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THE LONGITUDINAL FEEDBACK SYSTEM IN ADONE

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Summary. In the paper is described the feedback system, actually working on Adone, that acts on the longitudinal relative modes of oscillation of the two e^+, e^- beams, of three bunches each. The stability condition for small oscillations of rigid bunches is analitically found. The system, inplemented with two resonators tuned at the 8^{th} harmonic of the revolution frequency, is described. The results are discussed.

1. - Introduction

The instabilities of the synchrotron oscillations of particle bunches in a circular accelerator are known and their mechanism has been discussed by many authors[1,2,3] together with the means to counteract them. However the damping systems designed or inplemented for storage rings up to the present time were made for beams containing only one bunch. In this work we shall describe the damping system for the e^+, e^- beams in Adone, containing three bunches each.

As it is known, the longitudinal modes of oscillations of the bunches of a beam may be classified in center of mass mode and relative modes. The latter are the most difficult to deal with and may be damped in two ways. One way is to split the synchrotron frequencies of the various bunches by means of a sinusoidal voltage whose frequency is a multiple of the revolution frequency but not of the main radio-frequency. In Adone we have introduced 2 KV peak by means of a small resonator tuned at 71.4 MHz which is the 25^{th} harmonic of the revolution frequency. This voltage alone has not been sufficient to damp the instabilities of the relative modes, especially at low energy, and it has been necessary to introduce the feedback damping system illustrated hereafter.

The centre of mass mode of oscillation of one beam has been damped by a feedback system that uses the main accelerating cavities to interact with the beam. This system has already been mentioned elsewhere[4] and will not be described in this paper. The mode of oscillation of the centers of mass of the two beams in phase opposition cannot be acted upon by any of the systems here described. However it can be influenced by splitting the synchrotron frequencies of the two beams. This can be achieved in Adone by varying suitably the phases of the R.F. voltages on two adjacent main accelerating gaps.

2.1. - Description of the feedback system for relative modes.

The block scheme of the feedback system is shown in Fig. 1. For the symbols used see Appendix A.

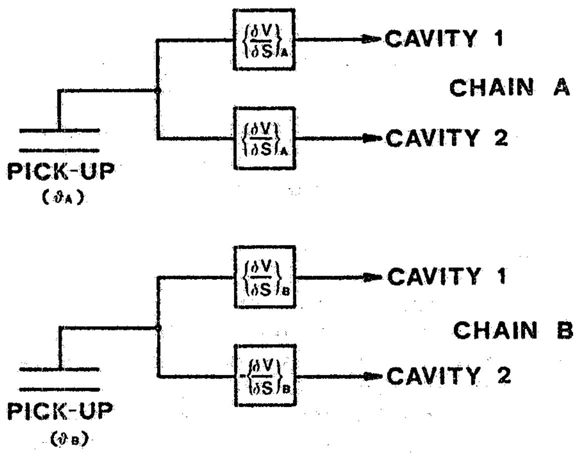

FIG. 1

As shown in the scheme, the system is made of two amplifying chains (A and B) and two cavities. The two amplifier chains correspond to two different equivalent positions of the pick-ups on the circumference of the accelerator and two different gains $(\Delta V/\Delta S)_A$ and $(\Delta V/\Delta S)_B$. From pick-up A the same signal is sent to the two cavities, from B the signal to cavity 2 is inverted w.r. to that sent to cavity 1. The signals from the two beams (e^+, e^-) are separated by a gate system, so that it is possible to shift the equivalent position of the pick-up for each beam merely by inserting delays. The pick-up gives a signal proportional to the radial displacement of the beams. There are various reasons for employing a transverse position pick-up instead of a longitudinal one. The most immediate one is that the radial displacement signal has a 90^o phase shift w.r. to the longitudinal displacement, so that its phase is already the correct one to be fed back on the beam. There is however[5] a fundamental reason that excludes the use of a longitudinal position pick-up in a feedback system on many bunches, because with such a signal it is impossible to damp simultaneously all the modes of oscillation.

2.2. - Equations of motion.

The linearized motion equations, in the absence of radiation damping and noise, written as iterative equations turn by turn, are[3]:

$$\phi^{\pm}_{l,m} - \phi^{\pm}_{l,m-1} = \varrho\, \varepsilon^{\pm}_{l,m} \qquad (1)$$

$$(l = 1, 2, 3, \ldots, \ -\infty < m < +\infty)$$

$$\varepsilon^{\pm}_{1,m} - \varepsilon^{\pm}_{1,m-1} = -\frac{\Omega^{2\pm}}{\varrho}\ \phi^{\pm}_{1,m} +$$

$$+ N \sum_{k=1}^{3} \sum_{n=-\infty}^{+\infty} (C^{\mp}_{k-1,n-m}\ \phi^{\mp}_{k,n} + C^{o}_{k-1,n-m}\ \phi^{\pm}_{k,n}) + \qquad (1)$$

$$+ N \sum_{k=1}^{3} \sum_{n=-\infty}^{+\infty} (S^{\mp}_{k-1,n-m}\ \varepsilon^{\mp}_{k,n} + D^{\pm}_{k-1,n-m}\ \varepsilon^{\pm}_{k,n})$$

The meaning of the symbols is the following:

$\phi^{\pm}_{k,m}$, $\varepsilon^{\pm}_{k,m}$ Displacement from synchronous phase and energy of the k^{th} bunch $e^{\pm}$ at the m^{th} turn.

$\varrho = \frac{2\pi\alpha}{E}$

α = momentum compaction
E = energy of synchronous particle
$\Omega^{\pm}$ = synchrotron frequency of $e^{\pm}$ beam in unities of the revolution frequency ω_o.
$C^{\pm}_{r,s}$, $C^{o}_{r,s}$ coefficients that give a weight to the signals left in the cavities in the preceding turns.
$D^{\pm}_{r,s}$, $S^{\pm}_{r,s}$ coefficients that take into account the action of feedback.

In Appendix A are reported the expression of:

$$\Omega^{2\pm},\ C^{\pm}_{r,s},\ C^{o}_{r,s},\ D^{\pm}_{r,s},\ S^{\pm}_{r,s}$$

Equations (1) are obtained using the approximation of rigid bunches. We also suppose that in the accelerator are circulating 3 bunches per beam with an identical number of particles, and that the synchronous phase is equal for the various bunches.

2.3. - Damping coefficients.

Neglecting the difference ($\Omega^{2+} - \Omega^{2-}$) and supposing the two feedback cavities identical, we obtain from equations (1) the damping coefficients (see again ref. 3) for the four relative modes:

$$\delta^{\pm}_{r} = 3N\Big\{ g_A\big[\cos(m_o(\theta_A-\theta_1)) + \cos(m_o(\theta_A-\theta_2))\big] + g_B\big[\cos(m_o(\theta_B-\theta_1)) - \cos(m_o(\theta_B-\theta_2))\big] + \gamma \mathrm{Re}(\alpha_r \pm \sqrt{\Delta_r})\Big\} \qquad r = 1,2 \qquad (2)$$

with: Re( ) = real part of ( ).

$$\Delta_r = \Big\{ g_A\big[\cos(m_o(\theta_A+\theta_1)) + \cos(m_o(\theta_A+\theta_2))\big] + g_B\big[\cos(m_o(\theta_B+\theta_1)) - \cos(m_o(\theta_B+\theta_2))\big] + \gamma\alpha^{c}_{r}\Big\}^2 + \Big\{ g_A\big[\mathrm{sen}(m_o(\theta_A+\theta_1)) + \mathrm{sen}(m_o(\theta_A+\theta_2))\big] + g_B\big[\mathrm{sen}(m_o(\theta_B+\theta_1)) - \mathrm{sen}(m_o(\theta_B+\theta_2))\big] + \gamma\alpha^{s}_{r}\Big\}^2 - \Big\{ g_A\big[\mathrm{sen}(m_o(\theta_A-\theta_1)) + \mathrm{sen}(m_o(\theta_A-\theta_2))\big] + g_B\big[\mathrm{sen}(m_o(\theta_B-\theta_1)) - \mathrm{sen}(m_o(\theta_B-\theta_2))\big]\Big\}^2 \qquad (3)$$

where:

$$g_{A,B} = \frac{e\psi}{4E}\left(\frac{\Delta V}{\Delta S}\right)_{A,B}, \qquad \gamma = \frac{e^2\ \omega_o\ \varrho}{2\ \Omega_o}$$

ψ = closed orbit function at the pick-up position;
$\theta_{1,2}$ = longitudinal abscissa of the feedback cavities 1, 2;
$\theta_{A,B}$ = equivalent longitudinal abscissa of pick-up for chains A, B;
$\Omega_o \simeq \Omega^{+} \approx \Omega^{-}$
m_o = harmonic of the revolution frequency to which are tuned the feedback cavities;
α_r, α^{c}_{r}, α^{s}_{r} (r = 1, 2) = coefficients that depend on the impedances of the resonant modes of the cavities interacting with the beam along its path; for their expressions see Appendix A.

In order that the four relative oscillation modes be stable, it is necessary that the four coefficients δ^{+}_{1}, δ^{-}_{1}, δ^{+}_{2}, δ^{-}_{2} be positive.

As the sign before the term $\sqrt{\Delta_r}$ is ambiguous, it is convenient to choose the parameters of the feedback system so that the dependence of Δ_r on g_A and g_B disappears.

If we choose:

$$\theta_A = \frac{\pi n}{m_o} + \frac{\theta_1+\theta_2}{2}$$

$$\theta_B = \frac{\pi s}{m_o} + \frac{\pi}{2m_o} + \frac{\theta_1+\theta_2}{2} \qquad (4)$$

$$g_A = (-1)^n g\, \mathrm{sen}\left(m_o \frac{\theta_1-\theta_2}{2}\right) \qquad n = 0, \pm 1, \pm 2, \ldots$$

$$g_B = (-1)^s g \cos\left(m_o \frac{\theta_1-\theta_2}{2}\right) \qquad s = 0, \pm 1, \pm 2, \ldots$$

we obtain for $\sqrt{\Delta_r}$ an expression that is independent from gain parameters:

$$\sqrt{\Delta_r} = \gamma \sqrt{(\alpha^{c}_{r})^2 + (\alpha^{s}_{r})^2}$$

and we have:

$$\delta^{\pm}_{r} = 3N\Big[2g\,\mathrm{sen}(m_o(\theta_2-\theta_1)) + \gamma \mathrm{Re}\Big(\alpha_r \pm \sqrt{(\alpha^{c}_{r})^2 + (\alpha^{s}_{r})^2}\Big)\Big] \qquad (5)$$

The physical meaning of the choice made above is that the equivalent pick-up for chain A is placed, with respect to the intermediate point between the two feedback cavities $(\theta_1+\theta_2)/2$, in a crossing point for the displacement of the bunches at the harmonic m_o of the revolution frequency, while pick-up B is placed in a non-crossing point. Thus chain A sees only the in-phase modes of the two beams, while B sees only the ones in phase opposition. The conditions on g_A and g_B exclude effects arising from interference between in-phase and counter-phase modes.

3.1. - Implementation of the system.

With reference to the scheme of Fig. 1, and to equations (4) the feedback system can be implemented with two cavities tuned at the harmonic m_o of the revolution frequency ω_o, where m_o is not a multiple of the main radio frequency and with two pick-ups placed one at a crossing point and the other one at a non crossing point at frequency $m_o\omega_o$. From the same equations we see that the gains of the two chains, g_A and g_B, are equal if the distance between two feedback cavities along the circumference is a whole number of half wavelengths plus one quarter wavelenght, still at frequency $m_o\omega_o$. Imposing the latter condition, as it simplifies the system, and given the physical distance L between the two gaps, the harmonic m_o is determined:

$$2\pi m_o \frac{L}{C} = \frac{\pi}{2} + k\pi \qquad k = 0, 1, \ldots$$

Where C is the perimeter of the accelerator. In Adone C = 105 m, L = 16.5 m, so, for k = 2 it results $m_o \approx 8$ with an error of about 3 degrees on the angular distance. As $\omega_o = 2.856$ MHz the resonant frequency of the cavities results $m_o\omega_o = 22.8$ MHz. Another condition on the cavities is that their bandwidth be large with respect to the synchrotron oscillation frequency.

Furthermore the pick-ups must behave as magnetic ones, i.e. the signal must depend only on transverse displacement and not on the sign of the particle. The two pick-ups can be reduced to only one by means of suitable delays added in the chains, as it was already mentioned. To see this let us choose the origin of coordinates in the mid point between feedback cavities as in Fig. 2, where θ_1, θ_2 are the coordinates of the cavities, θ_A, θ_B those of the pick-ups.

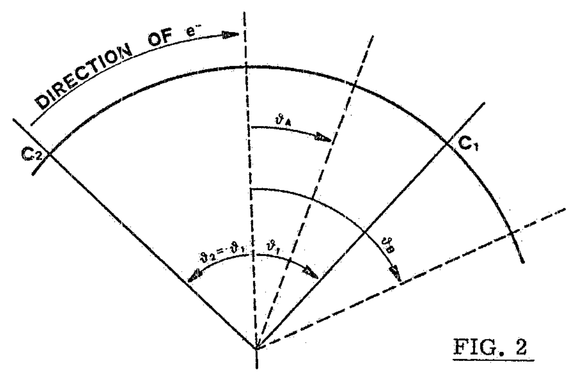

FIG. 2

We may write, for the signals from two e^+, e^- bunches on a pick-up situated at an abscissa θ^x, at frequency $m_o\omega_o$, using phasor notation:

$$w^+(\theta^x) = w^+ e^{i\varphi^+} e^{+im_o\theta^+} = W^+ e^{+im_o\theta^x},$$

$$w^-(\theta^x) = w^- e^{i\varphi^-} e^{-im_o\theta^x} = W^- e^{-im_o\theta^x}$$

where φ^+, φ^- are the phases of each bunch through the origin; w^+, w^- are proportional to the bunch intensities. Remembering that $\theta_A = n\pi/m_o$, $\theta_B = (s\pi/m_o) + (\pi/2m_o)$ we may write:

$$w^+(\theta_A) = W^+(-1)^n; \qquad w^-(\theta_A) = W^-(-1)^n$$

$$w^+(\theta_B) = +iW^+(-1)^s; \qquad w^-(\theta_B) = -iW^-(-1)^s$$

Calling v_1, v_2 the voltages on cavities C_1 and C_2 and choosing n, s both even or odd, we obtain:

$$v_1 \propto W^+(1+i) + W^-(1-i) \propto W^+ e^{+i\pi/4} + W^- e^{-i\pi/4}$$

$$v_2 \propto W^+(1-i) + W^-(1+i) \propto W^+ e^{-i\pi/4} + W^- e^{+i\pi/4}$$

We may rewrite these equations as follows:

$$v_1 \propto \left[W^+ e^{+im_o\theta^x} e^{-i(-\frac{\pi}{4}+m_o\theta^x)} + W^- e^{-im_o\theta^x} e^{-i(\frac{\pi}{4}-m_o\theta^x)}\right]$$

$$v_2 \propto -i\left[W^+ e^{im_o\theta^x} e^{-i(-\frac{\pi}{4}+m_o\theta^x)} - W^- e^{-im_o\theta^x} e^{-i(\frac{\pi}{4}-m_o\theta^x)}\right] \qquad (5)$$

If the signals due to e^+, e^- are separated by means of gates, they can be delayed independently and equations (5) can be translated into the block scheme of Fig. 3.

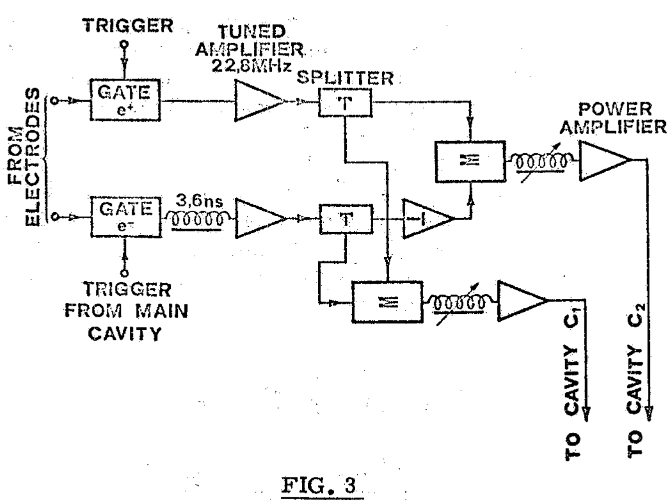

FIG. 3

In Fig. 3 the signals are taken from one only pick-up placed in θ^x, the e^+ and e^- channels are separated by gating and the signals are combined according to equations (5). Taking into account the effective position of the pick-up at it results from Fig. 4, the shown values of the delays turn out.

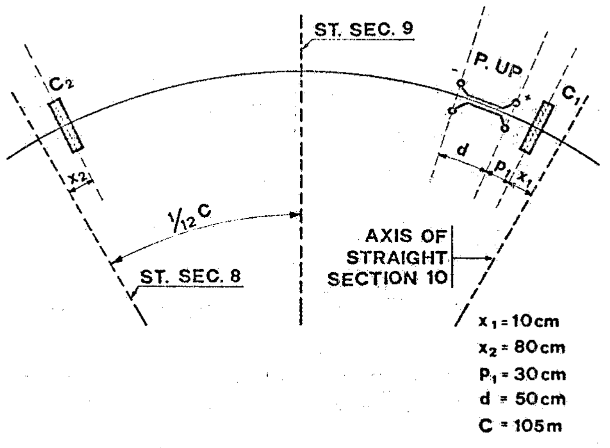

FIG. 4

3.2. - Pick-up sensitivity and loop gain.

The signals proportional to transverse displacement are taken from a strip-line pick-up similar to that already used in Stanford[6]. The difference between the signals from the two electrodes is made by means of hybrid junctions, as illustrated in Fig. 5.

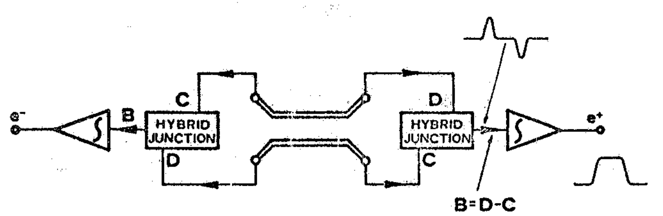

FIG. 5

The pick-up system therefore behaves as a magnetic pick-up, i.e. the polarity of the signal depends only on the transverse displacement and not on the sign of the particle. The wave form, after integration, is a train of approximately rectangular pulses, whose width is about 5 nsec (of the order of twice the transit time of a wavefront along the electrodes and whose period is 350 nsec. The peak voltage is $S = 4,2$ mV/ $\langle$ mA $\rangle$ bunch. The 8^{th} harmonic of such a waveshape is $\approx 2.7 \times 10^{-2}$ S, the gain of the chains has resulted about 100 db, therefore $(1/e\omega_0)(\Delta V/\Delta S) = 11.5$ V/mm $\langle$mA$\rangle$ bunch. Supposing the oscillations to be caused by one only resonant line at $\approx$100 MHz, and remembering that

$$g = \frac{e}{4}\left(\frac{\Delta V}{\Delta S}\right)\frac{\psi}{E} \gtrsim \gamma \alpha_s$$

with

$$\gamma = \frac{\pi \alpha e^2 \omega_0}{\Omega E}, \quad \left(hB - s - \frac{\Omega}{2\pi}\right) \approx 35$$

$\alpha_s \approx 35\, Z(35\,\omega_0)$, $\alpha = 10^{-2}$, $\Omega = 3,3 \times 10^{-3}$ at 1 GeV, $\omega_0 = 2\pi \times 3$ MHz, $\psi = 2$m. We obtain $Z(35\,\omega_0) = 17\, K\Omega$ i.e. the instability could be caused by a spurious mode of the cavities of shunt impedance 17 KΩ at 100 MHz.

4. - Conclusions.

The adjustment of the system is simple enough. It is made with only one beam and with only one feedback cavity working at a time, by acting separately on the variable delays shown on each chain.

The feedback system described is working on Adone. In Fig. 6 we show a photo of straight section 10 with one of the feedback cavities and the synchrotron frequency splitting cavity.

The system has resulted essential for a good injection and to avoid sudden beam losses, especially at low energies. Care must be taken that the signal level, depending on the stored current level, does not saturate the amplifiers.

Acknowledgments.

Thanks are due to all the Adone staff and particularly to D. Fabiani for collaboration.

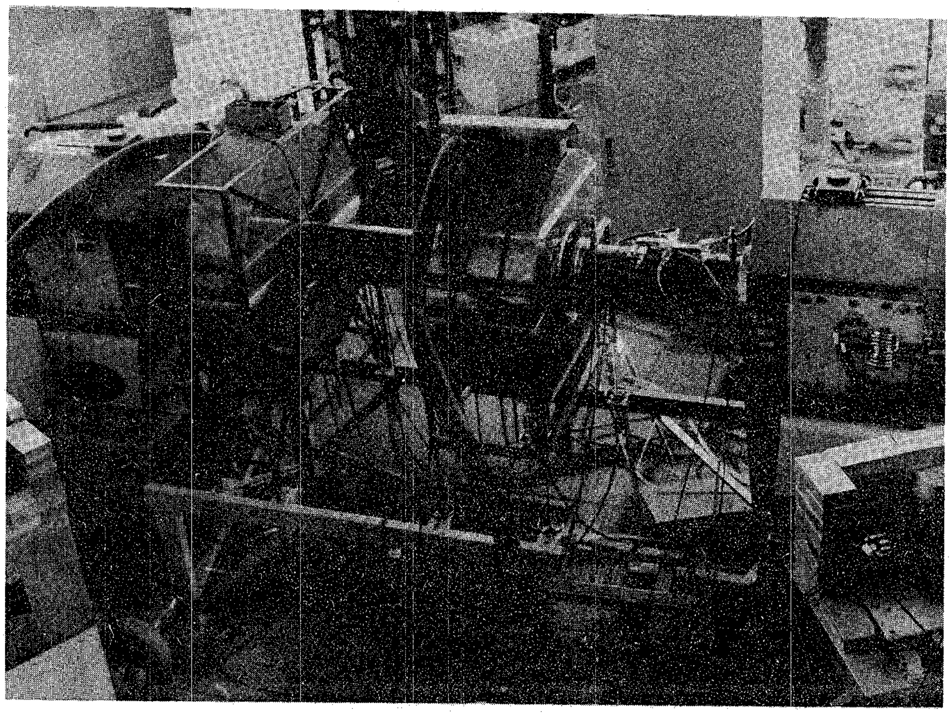

FIG. 6 - Straight section with 71.4 MHz cavity and one of the feedback cavities.

Appendix A.

Definitions:

B = n° of bunches per beam (in Adone B=3)
N = n° of particles per bunch
N_C = number of spurious cavities
N_{RF} = n° of accelerating cavities
v_j, ψ_j = peak voltage and phase of the j^{th} accelerating cavity.
$\varrho = 2\pi\alpha/E$, α = momentum compaction, E = energy of synchronous particle.
e = charge of the electron
ϕ_o = synchronous phase (supposed equal for all bunches)
$\theta_{A,B}$ = azimuth of pick-up of chain A, B.

$Z_j(\omega)$, θ_j = impedance and azimuth of j^{th} cavity (indexes 1, 2 refer to the feedback cavities).
$\omega_o = 2\pi/T$ = revolution frequency.
$i = \sqrt{-1}$

$$\mu_{A,B} = \frac{e\psi}{2E\,\omega_o Z^x}\left(\frac{\Delta V}{\Delta S}\right)_{A,B}$$

$\left(\frac{\Delta V}{\Delta S}\right)$ = voltage in cavity (chain A, B) for a radial displacement ΔS at the pick-up.
ψ = closed orbit function at the pick-up.
Z^x = real part of feedback-cavities impedance at their harmonic frequency.

From reference 3 we have:

$$\Omega^{2\pm} = -e\varrho B \sum_{j=1}^{N_{RF}} V_j \cos\left[B(\phi_o \pm \theta_j) - \psi_j\right] + ie\varrho BN \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} hB\,\omega_o Z_j(hB\omega_o)(1+e^{\mp 2ihB\theta_j})$$

$$C^o_{r,s} = \frac{ie^2}{\omega_o} \sum_{j=1}^{N_C} \int_{-\infty}^{+\infty} \omega\, d\omega\, Z_j(\omega)\, e^{i\omega(s+\frac{r}{B})T}$$

$$C^{\pm}_{r,s} = \frac{ie^2}{\omega_o} \sum_{j=1}^{N_C} \int_{-\infty}^{+\infty} \omega\, d\omega\, Z_j(\omega)\, e^{i\omega(s+\frac{r}{B}\pm\frac{\theta_j}{})T}$$

$$D^{\pm}_{r,s} = \sum_{j=1}^{2} \int_{-\infty}^{+\infty} d\omega\, Z_j(\omega) e^{i\omega(s+\frac{r}{B})T} \left(\mu_A e^{\pm i\omega\frac{\theta_A-\theta_j}{2\pi}T} - (-1)^j \mu_B e^{\pm i\omega\frac{\theta_B-\theta_j}{2\pi}T}\right)$$

$$S^{\pm}_{r,s} = \sum_{j=1}^{2} \int_{-\infty}^{+\infty} d\omega\, Z_j(\omega) e^{i\omega(s+\frac{r}{B})T} \left(\mu_A e^{\pm i\omega\frac{\theta_A+\theta_j}{2\pi}T} - (-1)^j \mu_B e^{\pm i\omega\frac{\theta_B+\theta_j}{2\pi}T}\right)$$

$$\alpha_r = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} \left(hB-r-\frac{\Omega_o}{2\pi}\right) Z_j\left[\left(hB-r-\frac{\Omega_o}{2\pi}\right)\omega_o\right]$$

$$\alpha^c_r = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} \left(hB-r-\frac{\Omega_o}{2\pi}\right) Z_j\left[\left(hB-r-\frac{\Omega_o}{2\pi}\right)\omega_o\right] \cos\left[2\left(hB-r-\frac{\Omega_o}{2\pi}\right)\theta_j\right]$$

$$\alpha^s_r = \sum_{j=1}^{N_C} \sum_{h=-\infty}^{+\infty} \left(hB-r-\frac{\Omega_o}{2\pi}\right) Z_j\left[\left(hB-r-\frac{\Omega_o}{2\pi}\right)\omega_o\right] \operatorname{sen}\left[2\left(hB-r-\frac{\Omega_o}{2\pi}\right)\theta_j\right]$$

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