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LNF-74/31(P)
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E. Etim and M. Greco: DUALITY SUM RULES IN e^+e^- ANNIHILATION FROM CANONICAL TRACE ANOMALIES.

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E. Etim and M. Greco: DUALITY SUM RULES IN e^+e^- ANNIHILATION FROM CANONICAL TRACE ANOMALIES.

ABSTRACT. -

In the general framework of the canonical trace anomaly of the energy momentum tensor we derive a complete set of sum rules relating the asymptotic value of the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ to the low energy moments of $\sigma(e^+e^- \to \text{hadrons})$. Experimental implications are also discussed.

One of the most puzzling results of deep inelastic experimentation in the last few years is the large cross section of e^+e^- annihilation into hadrons[(1)]. The almost constant cross section observed is in marked disagreement with the pointlike behaviour expected theoretically on the basis of scaling. The question thus arises as to whether there is complete break-down of scaling or that it is postponed to much higher energies. Adopting this latter point of view we show in this letter how in the framework of the canonical trace anomaly of the energy-momentum tensor the asymptotic value of the ratio $R = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-)$ can be related to the low energy behaviour of $\sigma(e^+e^- \to \text{hadrons})$, through a general duality principle which we derive explicitly.

Some time ago we proposed[2] a simple model in which scaling is built up by a series of resonances of Veneziano type in which this duality principle between R and low energy parameters is verified. Explicitly we found $R = 8\pi^2/f_\rho^2 \simeq 2.5$. A similar treatment by Sakurai[3] gives numbers between 3 and 5, but as we shall show later, a more accurate analysis will confirm tha value of R given above.

Recently from a completely different point of view Terazawa[4] has determined the anomaly of PCDC (partial conservation of dilatation current) Ward identity using conventional vector meson dominance of real photons and found for R a value of $16\pi^2/f_\rho^2$. This prediction, however, depends critically on the smoothness of the extrapolation of the amplitudes from $q^2 = 0$. The q^2 dependence of the matrix elements involved can be studied by examining in detail the energy momentum tensor trace anomaly.

Defining the Green functions

$$\Delta_{\mu\nu}(p,q) = \int d^4x\, d^4y\, e^{iqy} \langle 0 | T(\theta_\lambda^\lambda(x)\, J_\mu(y)\, J_\nu(0)) | 0 \rangle e^{ipx} , \qquad (1)$$

and

$$\pi_{\mu\nu}(q) = i \int d^4x\, e^{iqx} \langle 0 | T(J_\mu(x)\, J_\nu(0)) | 0 \rangle , \qquad (2)$$

where θ_λ^λ is the trace of the energy momentum tensor, Crewther[5] and Chanowitz and Ellis[6] have established the following anomalous trace identity

$$\Delta(q^2) = -2q^2 \frac{\partial \pi(q^2)}{\partial q^2} - \frac{e^2 R}{6\pi^2} , \qquad (3)$$

with

$$\Delta_{\mu\nu}(q,0) = (q_\mu q_\nu - g_{\mu\nu} q^2)\,\Delta(q^2) ,$$

$$\pi_{\mu\nu}(q) = (q_\mu q_\nu - g_{\mu\nu} q^2)\,\pi(q^2) .$$

Since $\pi(q^2)$ is regular at $q^2 = 0$ one recovers from (3) the familiar result:

$$\Delta(0) = -\frac{e^2 R}{6\pi^2} . \tag{4}$$

On very general grounds(7) $\Delta(q^2)$ is expected to vanish asymptotically for large q^{2}(*). One can therefore write for it an unsubtracted dispersion relation:

$$\Delta(q^2) = -\frac{2}{\pi} \int_{s_0}^{\infty} \frac{s \frac{\partial}{\partial s} \mathrm{Im}\,\pi(s)}{s - q^2}\, ds , \tag{5}$$

where $\mathrm{Im}\,\pi(s)$ is related to the total e^+e^- annihilation cross section into hadrons by

$$\sigma_{had}(s) = \frac{4\pi\alpha}{s}\, \mathrm{Im}\,\pi(s) . \tag{6}$$

The most general form of $\mathrm{Im}\,\pi(s)$ which scales asymptotically is

$$\mathrm{Im}\,\pi(s) = f(s)\,\theta(s - s_0)\,\theta(\bar{s} - s) + \frac{\alpha}{3} R\,\theta(s - \bar{s}) , \tag{7}$$

which inserted into (5) gives the following q^2 structure

$$\Delta(q^2) = -\frac{2}{\pi} q^2 \int_{s_0}^{\bar{s}} \frac{\mathrm{Im}\,\pi(s)\,ds}{(s - q^2)^2} - \frac{2\alpha}{3\pi} R\, \frac{\bar{s}}{\bar{s} - q^2} . \tag{8}$$

In agreement with (4) $\Delta(q^2 = 0)$ is independent of the low ener-

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(*) - The usual argument(7) is that of Weinberg's theorem or the identification of θ_λ^λ with generalized mass terms in the Lagrangian.

gy details of Im $\pi(s)$. On the contrary one finds from (8) that the difference

$$\Delta(q^2) - \Delta(0) = -\frac{2}{\pi} q^2 \int_{s_o}^{\bar{s}} \frac{\mathrm{Im}\,\pi(s)\,ds}{(s-q^2)^2} - \frac{2\alpha}{3\pi} R \frac{q^2}{\bar{s}-q^2} \tag{9}$$

is not only q^2 dependent but is critically sensitive to the low energy behaviour of Im $\pi(s)$. In particular the extrapolation from $q^2 = 0$ to $q^2 = m_\rho^2$, which is necessary to obtain the result of Terazawa, involves the unknown of the r.h.s. of eq. (9).

A more general formulation of the approach to scaling is obtained by considering the large q^2 behaviour ($q^2 >> \bar{s}$) of $\Delta(q^2)$. Expanding both sides of eq. (8) in inverse powers of q^2 and comparing coefficients one gets:

$$\int_{s_o}^{\bar{s}} \mathrm{Im}\,\pi(s)\; s^n\, ds = \frac{\alpha R}{3} \frac{\bar{s}^{n+1}}{n+1} - \frac{c_n}{n+1}, \tag{10}$$

where

$$q^2 \Delta(q^2) \sim \frac{2}{\pi} \sum_{n=0}^{\infty} c_n \left(\frac{1}{q^2}\right)^n . \tag{11}$$

Eqs. (10) are nothing but the analogue of the familiar duality sum rules of strong interactions apart from the constants c_n. This is the exact mathematical formulation of duality in e^+e^- annihilation, where asymptotic scaling replaces the usual assumption of Regge behaviour. As can be seen from (10) the asymptotic value of R is directly connected to the low energy region of e^+e^- annihilation into hadrons.

The exact validity of duality, as in the case of strong interactions, requires $c_n = 0$ for all n, and therefore a maximally soft trace of the energy momentum tensor.

It is easily verified that the model of ref. (2) satisfies the duality sum rules of zeroth and first order with $c_o = c_1 = 0$. In fact, for n = 0, we have

$$\int_{s_o}^{\bar{s}} \left(\mathrm{Im}\,\pi(s) - \frac{\alpha R}{3}\right) ds = 0 \ , \qquad (12)$$

as an identity in this model which is valid also locally. This follows from the fact that $\mathrm{Im}\,\pi(s)$ around any resonance of the series is given by

$$\mathrm{Im}\,\pi(s) \sim \frac{m_n^2}{f_n^2}\, 4\pi^2\alpha\, \delta(s-m_n^2) = \frac{m_\rho^2}{f_\rho^2}\, 4\pi^2\alpha\, \delta(s-m_n^2)$$

and the effective range of integration is $2m_\rho^2$ as given by the spacing of the mass spectrum. The value of R found this way ($R = 8\pi^2/f_\rho^2$) agrees with that previously obtained by summing the entire series[(2)]. This result is to be compared with that by Sakurai[(3)] in which the uncertainties in the threshold factors of his $\sigma_{comp}(s)$ give rise to corresponding uncertainties in R ($R \approx 3 - 5$).

In the light of the above considerations it is interesting to comment on the experimental situation of e^+e^- annihilation[(1)]. Above the prominent resonances ($\rho, \omega, \varphi, \rho', \ldots$) and up to $q^2 \approx 10\ \mathrm{GeV}^2$, R is approximately constant and ~ 2.5. This is in agreement with our prediction and with the duality sum rule, thus suggesting $c_o = 0$. From $q^2 \approx 10$ to about $25\ \mathrm{GeV}^2$ R apparently increases linearly with q^2. If scaling is reestablished and c_o is zero as suggested by the low energy comparison, then the duality sum rule implies that the present trend of the data has to turn over in order to satisfy eq. (12). Future e^+e^- colliding beam experiments at very high energies will be decisive from this point of view.

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