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INTRODUCTION. -

Medium energy pion beams can be obtained by electron linear accelerators in a two steps process, consisting in production of a Bremsstrahlung gamma ray beam followed by the subsequent photo productions of pions. The two processes can either be made to occur in the same target(1) or in two separate targets - the radiator and the pion source - as is the case of the geometry adopted at the LEALE Laboratory in Frascati(2). In this geometry radiator and pion source are 35 cm apart, and a magnet placed between them sweeps away electrons which could contribute to contaminate the π^- beam.

High density materials with low A and low Z are generally choosen as pion sources, since photoproduction is a surface phenomenon which shows an $A^{2/3}$ dependence so that the yield will vary as $A^{-1/3}$. In our Laboratory a Carbon target has been preferred, since it allows for rather similar yields of positive and negative pions.

On the other hand, high Z materials must be choosen as radiators, in order to enhance the Bremsstrahlung yield. Moreover, due to the small electron beam cross section, a large amount of power impinges on a restricted area of the radiator and can result in a local or general melting of it. Therefore high melting point and good thermal conductivity place a further constraint on the selection of suitable materials.

In this paper, we study the temperature field established by the primary electron beam throughout the radiator, for various dimensions, thicknesses and materials, also in order to find out criteria for the choose of the material and for a possible cooling of it.

2. - THE STEADY STATE TEMPERATURE EQUATION. -

The equation for the steady state heat conduction through the radiator, keeping in account the thermal radiation from the surface, can be established as follows[3]:

(1) $$\mathrm{div}\,(\lambda\, \vec{\mathrm{grad}}\, T) + q_E - q_I = 0$$

where

$\lambda = \lambda(T)$ is the thermal conductivity of the material, q_E and q_I are the quantities of heat supplied and removed per unit volume of the medium and per unit time.

Let the radiator be a disc of radius R and thickness h. The electron beam can be schematized as having a circular cross section with radius R_o ($R_o < R$) and a uniform density within R_o. In this case we have

(2) $$q_E = \frac{W}{h \pi R_o^2} \qquad q_I = \frac{2}{h} \varepsilon(T)\, \sigma\, (T^4 - T_e^4)$$

where:

W is the power deposited in the radiator by the electron beam;

$\varepsilon(T)$ is emissivity of the radiator's material;

σ is the Stefan - Boltzmann constant;

T_e is the temperature of the external surroundings.

We then write the Laplace operator of (1) in cylindrical coordinates and impose the circular simmetry and the independence of the solution from the thickness h (since $h \ll R$). Eq. (1) can therefore be rewritten as

(3) $$\frac{d^2T}{dr^2} + \frac{1}{r}\frac{dT}{dr} + \frac{1}{\lambda}\frac{d\lambda}{dr}\frac{dT}{dr} - \beta(T)(T^4 - T_e^4) = -\alpha(T)\, f(r)$$

where r is the radial coordinate measured from the center of the disc and

(4) $$\alpha(T) = \frac{q_E}{\lambda(T)}; \qquad \beta(T) = \frac{2\sigma}{h}\frac{\varepsilon(T)}{\lambda(T)}; \qquad f(r) = \begin{cases} 1 & 0 \leq r \leq R_o \\ 0 & r > R_o \end{cases}$$

3. - BOUNDARY CONDITIONS. -

In order to fix the boundary conditions of eq. (3), we will consider three separate cases:

a) The radiator is completely in vacuo; heat is exchanged only through thermal radiation;

b) The radiator is cooled along its circumference; heat is exchanged both by thermal radiation and convection.

c) Thermal radiation is completely neglected; heat is exchanged only by means of circumferential cooling ($\beta(T) \equiv 0$).

In case a), there is no heat conduction at r = R, so that

(5) $$\left[\lambda\, \overrightarrow{\text{grad}}\, T\right]_{r=R} = 0$$

which in our hypothesis becomes

(6) $$\left(\frac{dT}{dr}\right)_{r=R} = 0$$

Another condition on T can be found by integrating eq. (1) to all the volume of the radiator. We then observe that

(7) $$\int_V \operatorname{div}(\lambda\ \overrightarrow{\text{grad}}\, T)\, dV = \int_S (\lambda\ \overrightarrow{\text{grad}}\, T \cdot \vec{n})\, dS = 0$$

for the condition (5), and that the solution for T must therefore satisfy the following integral condition

(8) $$\frac{2\sigma}{h} \int_V \varepsilon(T)(T^4 - T_e^4)\, dV = \int_{V_o} q_E\, dV \quad \left\{V_o = \pi h R_o^2\right\}$$

which can finally be written:

(9) $$4\pi\sigma \int_o^R \varepsilon(T)(T^4 - T_e^4)\, r\, dr = W$$

In the case b), we call μ the heat transfer coefficient between the lateral surface of the disc and the cooling medium and remark that

(10) $$\lambda(T_R)\left[\frac{dT}{dr}\right]_R = -\mu(T_R - T_C)$$

where T_R is the temperature of the disc at $r = R$ and T_C is the temperature of the cooling medium. Integration of equation (1) with the condition (10) now yields

(11) $$4\pi\sigma \int \varepsilon(T)(T^4 - T_e^4)\, r\, dr = W - 2\pi\mu\, hR(T_R - T_C)$$

In case c) eq. (10) remains valid and eq. (11) becomes

(12) $$W = 2\pi\mu h R \left[T_R - T_C\right]$$

4. - PARAMETERS OF THE TEMPERATURE EQUATION. -

4.1. - Power absorption in the radiator. -

The power lost by the beam by ionization per unit thickness of the radiator is given by

(13) $$\frac{W}{h} = 10^6 \left[\frac{dE}{dx}\right]_{min} i_p f T \qquad \text{(Watt/cm)}$$

where

$\left[\frac{dE}{dx}\right]_{min}$ is the energy loss per unit length (in MeV/cm) at minimum of ionization, i_p (in A), f (in sec^{-1}), T (in sec) are respectively the peak current, the frequency and the pulse width of the beam. The values of $\left[dE/dx\right]_{min}$ are reported in Table I, together with values of W/h calculated for the standard operating conditions of our accelerator (i_p = 100 mA, f = 250 sec^{-1}, T = 3.5 μ sec).

For the sake of simplicity, energy carried away from the radiator by secondary electrons and density effect have been neglected in this calculation.

4.2. - Radius of the electron beam. -

The actual values of the electron beam radii to be inserted in the temperature equation depend from a number of factors: the peculiar features of our accelerator and of the magnetic transport, the

TABLE I

Material	Z	Melting Point (^oK)	λ_0 ($W cm^{-1} {}^oK^{-1}$)	λ_1 ($W cm^{-1} {}^oK^{-2}$)	λ_2 ($W cm^{-1} {}^oK^{-3}$)	L ($Volt^2 {}^oK^{-2}$)	Validity range (in ^oK)$^{(x)}$	$[dE/dx]_{min}$ (MeV/cm)	W/h (Watt/cm)
W	74	3673	1,6521	$-5,4305 \cdot 10^{-4}$	$9,71 \cdot 10^{-8}$	$39,3 \cdot 10^{-9}$	800÷3300	22,6	1977
Ta	73	3123	0,52	$(7,3 \pm 1,5) \cdot 10^{-5}$	0	$39,3 \cdot 10^{-9}$	1500÷2800	19,4	1699
Cu	29	1356	4,068	$-5,9774 \cdot 10^{-4}$	$-7,08 \cdot 10^{-8}$	$22,3 \cdot 10^{-9}$	300÷1200	12,9	1129

(x) - Interval within which the best fit formula is in good agreement with experimental data.

optical characteristics of the produced pion beam, and the geometrical arrangement of the radiator-pion source assembly. In the case when radiator and pion source are tied together (or are physically the same target), the e⁻ beam emittance places a lower limit to its radius, whereas an upper limit is set by the transverse dimensions one can require for the secondary pion beam.

When radiator and pion source are physically separated, as it is in our pion channel, the lower limit to the radius is still dictated by the acceleration and magnetic transport features, and turns out to be $\approx$ 1 mm in our case; on the other hand, the upper limit is determined by the radiator-pion source distance, the source dimensions, the angular divergence of the e⁻ beam and its multiple scattering inside the radiator. Our pion beam is used in connection with a magnetic spectrometer[(4)], whose first order momentum resolution depends upon the pion source's dimensions. In our case the pion source is a cylinder 5 cm long and with a radius of 0.6 cm, and gives rise to a momentum resolution of about 1%. Once the source radius has been fixed, the size of the e⁻ beam to be focussed on the radiator must allow for intercepting the maximum Bremsstrahlung fraction by the source itself. This fraction turns out to be a decreasing function of the e⁻ beam radius. Detailed calculations carried out for our geometry (radiator and pion source 35 cm apart), for various radiators' thicknesses and different beam radii are reported elsewhere [(5)]: here we just want to recall that for a beam radius of 3 mm, a radiator thickness of 0.25 r.l., the fraction of intercepted Bremsstrahlung is of the order of 46%, and becomes about 40% for a radius of 5 mm. The useful range of beam radii is therefore 1 to 5 mm in our case. As a consequence we performed all the calculations of this paper for standard value of 3 mm for the e⁻ beam radius.

4.3.- Thermal conductivity and hemispherical emissivity.-

Both the thermal conductivity λ and the hemispherical emissivity ε of materials are function of the temperature. The former has been represented by a best fit equation of the type

(14) $$\lambda(T) = \lambda_o + \lambda_1 T + \lambda_2 T^2$$

extracted from experimental values quoted in textbooks[6]. The coefficients of the best fit are shown in Table I, together with other quantities related to the materials we took in consideration for this work. The emissivity, according to theoretical considerations[7], has the following expression

(15) $$\varepsilon(T) = f\sqrt{T\,r(T)} - g\,T\,r(T)$$

with

(16) $$r(T) = \frac{LT}{\lambda(T)} \qquad f = \begin{cases} 0,751\ \Omega^{-1/2}\ \mathrm{cm}^{-1/2}\ {}^oK^{-1/2} & \text{for } 0 \leq r(T)T \leq 0,2\ \Omega\,\mathrm{cm}\,{}^oK \\ 0,698\ \Omega^{-1/2}\ \mathrm{cm}^{-1/2}\ {}^oK^{-1/2} & \text{for } 0,2 \leq r(T)T \leq 0,5\ \Omega\,\mathrm{cm}\,{}^oK \end{cases}$$

$$g = \begin{cases} 0,396\ \Omega^{-1}\ \mathrm{cm}^{-1}\ {}^oK^{-1} & \text{for } 0 \leq r(T)T \leq 0,2\ \Omega\,\mathrm{cm}\,{}^oK \\ 0,266\ \Omega^{-1}\ \mathrm{cm}^{-1}\ {}^oK^{-1} & \text{for } 0,2 \leq r(T)T \leq 0,5\ \Omega\,\mathrm{cm}\,{}^oK \end{cases}$$

The parameter L can be adjusted to fit the available experimental data (see Table I). Fig. 1 shows the temperature dependence of conductivity and emissivity resulting from equations (14) and (15) for Cu, Ta, W.

4.4.- The coefficient of heat transfer.-

The heat transfer coefficient μ is a quantity which can hardly be known "a priori", since it is a function of many variables which are specific both for the flow and heat conduction of the cooling fluid and for the mechanical design of the heat exchanger.

Moreover, the heat exchange coefficient depends also upon the local temperature of the cooled surface.

Leaving to the engineer the task of practically achieving a given coefficient of heat transfer, we limited ourselves to evaluate its influence on cooling, by varying its value in a wide range.

5. - THE SOLUTIONS OF THE TEMPERATURE EQUATION. -

Boundary conditions (6) to (12) allow us to solve the temperature equation (3) for all the three cases we took in consideration. As far as the authors know, no analytical general solution of this type of equation has been found out till now, even in the simple approximation $\lambda(T) = \text{const}$, $\beta(T) = \text{const}$ and $\alpha(T) = \text{const}$. Numerical solutions instead can be found, by repeatedly applying the standard integration method of Runge-Kutta-Gill[8], and imposing the given integral boundary conditions.

We finally remark that in case c) ($\beta(T)=0$) eq. (3) can be analytically solved if $\lambda(T) = \bar{\lambda} = \text{const}$. Boundary conditions are given by equations (10) and (12) and the radial dependence of temperature becomes

$$T(r) - T_C = \frac{1}{2\pi} \frac{W}{h} \left[\frac{1}{\mu R} + \frac{1}{\bar{\lambda}} \ln \frac{R}{R_o} + \frac{1}{2\bar{\lambda}} \left(1 - \frac{r^2}{R_o^2}\right) \right] \qquad 0 \leq r \leq R_o \tag{17}$$

$$T(r) - T_C = \frac{1}{2\pi} \frac{W}{h} \left[\frac{1}{\mu R} + \frac{1}{\bar{\lambda}} \ln \frac{R}{r} \right] \qquad R_o \leq r \leq R \tag{18}$$

6. - CONCLUSIONS. -

The radial temperature distribution has been plotted for the three different boundary heat exchange mechanisms we took in account: heat loss by thermal radiation only, by circumferential (convective) cooling only and by both.

a) - Thermal radiation. -

Fig. 2 shows the radial temperature distribution, in W, for two different radiator's radii (R=1 cm and 1.5 cm) and four thicknesses (h=0.025, 0.075, 0.1 and 0.25 radiation lengths). In Fig. 3 and 4 similar temperature distributions are depicted for Ta and for a Copper radiator with a radius of 1.5 cm. An electron beam radius R_o=3 mm has been adopted everywhere. A sketch of the temperature dependence on the electron beam radius R_o is given in Fig. 8 for a Tantal radiator (R=1 cm and h=0.25 r.l.) and for three different beam radii: R_o=1, 3 and 5 mm.

b) - Convective cooling. -

The radial temperature distribution for a Copper radiator with R=1 cm and h=0.25 r.l. and two different heat transfer coefficients (μ = 10 W cm^{-2} $^oK^{-1}$ and $\mu = \infty$) is given in Fig. 5. Here again the beam radius is 3 mm.

c) - Thermal radiation plus convective cooling. -

The combined effect of the two heat losses can be observed in Figg. 6 and 7. Fig. 6 gives the temperature distribution, in Copper (R= 1 cm, h= 0.25 r.l., $R_o \doteq$ 3 mm) for increasing values of the heat exchange coefficient (μ = 0.01, 0.1, 1 Wcm^{-2} $^oK^{-1}$). In Fig. 7 all the possible heat exchange mechanisms have been considered for a Tantal disc (R = 1.5 cm, h= 0.25 r.l.), starting from the pure heat radiation

($\mu = 0$) up to the pure circumferential heat convection ($\mu = \infty$), through various situations of combined heat losses ($\mu = 0.01$, 0.1, 1 and 10 W cm^{-2} ^{o}K^{-1}).

From the quoted temperature distributions, we can conclude that heat radiation is sufficient to keep the temperature all along the radius throughout the radiator well below the melting point in high melting point materials, such as Tantal and Tungsten, at least in the limits of radii, thicknesses and beam dimensions we have choosen for the calculations, which are typical in radiators' design. We notice also that, despite of the low thermal conductivities of such materials, the pure radiation heat loss can safely prevent from melting even in the central region of the radiator.

On the other hand, high conductivity and low melting point materials such as Copper, show a temperature which is well above the melting point everywhere along the radius, even in "optimum" experimental conditions, viz. maximum radiator's radius and minimum thickness.

Convective cooling is obviously possible for both types of materials, but it is mandatory for the low melting point ones. Moreover it is more efficient, with respect to thermal radiation, in high conductivity materials, where heat can flow more easily from the center to the cooled boundaries. This is clearly shown in Fig. 9, where the percentage of heat lost by circumferential cooling is plotted vs. μ for a Cooper and a Tantalum radiator both with $R = 1$ cm, $h = 0,25$ r.l. and for $R_o = 3$ mm.

REFERENCES. -

(1) - See for instance, P.J. Bertin et al., Saclay Internal Report DPHN/HE/71/3 (1971).

(2) - R.Barbini, S.Faini, C.Guaraldo, C.Schaerf and R.Scrimaglio, Nuclear Instr. and Meth. 115, 85 (1974).

(3) - S.S.Kutateladze, Fundamental of Heat Transfer (E.Arnold Ltd., London, 1963).

(4) - R.Barbini, S.Faini, C.Guaraldo, C.Schaerf and R.Scrimaglio, Frascati Report LNF-73/36 (1973).

(5) - R.Barbini, C.Guaraldo and R.Scrimaglio, Frascati Internal Report LNF-74/2(Int.) (1974).

(6) - a) Landolt-Börnstein, Zahlenwerte und Funktionen aus Physik, Chemie, Astronomie, Geophysik und Technik, II Band, 5 Teil, Bandteil b (Springer Verlag, Berlin, 1968) ;

b) Handbook of Chemistry and Physics, Editor R.C. Weast (Chemical Rubber Co., New York, 1971/72).

(7) - M.Jacob, Heat Transfer (J.Wiley and Sons, New York, 1962), vol. I and II.

(8) - G.A.Korn and T.M.Korn, Mathematical Handbook for Scientists and Engineers (McGraw-Hill Book Co., New York, 1968).

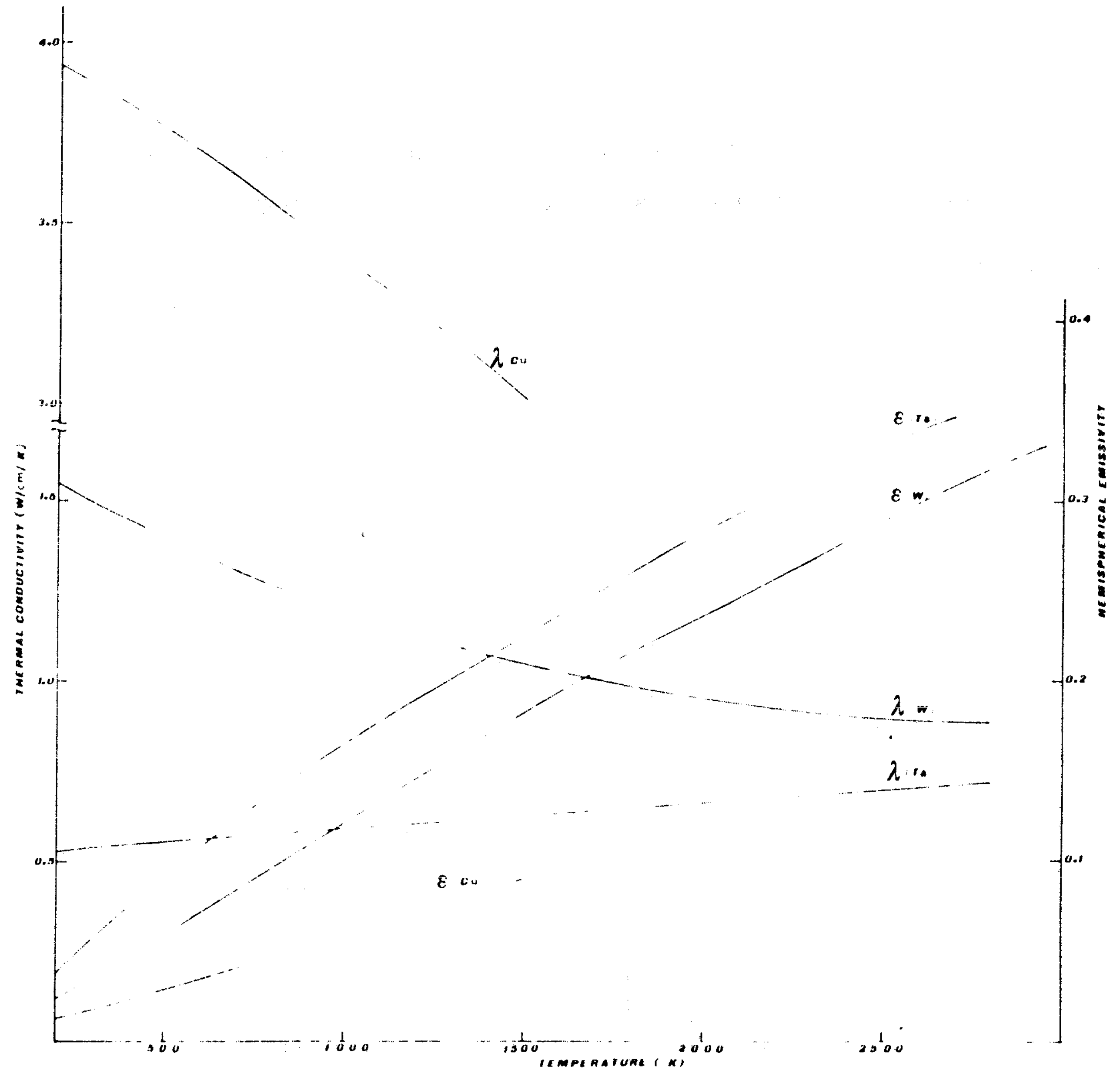

FIG. 1 - Thermal conductivity (λ) and hemispherical emissivity (ε) of Cu, W and Ta vs. temperature.

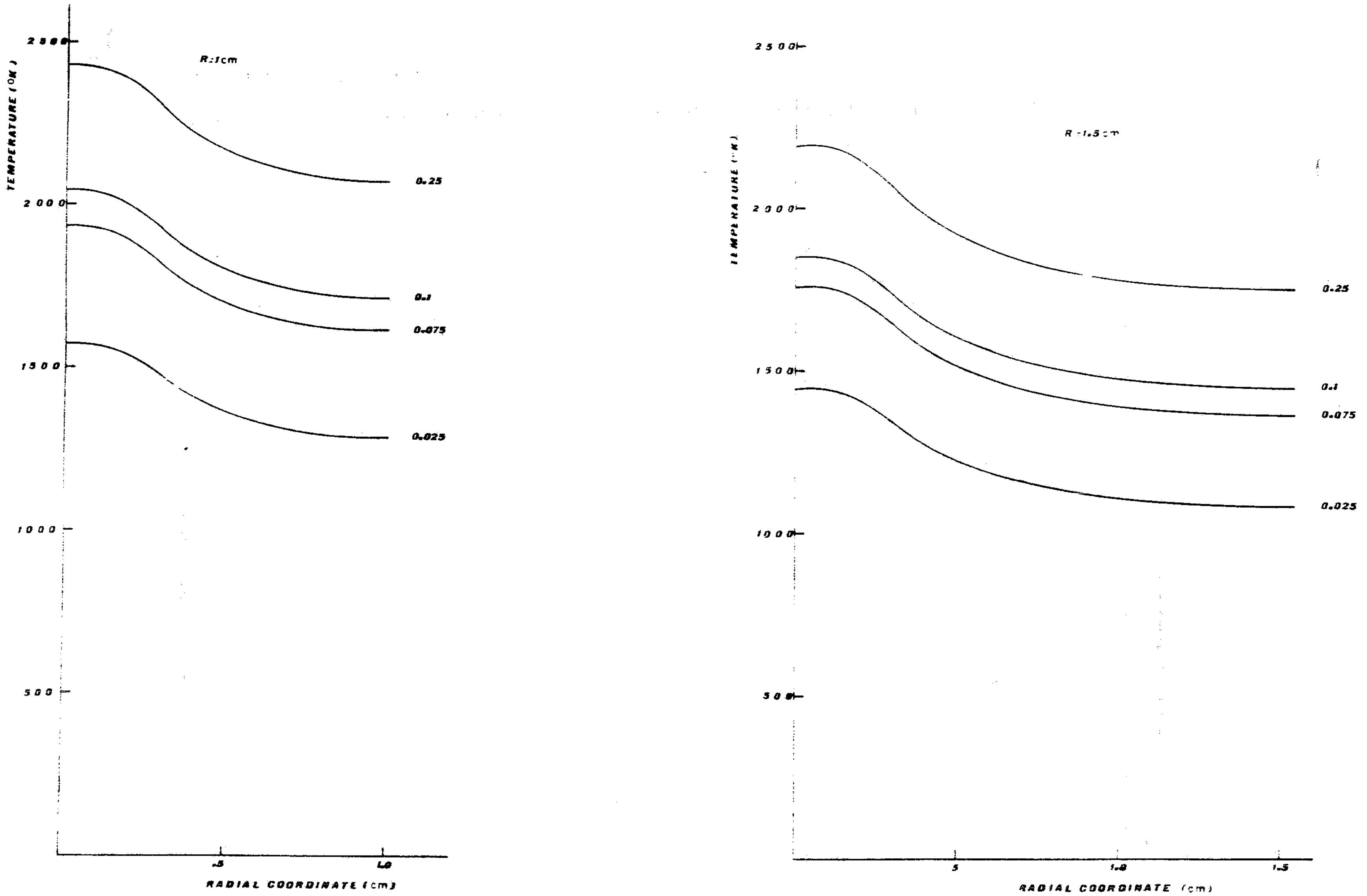

FIG. 2 - Radial temperature distribution for a W disc with radius R=1 cm (left) and R=1,5 cm (right). Curves are labelled by thickness' values in radiation lenghts. Beam radius $R_o = 3$ mm.

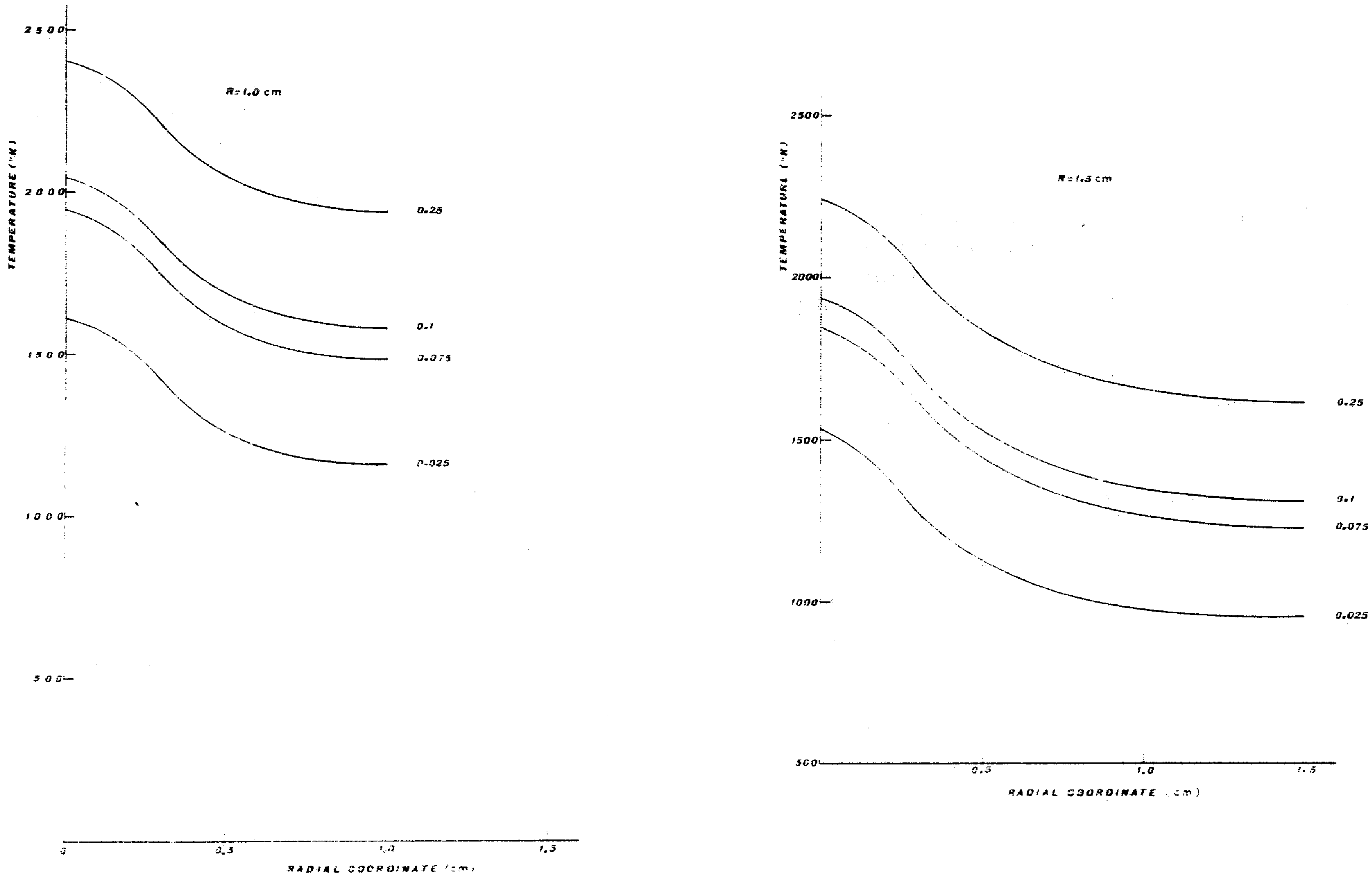

FIG. 3 - Radial temperature distribution for a Ta disc with radius R = 1 cm (left) and R = 1.5 cm (right). Curves are labelled by thickness' values in radiation lengths. Beam Radius R_o = 3 mm.

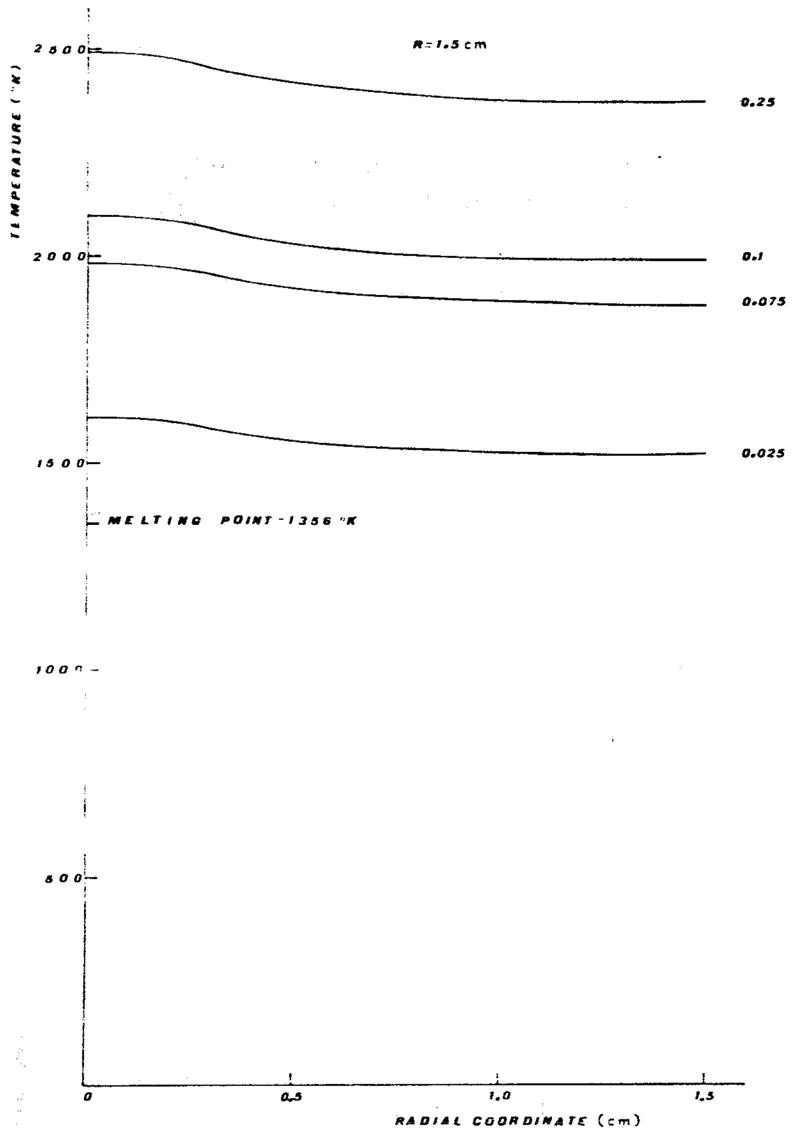

FIG. 4 - Radial temperature distribution for a Cu disc with radius R = 1.5 cm. Curves are labelled by thickness' values in radiation lengths. Beam radius R_o = 3 mm.

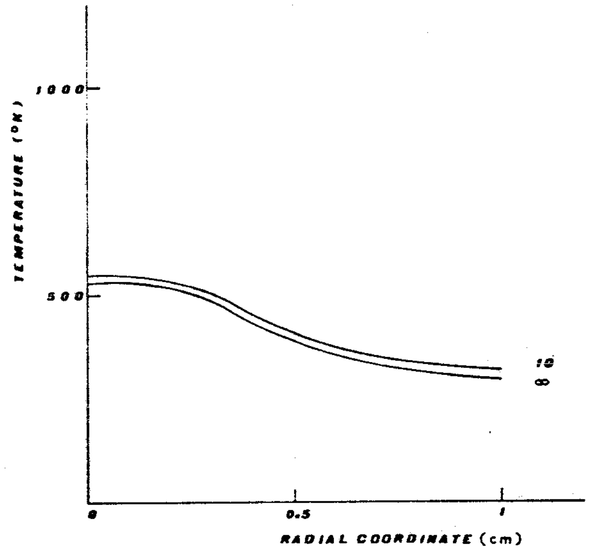

FIG. 5 - Radial temperature distribution for a Cu disc with R = 1 cm, h = 0.25 r.l. Curves are labelled with μ values (in W cm^{-2} $°K^{-1}$). Beam radius R_o = 3 mm.

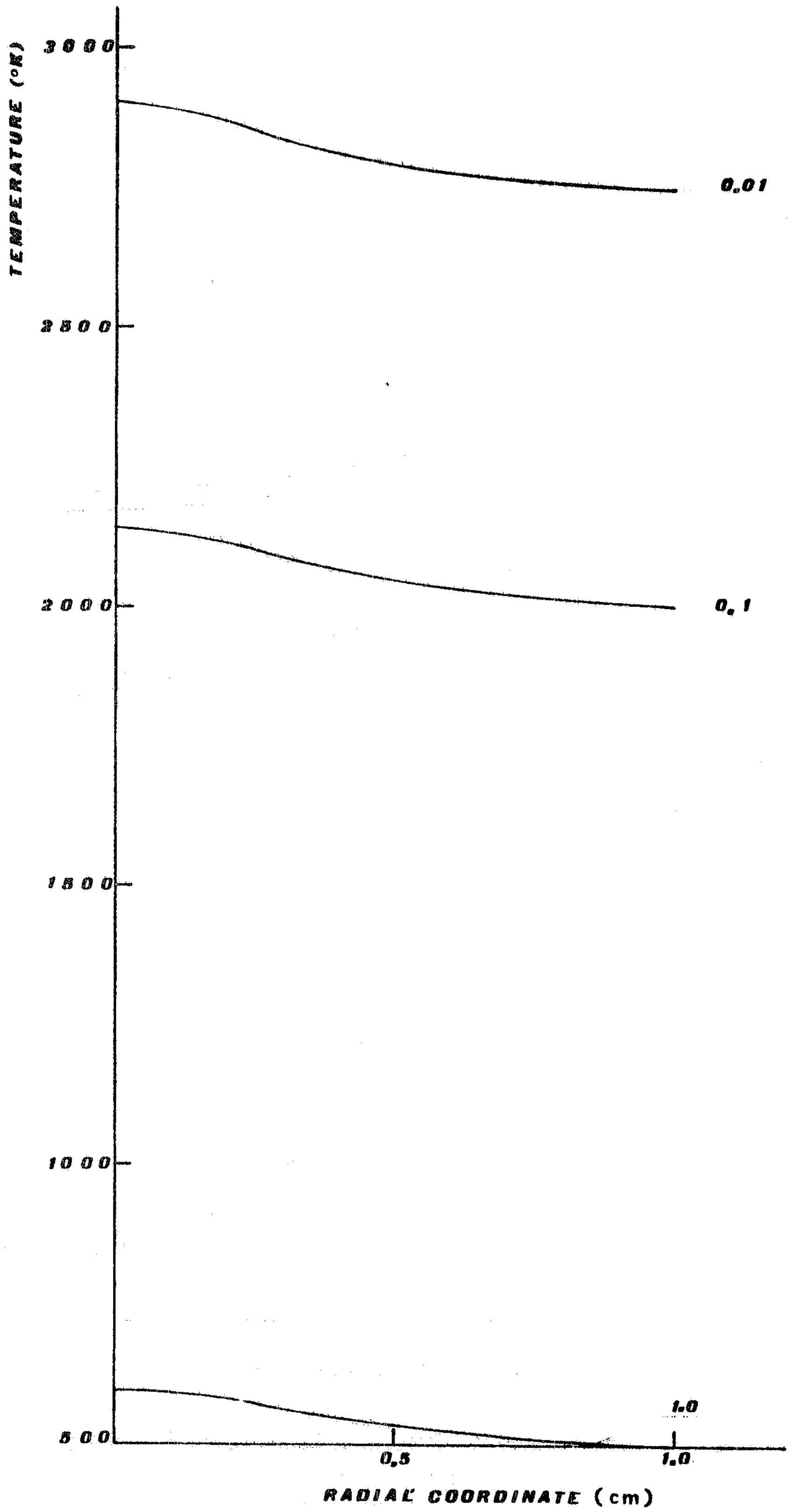

FIG. 6 - Radial temperature distribution for a Cu disc with R = 1 cm, h = 0.25 r.l. Curves are labelled with μ values (in W cm^{-2} $^{o}K^{-1}$). Beam radius R_o = 3 mm.

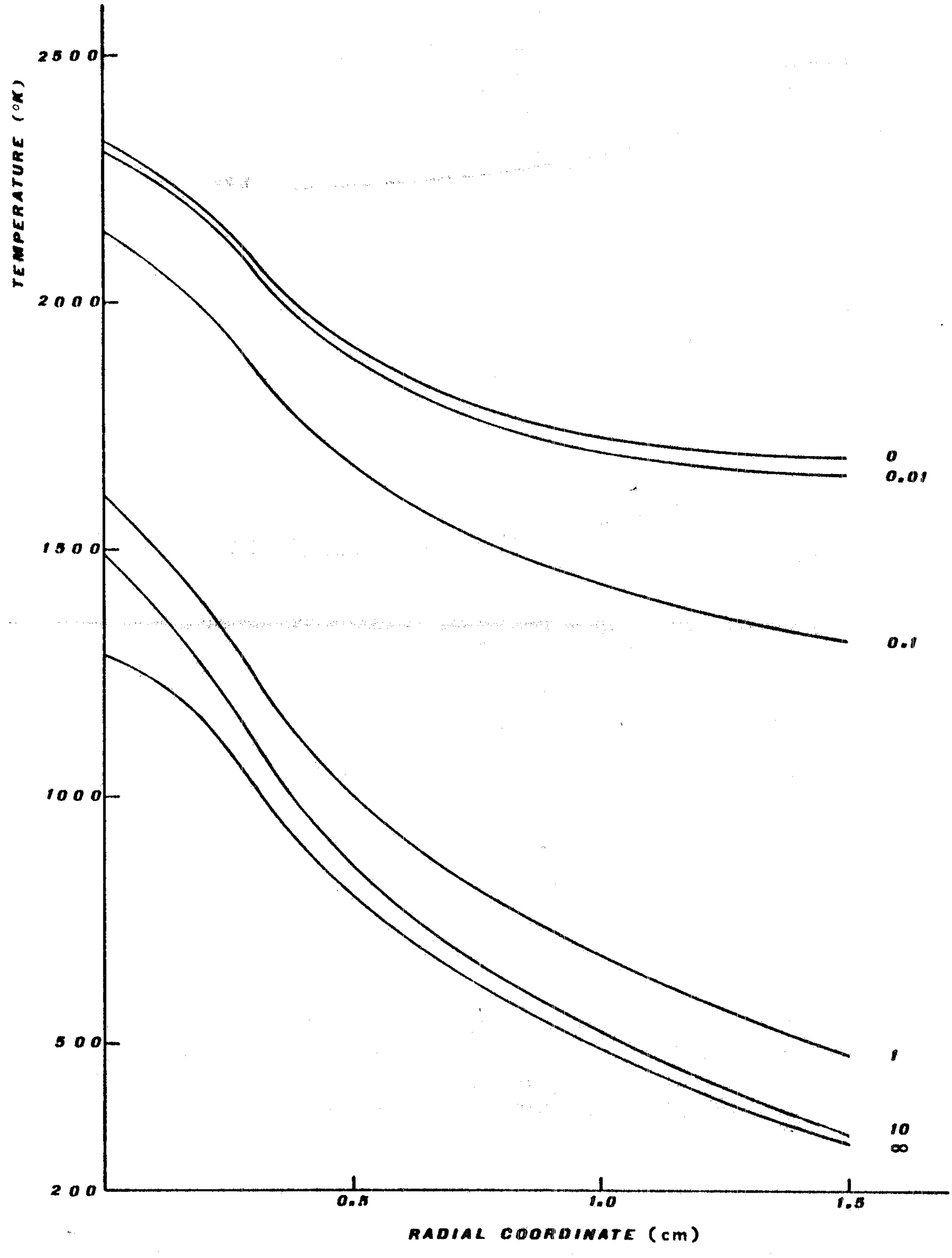

FIG. 7 - Radial temperature distribution for a Ta disc with $R = 1.5$ cm, $h = 0.05$ r.l. Curves are labelled with μ values (in W cm^{-2} $^{o}K^{-1}$). Beam radius R_o = 3 mm.

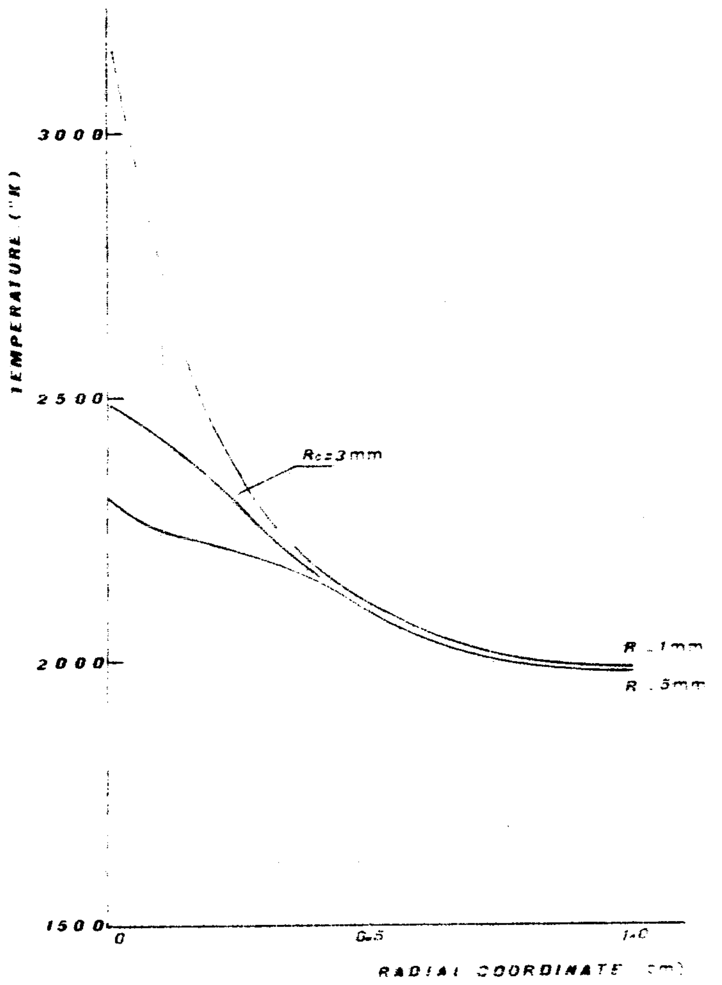

FIG. 8 - Radial temperature distributions for a Ta disc with R = 1 cm, h = 0.25 r.l., and three different beam radii (R_o = 1.3 and 5 mm).

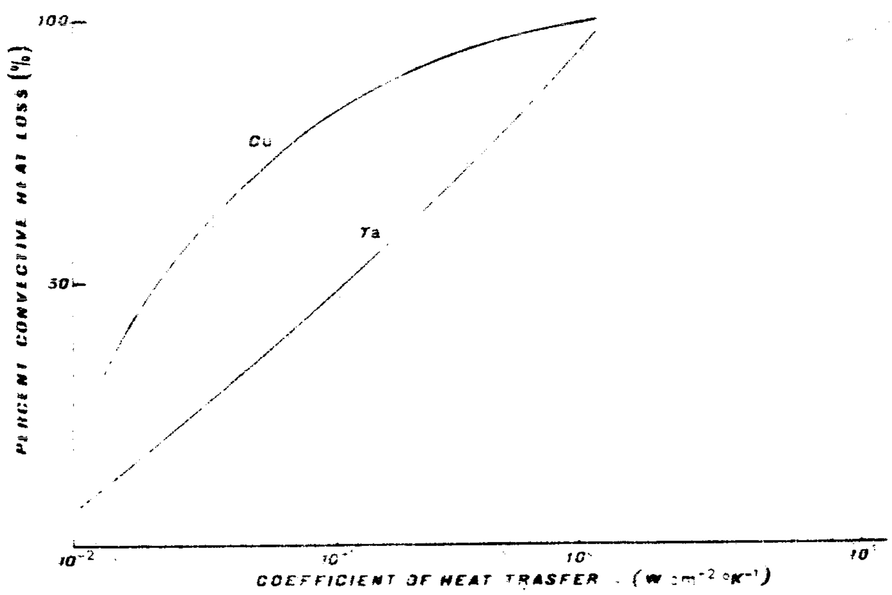

FIG. 9 - Percentage of heat lost by circumferential cooling vs. μ for a Cu and Ta disc with R = 1 cm and h = 0.25 r.l. Radius of the beam R_o = 3 mm