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E. Etim: LOCAL DUALITY IN e^+e^- ANNIHILATION FROM CANONICAL TRACE ANOMALIES.

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ABSTRACT. -

Using familiar techniques of the renormalization group we establish the local version of the duality sum rules shown recently, in the framework of the canonical trace anomaly of the energy momentum tensor, to relate the asymptotic value of the ratio $R = \sigma(e^+e^- \to \text{hadrons})/ / \sigma(e^+e^- \to \mu^+ \mu^-)$ to the low energy moments of $\sigma(e^+e^- \to \text{hadrons})$. In order of magnitude the experimental data are well followed by the duality average up to the SLAC-LBL and CEA energy range.

The idea[1] that neutral vector mesons lying on linear trajectories of Veneziano type build up an asymptotically scaling contribution to the total hadronic cross section in e^+e^- annihilation is very interesting. This new form of duality has been established formally by us in a recent note[2] as a set of sum rules in the framework of the canonical trace anomaly of the energy momentum tensor. The merit of this approach lies not only in its generality but also in the rather simple way in which the breaking of scaling can be related to the breaking of asymptotic scale invariance. By means of this relationship it is possible to gain some understanding of the puzzling (almost constant) behaviour of the total cross section in the SLAC-LBL[3] and CEA[4] energy range and perhaps other surprises which future experiments may reveal. The purpose of this letter is to show how this connection energies and indicate the precise manner of descent to the local version of the duality sum rules for both the scaling and non-scaling contributions to the total cross section. We thus learn from this exercise that duality is a rather general principle of the hadronic world.

The mechanism for breaking scale (and conformal) invariance is the usual one dictated by the non-vanishing of the trace, $\theta^\lambda_\lambda(x)$, of the energy momentum tensor. Defining the Green functions

$$\Delta_{\mu\nu}(p,q) = \int d^4x d^4y\, e^{ip+iqy} \langle 0 | T(\theta^\lambda_\lambda(x) J_\mu(y) J_\nu(0)) | 0 \rangle \tag{1}$$

$$\pi_{\mu\nu}(q) = i \int d^4y\, e^{iqy} \langle 0 | T(J_\mu(y) J_\nu(0)) | 0 \rangle \tag{2}$$

where $J_\mu(y)$ is the hadronic electromagnetic current, formal manipulations, based on canonical light cone singularities, allow one to derive the anomalous trace identity[5,6]

(3) $$\Delta(q^2) = -2q^2 \frac{\partial \pi(q^2)}{\partial q^2} - \frac{2\alpha}{3\pi} R$$

where

(4) $$\Delta_{\mu\nu}(q, 0) = (q_\mu q_\nu - g_{\mu\nu} q^2)\, \Delta(q^2)$$

$$\pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\, \pi(q^2)$$

and the constant R is the asymptotic value of the ratio $\sigma(e^+e^- \to \text{hadrons})/$ $/\,\sigma(e^+e^- \to \mu^+\mu^-)$. Since, by Weinberg's theorem[7], $\Delta(q^2)$ is expected to vanish asymptotically one can write for it an unsubtracted dispersion relation

(5) $$\Delta(q^2) = -\frac{2}{\pi} \int_{s_0}^{\infty} ds \frac{s \frac{\partial}{\partial s} \operatorname{Im} \pi(s)}{s - q^2}$$

where $\operatorname{Im}\pi(s)$ is related to the total e^+e^- annihilation cross section into hadrons by

(6) $$\sigma_{had}(s) = \frac{4\pi\alpha}{s} \operatorname{Im}\pi(s)$$

We assume that asymptotically $\sigma_{had}(s)$ approaches a scaling limit when $s \gtrsim \bar{s}$ where $\bar{s}$ is some cut-off. Making use of this fact in eq. (4) leaves us with

(7) $$\Delta(q^2) = -\frac{2}{\pi} q^2 \int_{s_0}^{\bar{s}} ds \frac{\operatorname{Im}\pi(s)}{(s-q^2)^2} - \frac{2\alpha}{3\pi} R \frac{\bar{s}}{\bar{s} - q^2}$$

In the large q^2 limit ($q^2 \gg \bar{s}$), eq. (7) gives an asymptotic expansion of $\Delta(q^2)$ in inverse powers of q^2. Carrying out this expansion with

(8) $$q^2 \Delta(q^2) \sim \frac{2}{\pi} \sum_{n=0}^{\infty} C_n (q^2)^{-n}$$

and comparing coefficients yields the set of sum rules

$$\int_{s_o}^{\bar{s}} ds\, s^n \operatorname{Im} \pi(s) = \frac{\alpha}{3} R \frac{\bar{s}^{n+1}}{n+1} - \frac{C_n}{n+1}\,; \qquad n = 0, 1, 2 \ldots.. \tag{9}$$

By their definition the coefficients C_n have the dimension of mass to the power $2(n+1)$. The available mass parameters setting their scale are those which break scale invariance and keep $\theta^{\lambda}_{\lambda}(x)$ non-vanishing. According to eq. (9) the same parameters break scaling and prevent $\operatorname{Im}\pi(s)$ from averaging to a constant. Above the prominent resonances ($\varrho, \omega, \phi, \varrho' \ldots$), and for s up to about 9 GeV^2, $\operatorname{Im}\pi(s)$ is approximately constant at about $(2\text{-}2.5)\alpha/3$, in agreement with the prediction of the extended vector meson dominance model[1]. This suggests a distinction between two different types of contributions to $\operatorname{Im}\pi(s)$:

i) a precocious component, $\operatorname{Im}\pi_p(s)$, built up by the neutral vector mesons and locally dual to a constant term $\alpha R_p/3$[1,2];

ii) a non-precocious component with an asymptotic form $\operatorname{Im}\pi^A_{NP}(s)$ which tends to a constant, $\alpha R_{NP}/3$, at infinity and averages at low energies the contributions $\Sigma_B \operatorname{Im}\pi_B(s)$ due to non-negligible thresholds of the final states B (and to all other dimensional parameters). For simplicity we consider only these thresholds as the mass parameters with which the coefficients C_n scale. If $\bar{s}$ is not taken in the scaling region we include it as one of the mass parameters in C_n.

The strategy is now to use the scale operator $\Sigma_B s_B(\partial/\partial s_B)$ to project out C_n from eq. (9) and observe that all the elements for breaking scale invariance are already contained in $\operatorname{Im}\pi^A_{NP}(s)$. In this way one finds as a reasonable approximation

$$\Sigma_B S_B \frac{\partial}{\partial S_B}\left[\frac{C_n}{n+1}\right] = C_n = \frac{\alpha}{3} R_{NP} \bar{S}^{n+1} - \int_{s_o}^{\bar{s}} ds\, s^n \operatorname{Im}\pi^A_{NP}(s) \tag{10}$$

valid for all $\bar{s}$ asymptotic or not. Substituting from here into eq. (9) and defining $\mathrm{Im}\,\pi^A(s)$ by

$$\mathrm{Im}\,\pi^A(s) = \left(\frac{\alpha}{3} R_P + \mathrm{Im}\,\pi^A_{NP}(s)\right) \tag{11}$$

one gets the sum rules

$$\int_{s_o}^{\bar{s}} ds\, s^n \left(\mathrm{Im}\,\pi(s) - \mathrm{Im}\,\pi^A(s)\right) = 0; \quad n = 0, 1, \ldots \tag{12}$$

which constitute our main result. Thus given $\mathrm{Im}\,\pi^A(s)$ in $s \gtrsim \bar{s}$ its extrapolation into the region $s \lesssim \bar{s}$ represents $\mathrm{Im}\,\pi(s)$ on the average. Actually eq. (12) is valid component-wise as given by the equations

$$\int_{s_o}^{\bar{s}} ds\, s^n \left(\mathrm{Im}\,\pi_P(s) - \frac{\alpha}{3} R_P\right) = 0; \quad n = 0, 1, \ldots \tag{13 a}$$

$$\int_{s_o}^{\bar{s}} ds\, s^n \left(\mathrm{Im}\,\pi_{NP}(s) - \mathrm{Im}\,\pi^A_{NP}(s)\right) = 0; \quad n = 0, 1, \ldots \tag{13 b}$$

The first of these, especially for $\bar{s} < \min(S_B)$ when it becomes

$$\int_{s_o}^{\bar{s}} ds\, s^n \left(\mathrm{Im}\,\pi(s) - \frac{\alpha}{3} R_P\right) = 0; \quad n = 0, 1, \ldots \tag{14}$$

is what the Frascati data[8] and precocious scaling[1] would lead one to expect.

Up to this point we have tacitly assumed that in one way or another it is possible to determine $\mathrm{Im}\,\pi^A_{NP}(s)$ for large s. While this was easy for $\mathrm{Im}\,\pi_P(s)$ by appeal to precocious scaling, we have no general principle to guide our search for $\mathrm{Im}\,\pi^A_{NP}(s)$. If one is willing, however, to accept the idea of asymptotically point-like structure for hadrons, which in effect guarantees a scaling total cross section at infinity, then the determination of $\mathrm{Im}\,\pi^{A}_{NP}(s)$ is quite simple. For purposes of comparison with experiment we identify the thresholds s_B with the production of point-like baryon-antibaryon pairs of the octet and decuplet representations of SU(3)[9]. One finds for $\mathrm{Im}\,\pi^{A}(s)$ in this case

$$\mathrm{Im}\,\pi^{A}(s) = \frac{\alpha}{3}\left(R_P + \Sigma_B R_B \frac{(2J_B+1)(J_B+1)}{6\,J_B}\left(1-\frac{s_B}{s}\right)^{1/2}\right) \tag{15}$$

where $J_B(=1/2,\ 3/2)$ is the spin of the baryon of mass $\sqrt{s_B/2}$ and effective charge $\sqrt{R_B}$ (in units of that of the electron). For ease of computation we average over the baryon multiplets and rewrite eq. (15) as

$$\mathrm{Im}\,\pi^{A}(s) = \frac{\alpha}{3}\left(R_P + R_{B_o}\left(1-\frac{s_{B_o}}{s}\right)^{1/2} + \frac{10}{9} R_D\left(1-\frac{s_D}{s}\right)^{1/2}\right) \tag{16}$$

where for the octet and decuplet thresholds we take $s_{B_o} = 5.29\ \mathrm{GeV}^2$ and $s_D = 6.76\ \mathrm{GeV}^2$ respectively. Eq. (16) is compared with experiment[3,4,8] in Fig. 1 where for curve (a) we have taken $R_P = 8\pi^2/f_\varrho^2 = 2.5$[1], $R_{B_o} = R_D = 1$ and for (b) $R_P = R_{B_o} = R_D = 2$. There is a strong indication therefore that the duality average does in fact follow the data locally in order of magnitude over a wide range of energy.

In conclusion it is interesting to remark that, although it was not stated explicitly in the beginning, our derivation of eq. (12) from eq. (9) amounts to solving a renormalization group equation of the Callan-Symanzik type[10] with mass but no coupling constant derivatives as expected of a theory which scales at infinity. The duality sum rules provide an alternative way to exhibit the approximate solutions of the renormalization group differential equations for the low energy moments of $\mathrm{Im}\,\pi(s)$.

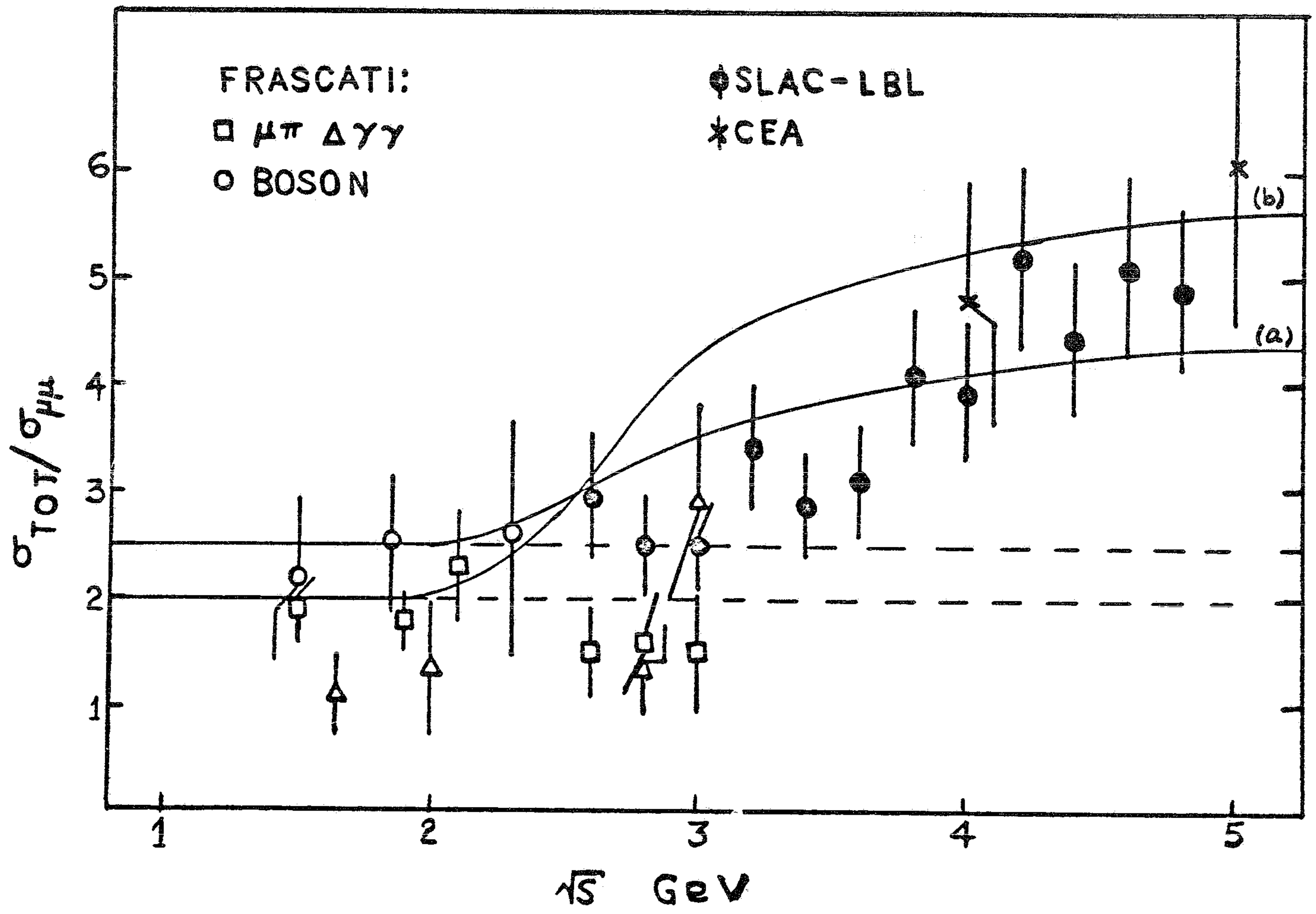

FIG. 1 - Plot of $\sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ against $\sqrt{s}$ as given by eq. (16). For curve (a) $R_P = 2.5$, $R_{B_o} = R_D = 1$ and for (b) $R_P = R_{B_o} = R_D = 2$.

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