_1\lambda_2}(J,t)\, B^{I_t^{*}}_{\lambda_3\lambda_4}(J,t)$$

where $\varrho(t)=\sqrt{\frac{t-4\mu^2}{t}}$ is the 2π -phase factor, $\lambda_i^{'}$s are the photon helicity indices.

Then, the satisfaction of the last two unitarity equations is achieved by writing

$$(7)\qquad C_{\lambda_3\lambda_4;\lambda_1\lambda_2}(J,t)= \hat{C}_{\lambda_3\lambda_4;\lambda_1\lambda_2}(J,t)+ \sum_{I_t=0,2} \frac{B^{I_t}_{\lambda_3\lambda_4}(J,t)\, B^{I_t}_{\lambda_1\lambda_2}(J,t)}{A^{I_t}(J,t)}$$

where $\hat{C}_{\lambda_3\lambda_4;\lambda_1\lambda_2}(J,t)$ stands for contributions from the left-hand cuts (and, if one wishes to generalize, from inelastic right hand cuts). Eq. (7) is useful because we know that $A^{I_t=2}(J=0,t)$ has a zero at $t\simeq 2\mu^2$, the Adler zero (discovered by Weinberg). It follows that $B^{I_t=2}(J=0,t)$ (i.e. the J=0 partial wave of the previous B amplitude) must vanish at the same point, or the C amplitude will have a pole there. But C is supposed to be free of singularities in the region $0\leq t\lesssim 4\mu^2$ and this pole cannot be cancelled by $\hat{C}$, since $\hat{C}$ in this region is free of singularities (it has only left-hand and possibly inelastic right-hand cuts). Thus, we conclude that

$$(8)\qquad B^{I_t=2}(J=0,\ t\simeq 2\mu^2) = 0$$

i.e. the $\pi\gamma$ amplitude inherits the corresponding Weinberg-Adler zeroes of $\pi\pi$ scattering. It is useful to discuss the above result before using it. First, as far as I_t=2 channel is concerned, the use of two pion elastic unitarity is probably not unreasonable since the other contributing (2 particle) channels are quite massive and thus should be appropriately accounted, on an average, by $\hat{C}$. Secondly, for applications which follow, luckily it turns out that the precise position of the zero is not important. All we really need is a zero somewhere in the vicinity of t near zero (of order μ^2). A third point which ought to be mentioned is that the corresponding zero in $I_t=0$ amplitude is useless for our purposes since we cannot write unsubtracted dispersion relations for it.

We could convert eq. (8) into a sum rule by partial waving eq. (1 b) in the t-channel and imposing (8) to the J=0 partial wave. We would (incorrectly!) obtain

$$4\mu^2 e^2 \int_{-1}^{+1} \frac{dz_t}{\mu^4+\mu^4 z_t^2} = \tag{9}$$

$$= -\frac{2}{\pi}\int_{\nu_0}^{\infty} \nu' d\nu' \quad m B(\nu', t=2\mu^2) \int_{-1}^{+1} \frac{dz_t}{\nu'^2+\mu^4 z_t^2}$$

Then, saturating the rh side of eq. (9) with ω, A_1 and A_2 resonances, we would obtain (again incorrectly)

$$\frac{\alpha}{6} \simeq \frac{4}{\pi}\frac{\mu^2}{m_\omega^2}\left[\frac{\Gamma_{\omega\to\pi^0\gamma}}{m_\omega} - \frac{\Gamma_{A_1\to\pi\gamma}}{m_{A_1}}\left(\frac{m_\omega}{m_{A_1}}\right)^2 - 5\frac{\Gamma_{A_2\to\pi\gamma}}{m_{A_2}}\left(\frac{m_\omega}{m_{A_2}}\right)^2\right] \tag{10}$$

neglecting, whenever possible, the pion mass.

The above sum rule is obviously unsatisfied. The discrepancy here is much more pathological than in Harari and Pagels sum rule (eq. (3)) due to the extra factor μ^2/m_ω^2 in front of the brackets at the right hand side.

It is clear that the zero required by t-channel unitarity at $t \simeq 2\mu^2$ cannot be obtained by balancing, in $B(J=0,\ t \simeq 2\mu)$, the contribution of the Born term with that of the dispersive integral as they are of a different order of magnitude. Furthermore (and consistently, the remedy, which cured the discrepancy between eqs. (3) and (4), works here as well. We postulate that i) there is a J=0 Kronecker delta singularity which is of course not picked up by the Frissart-Gribow continuation und ii) that it is "dual" to the pion pole. The correct sum rule (called YG from now on) now reads

$$0 = \int_{\nu_0}^{\infty} \nu' d\nu' \ \mathrm{Im}\ B(\nu', t \simeq 2\mu^2) \int_{-1}^{+1} \frac{dz_t}{\nu'^2 + \mu^4 z_t^2}$$

and (10) is changed into

$$(11) \qquad 0 \ \frac{\Gamma_{\omega \to \pi^0 \gamma}}{m_\omega} - \frac{\Gamma_{A_1 \to \pi\gamma}}{m_{A_1}} \left(\frac{m_\omega}{m_{A_1}}\right)^2 - 5 \frac{\Gamma_{A_2 \to \pi\gamma}}{m_{A_2}} \left(\frac{m_\omega}{m_{A_2}}\right)^2$$

We thus have three relations AG sum rule eq. (4); modified HP sum rule eq. (6) and YG sum rule eq. (11) for three unknowns. The answer is in the Table I along with the experimental value (for $\Gamma_{\omega \to \pi\gamma}$) and the VMD predictions.

TABLE I

	(HP)'+ (AG) + (YG) sum rules estimates	Exp	VMD estimates
$\Gamma_{\omega \to \pi^0 \gamma}$	0.95 MeV	0.87 MeV	$\simeq$ 0.87 MeV
$\Gamma_{A_1 \to \pi\gamma}$	0.11 MeV	—	
$\Gamma_{A_2 \to \pi\gamma}$	0.92 MeV	—	0.85 MeV

Thus a consistent and hopefully realistic picture seems to emerge giving us confidence in our hypothesis.

We want to point out that the introduction of the $J=0$ Kronecker-delta singularity is not an "ad hoc" hypothesis made to "fix up" a sum rule like (10) which is not working. Rather, if a $J=0$ fixed pole is present in $T^s_{1,0;1,0}$, it would be awkward not to require a δ_{J0} in $T^s_{1,0;-1,0}$. In fact if we say that

$$T^s_{1,0;1,0}(s,t,u) \xrightarrow[s\to\infty]{} s^{\alpha_{I=2}(t)} + c_1(t)$$

i.e. that the (real part of the) s-channel non-spin-flip amplitude goes to a constant at large s, one should postulate the same behaviour for the (real part of the) other helicity amplitude. One must write

$$T^s_{1,0;-1,0}(s,t,u) \xrightarrow[s\to\infty]{} s^{\alpha_{I=2}(t)} + c_2(t)$$

and then if, for the A amplitude, the fixed pole term represents in a loose sense how much of the Born term survives at high energy(2), we see that through unitarity a similar role is played by the Kronecker--delta term in the B amplitude. Notice in fact that even though the Born term seems to disappear at high energy (it goes down like s^{-2}) its memory, again in a loose sense, is kept in the δ_{J0} term.

To recapitulate: on the basis of some old and newly derived sum rules, we claim the existence of a J=0 fixed pole (in the non-sense channel) and a Kronecker-delta singularity at J=0 (in the sense channel) in pion Compton scattering. Moreover both these singularities are found to be "dual" to the Born terms. These will in turn show up in elastic photon-photon scattering as well (in both its real and imaginary part) thus vitiating many finite energy sum rules which have been assumed. A detailed analysis of this reaction is in progress and shall be presented elesewhere.

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