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ANGLE MECHANISMS IN HIGH ENERGY HADRON SCATTERING  
FROM NUCLEI. -

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R. Dymarz<sup>(x)</sup>, A. Małecki<sup>(x)</sup> and P. Picchi<sup>(o)</sup>: FORWARD AND LARGE  
ANGLE MECHANISMS IN HIGH ENERGY HADRON SCATTERING FROM  
NUCLEI. -

ABSTRACT. -

A multiple collision model, including the contributions of forward and large angle scatterings, is developed. The model is applied with success to p-<sup>4</sup>He scattering at 1 GeV.

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In this letter we are presenting a model of high energy hadron scattering from nuclei, which includes the two basic mechanisms of the multiple collisions:

- i) all scatterings from target sub-units are near forward direction, hence the projectile trajectory is almost rectilinear;
- ii) one scattering from a sub-unit occurs at a large angle; such a "violent" event may be associated with a number of forward scatterings on the remaining sub-units, along the initial and final projectile direction.

The dominance of these two contributions was established many years ago by Schiff<sup>(1)</sup>. The Glauber model<sup>(2)</sup> takes into account only forward scatterings. Our aim is to develop a "forward+large angle" model which includes both mechanisms.

Let us split the nuclear potential acting on the projectile into the two parts:  $V = V_{\text{forward}} + V_{\text{large}}$ , which are responsible for forward and large angle scatterings, respectively. In practice such a division will be specified for elementary scattering amplitudes, not for the potentials. The nuclear amplitude is:

$$(1) \quad F_p^{i \rightarrow f} = - \frac{E}{2\pi} \left[ \langle \Phi_f | V_{\text{forward}} | X_i^{(+)} \rangle + \langle \Psi_f^{(-)} | V_{\text{large}} | X_i^{(+)} \rangle \right]$$

where  $\Phi$  are the plane waves,  $X^{(\pm)}$  are the scattering states corresponding to  $V_{\text{forward}}$  alone while  $\Psi^{(\pm)}$  is the exact stationary wave yielded by the total nuclear potential. The subscripts  $i$  and  $f$  refer to the direction of the initial and final momenta,  $p$  and  $E$  being the momentum and energy of the projectile.

We will approximate Eq. (1) as follows. First, we treat  $V_{\text{large}}$  as a perturbation; indeed at high energies scattering is strongly peaked at forward direction. We can therefore replace  $\Psi_f^{(-)}$  by  $X_f^{(-)}$ . The large angle term is thus given by the Born approximation with waves

distorted by forward scatterings. Secondly, for  $X^{(\pm)}$  we use the eikonal approximation, justified for high energy, small angle collisions:

$$(2) \quad X_i^{(+)} = e^{i\vec{p}_i \cdot \vec{r}} \exp \left[ -iE/p \int_{-\infty}^0 dt V_{\text{forward}} \left( \vec{r} + \frac{\vec{p}_i}{p} t \right) \right]$$

$$X_f^{(-)} = e^{i\vec{p}_f \cdot \vec{r}} \exp \left[ +iE/p \int_0^{\infty} dt V_{\text{forward}} \left( \vec{r} + \frac{\vec{p}_f}{p} t \right) \right]$$

Further, we assume the additivity of the potentials produced by single nucleons:

$$(3) \quad V_{\text{forward}}(\vec{r}) = \sum_{j=1}^A v_{\text{forward}}(\vec{r} - \vec{r}_j)$$

An equation similar to (3) holds also for  $V_{\text{large}}$ .

Let us define for each nucleon in the target the shadow and antishadow functions<sup>(3)</sup>:

$$(4) \quad B(\vec{r} - \vec{r}_j) = 1 - \exp \left[ -iE/p \int_{-\infty}^0 dt v_{\text{forward}} \left( \vec{r} - \vec{r}_j + \frac{\vec{p}_i}{p} t \right) \right]$$

$$\bar{B}(\vec{r} - \vec{r}_j) = 1 - \exp \left[ -iE/p \int_0^{\infty} dt v_{\text{forward}} \left( \vec{r} - \vec{r}_j + \frac{\vec{p}_f}{p} t \right) \right]$$

Applying (1)-(4) we obtain the amplitude (meant as the operator acting between nuclear states) of nuclear scattering at momentum transfer  $\vec{q}$ :

$$(5) \quad \hat{F} = \frac{ip}{2\pi} \int d^3 r e^{i\vec{q} \cdot \vec{r}} \sum_{j=1}^A f_{\text{forward}}(\vec{r} - \vec{r}_j) \frac{A}{A - k(\neq j)} \left[ 1 - B(\vec{r} - \vec{r}_k) \right] +$$

4.

$$(5) \quad + \frac{ip}{2\pi} \int d^3 r e^{i\vec{q}\vec{r}} \sum_{j=1}^A f_{\text{large}}(\vec{r} - \vec{r}_j) \frac{A}{k(\neq j)} (1 - \bar{B}_k)(1 - B_k)$$

where:

$$(6) \quad f_{\text{forward}}(\vec{b}, z) = \frac{\partial}{\partial z} B(\vec{b}, z); \quad Oz \parallel \vec{p}_i$$

$$f_{\text{large}}(\vec{b}, z) = i \frac{E}{p} v_{\text{large}}(\vec{b}, z)$$

Though our starting point was based on the potentials, we will eliminate them by a suitable interpretation of various terms in Eq. (5). It is essential to observe that the function:

$$(7) \quad f(\vec{b}, z) = f_{\text{forward}} + f_{\text{large}}$$

is nothing else but the three-dimensional Fourier transform of the elementary amplitude for elastic scattering - we call it the scattering shape function<sup>(3)</sup>. The shape functions are to be deduced from the elementary scattering amplitudes; in particular the knowledge of their splitting into the forward and large angle parts is very important. The shapes for forward and large angle scattering represent the basic input of multiple scattering calculations.

The shadows are simply related to the shape for forward scattering; from (6) and (4) we have:

$$(8) \quad B(\vec{b}, z) = \int_{-\infty}^z d\xi f_{\text{forward}}(\vec{b}, \xi)$$

$$\bar{B}(\vec{b}, z) = \int_{z'}^{+\infty} d\xi f_{\text{forward}}(\vec{b}', \xi)$$

where the primed coordinates refer to the frame which is rotated with respect to the natural frame ( $Oz \parallel \vec{p}_i$ ) through the scattering angle  $\theta$ .

The forward term in Eq. (5) is further elaborated in the standard way. Neglecting the longitudinal momentum transfer, one is able to perform the integration over  $z$  which secures the invariance under time-reversal. The result is the Glauber formula<sup>(2)</sup>:

$$(9) \quad \hat{F}_{\text{forward}} = \frac{ip}{2\pi} \int d^2 b e^{i\vec{q}\cdot\vec{b}} \left\{ 1 - \prod_{j=1}^A \left[ 1 - \gamma(\vec{b} - \vec{s}_j) \right] \right\}$$

with  $\gamma(b) = B(b, +\infty)$  called the profile function.

The large angle term is much more complicated, because of the integrals which occur in the shadow functions. We will approximate this term as follows:

$$(10a) \quad \hat{F}_{\text{large}} = \frac{ip}{2\pi} \int d^3 r e^{i\vec{q}\cdot\vec{r}} \sum_{j=1}^A f_{\text{large}}(\vec{r} - \vec{r}_j) \prod_{k(\neq j)}^A (1 - B_k - \bar{B}_k)$$

$$(10b) \quad = \frac{ip}{2\pi} \int d^3 r e^{i\vec{q}\cdot\vec{r}} \sum_{j=1}^A f_{\text{large}}(\vec{r} - \vec{r}_j) \prod_{k(\neq j)}^A \left[ 1 - \gamma(\vec{b} - \vec{s}_k) \right]$$

The first approximation is justified since the shadows are small compared with unity. The second approximation is more thoroughgoing. It assumes an independence of the shadows of the path of integration in Eqs (8). It is instructive to observe the structure of the multiple scattering series given by (10b). E.g., the double scattering term reads:

$$(11) \quad \hat{F}_{\text{large}}^{(2)} = -\frac{1}{2\pi ip} \sum_{j \neq k}^A \int d^3 q_j d^3 q_k f_{\text{large}}(\vec{q}_j) f_{\text{forward}}(\vec{q}_k) \times \\ \times e^{i\vec{q}_j \cdot \vec{r}_j} e^{i\vec{q}_k \cdot \vec{r}_k} \delta^{(3)}(\vec{q} - \vec{q}_j - \vec{q}_k) \delta(q_k^{\parallel})$$

6.

Eq. (11) may be considered as a straightforward extrapolation of the analogous term in the Glauber formula. The modification consists in the inclusion of longitudinal momentum transfer and in the distinction between forward and large angle scatterings.

In order to compute the elastic scattering on the basis of Eqs (9) and (10) we need a suitable representation of the elementary amplitude and the nuclear density for averaging out the positions of the target nucleons. In calculations presented in Figs 1 and 2 we have used the following form of the nucleon-nucleon amplitude ( $q_{\perp}$  and  $q_{\parallel}$  being transverse and longitudinal momentum transfers):

$$(12) \quad f(q_{\perp}, q_{\parallel}) = \frac{ip\sigma(1-i\alpha)}{4\pi} \sum_{j=1}^3 f_j(q_{\parallel}) e^{-\frac{1}{2} a_j q_{\perp}^2} e^{-\frac{1}{2} b_j q_{\parallel}^2}$$

$$f_1 = 1, \quad f_2 = \sqrt{c_2} q_{\parallel}, \quad f_3 = (c_3 + id_3) q_{\parallel}^2$$

with the parameters obtained by the MINUIT adjustment to the nucleon-nucleon data in the range 1-20 GeV. The parameters are (in  $\text{GeV}^{-2}$ ):  $a_1 = b_1 = 9.42$ ,  $a_2 = 8.67$ ,  $b_2 = 1.38$ ,  $a_3 = 2.16$ ,  $b_3 = 0.23$ ,  $c_2 = 8.92$ ,  $c_3 = 3.47 \times 10^{-3}$ ,  $d_3 = 2.46 \times 10^{-2}$ . For 1 GeV protons we have taken  $\sigma = 43.8$  mb,  $\alpha = -0.27$ . The first element of the sum in Eq. (12) determined the term "forward", the remaining two the term "large".

For the ground state density we have used the pair correlation model<sup>(4)</sup>:

$$(13) \quad |\Psi|^2 = \frac{A}{\pi} \rho(r_j) \left\{ 1 + \sum_{l=1}^{A/2} (2^{1/l})^{-1} \sum_{j_1 \neq k_1 \neq \dots \neq j_l \neq k_l} \left[ \Delta(j_1, k_1) \dots \Delta(j_l, k_l) \right] \right\}$$

$$\Delta(j, k) = G^2(r_{jk}) - 1$$

which is capable of accounting for the short-range interactions between

the target nucleons. The successive terms of (13) correspond to the expansion in numbers of correlated pairs: independent particles, one correlated pair, etc. The two inputs of the model: the single particle density  $\rho(r)$  and the correlation operator  $G(r_{jk})$ , have been chosen as follows:

$$(14) \quad \rho(r) = \pi^{-3/2} R^{-3} \exp\left(-\frac{r^2}{R^2}\right), \quad G^2 = \frac{g^2 + (M-1)g}{M},$$

$$g(r_{12}) = 1 - \exp\left(-\frac{\lambda r_{12}^2}{R^2}\right)$$

with the parameters for the  ${}^4\text{He}$  nucleus<sup>(4)</sup>:  $R = 1.265$  fm,  $\lambda = 0.652$ , the coefficient  $M$  being determined by the normalization.

In Fig. 1 we have presented the elastic cross-section for  $p$ - ${}^4\text{He}$  scattering at 1 GeV, calculated in various models of multiple scattering. Here, the uncorrelated form of the nuclear density ( $G \equiv 1$ ) was used. We see that the large angle contribution already begins to be important in the region of the minimum. It makes this minimum rather shallow, which effect is especially evident in the calculations with shadows. This remarkable feature of diffraction minima has recently been widely discussed in connection with some discrepancies between experimental data<sup>(5, 6)</sup>. We are of the opinion that the shallowing of a minimum is mainly due to large angle scattering which, in the region where a destructive interference of single and double scattering is most effective, successfully competes with forward collisions. Going outside the minimum, the role of the large angle term is still increasing, although its effect may be, to some extent, simulated by forward scatterings. In fact, the Glauber model and ours give here similar results. We wish, however, to stress that the two features of recent experimental data<sup>(6)</sup>, viz. the shallow minimum and the structureless fall of the cross-section down the second maximum;



8.

which are well reproduced by our model, seem to be in conflict with the Glauber model.

In Fig. 2 we have illustrated the influence of short-range correlations on multiple scattering. Because of numerical difficulties, only the simpler version of our model (no shadows - Eq. (10b)) was considered. The increase in the cross-section, caused by the correlations near  $50^\circ$ , suggests that the fit given by our model with shadows (solid line in Fig. 1) might be improved. This problem and attempts at other representations of elementary amplitudes are now being investigated.

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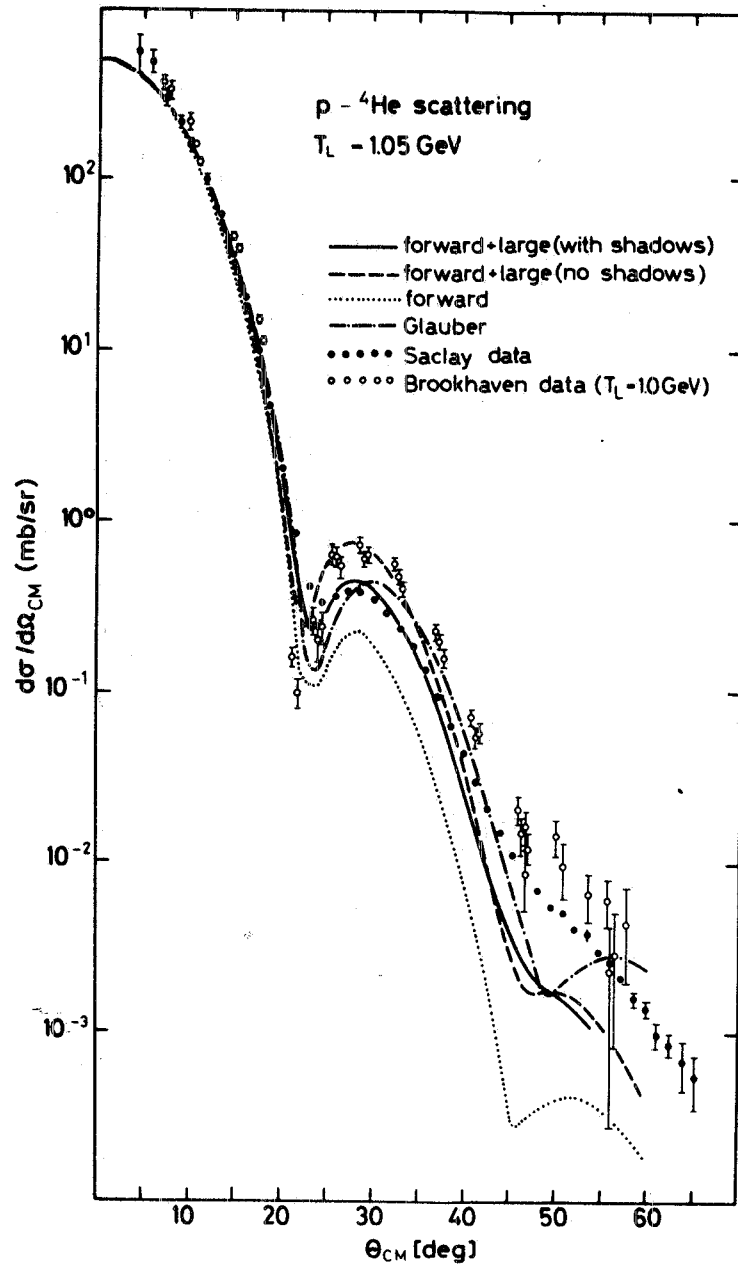


FIG. 1 - Comparison of various models of multiple scattering. The curves denoted "forward+large" are calculated either from Eqs. (9) and (10a) (with shadows), or from Eqs. (9) and (10b) (no shadows). The curves "forward" and "Glauber" are given by Eq. (9); they differ by a value of the slope parameter in the first term of Eq. (12) which for the Glauber calculations at 1 GeV has been chosen  $a = 4.7 \text{ GeV}^{-2}$ . The experimental data are from Refs (5) and (6).

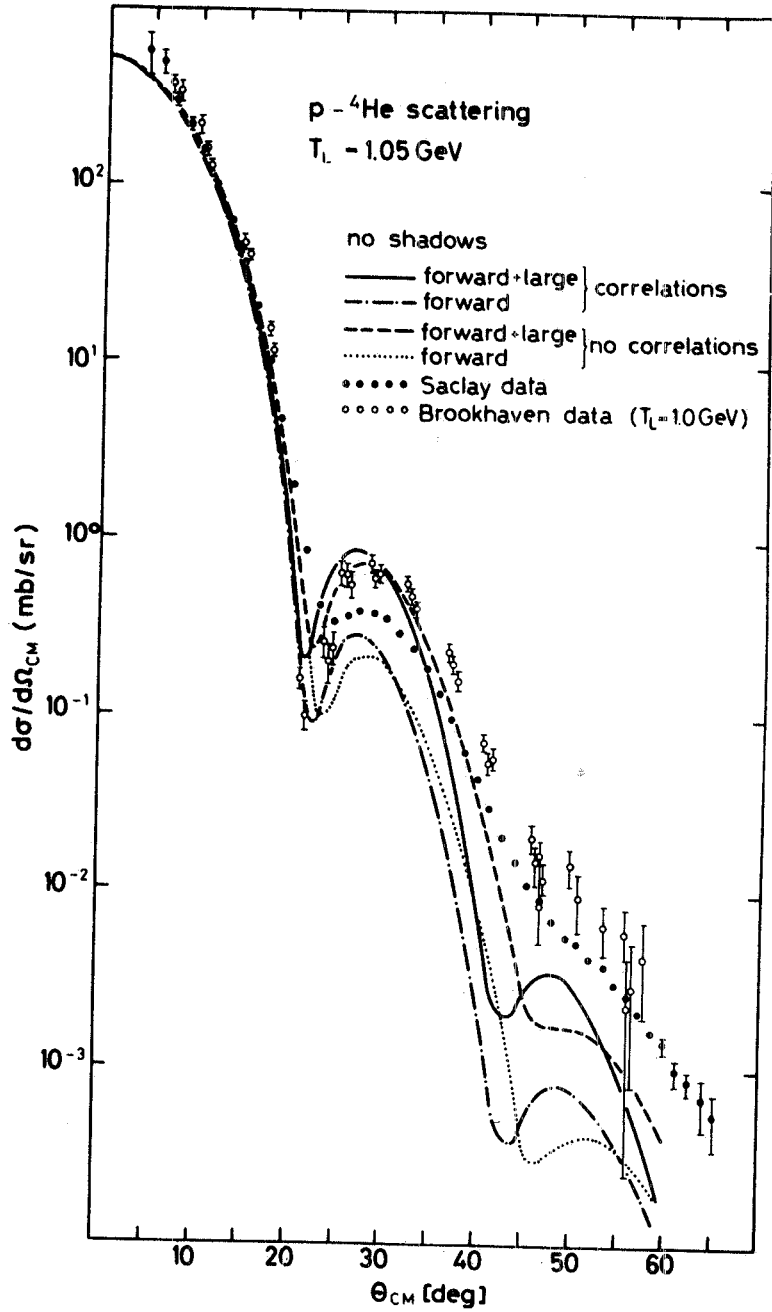


FIG. 2 - The role of short-range correlations in multiple scattering.