Laboratori Nazionali di Frascati

LNF-74/8(P)

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Phys. Letters 48B, 137 (1974)

QUARK MODEL PREDICTIONS FOR RADIATIVE DECAYS OF MESONS

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Received 16 October 1973

From simple quark model arguments and assuming a quadratic η–η' mixing we predict the radiative decays of the vector and pseudoscalar mesons, in good agreement with experiments. Our results strongly favour the new determination of the $\eta \to \gamma\gamma$ partial width.

A great deal of theoretical work has been dedicated for long time to the problem of understanding the radiative decays of mesons. Unitary-symmetry ideas, combined with the vector meson dominance hypothesis, have provided the most popular framework for such a study [e.g. 1]. In order to achieve a more quantitative agreement with experiments, further assumptions, however, had to be included, with a consequent reduction of predictive power, the inclusion of SU(3) breaking effects having been the main tendency [2]. A different approach was recently proposed [3], based on the application of the naive quark model in relating all different coupling constants involved in the decays. The lack of knowledge of the relative strange-non strange quark content of the η particle poses the only problem to face. In the assumption of η–η' mixing, the solution to the problem is simply shifted to the determination of the η–η' mixing angle θ_p.

Two new experimental results have been recently added to the so far known picture of meson decays: the absence of splitting of the A_2 meson [4], and a new determination, via the Primakoff effect, of the $\eta \to \gamma\gamma$ partial width [5]. It is our claim that those new sources of information allow the determination of the mixing angle θ_p, leading also to a final clarification of the whole picture of the radiative meson decays.

In the present paper, within the framework of the naive quark model, and assuming a η–η' mixing angle according to the quadratic Gell–Mann–Okubo mass formula, we relate successfully together different radiative decays of mesons with the same number of photons appearing in the final state. We are able then to predict the $\eta \to \gamma\gamma$ partial width in two distinct ways, favouring the new determination of this rate. We also predict the radiative partial widths of the η' meson, leading to $\Gamma(\eta' \to \text{all}) \approx 400$ keV. Furthermore we discuss the absolute links of one- and two-photon radiative decays with purely strong decays, according to extended vector meson dominance, along the lines described in ref. [3].

In the simplest quark model, by assuming η–η' mixing one has:

$$\eta \sim \frac{\alpha}{\sqrt{2}}(p\bar{p} + n\bar{n}) - \sqrt{1-\alpha^2}\lambda\bar{\lambda},$$
$$\eta' \sim \frac{\sqrt{1-\alpha^2}}{\sqrt{2}}(p\bar{p} + n\bar{n}) + \alpha\lambda\bar{\lambda}, \qquad (1)$$

where $\alpha = \sin(\theta_0 - \theta_p)$, θ_p is the usual mixing angle and θ_0 is the "ideal" mixing angle, with $\tan\theta_0 = 1/\sqrt{2}$. The experimental informations now available enable us to estimate the mixing parameter α. In fact, from the absence of the splitting of A_2, which was a considerable source of troubles for the quark model, one can deduce [4]:

$$g^2_{A_2\eta\pi}/g^2_{f_{\pi^0\pi^0}} = \alpha^2 = 0.42 \pm 0.10. \qquad (2)$$

Furthermore, by comparing the η and η' production cross sections in the reactions $\pi N \to \eta(\eta')N, \Delta^*$ at high energies, using the quark model relation

$$\frac{\sigma(\eta)}{\sigma(\eta')} = \frac{\alpha^2}{1-\alpha^2} = \tan^2(\theta_0 - \theta_p) \qquad (3)$$

one finds [6]:

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$$\alpha^2 = 0.52 \pm 0.09, \qquad \alpha^2 = 0.58 \pm 0.12, \tag{4}$$

depending on the prescription used for accounting the η and η' mass differences. Eqs. (2) and (4) have to be compared with the values $\alpha^2 = 0.5$ and $\alpha^2 = 0.73$ obtained from a quadratic or linear Gell-Mann–Okubo mass formula, respectively. The quadratic mixing seems therefore to be favoured. We will take this attitude in the following. This corresponds to a $\eta - \eta'$ mixing angle $\theta_p \approx -10°$, and to the simple relations:

$$\eta \sim \tfrac{1}{2}(p\bar{p}+n\bar{n}) - \frac{1}{\sqrt{2}}\lambda\bar{\lambda}, \quad \eta' \sim \tfrac{1}{2}(p\bar{p}+n\bar{n}) + \frac{1}{\sqrt{2}}\lambda\bar{\lambda}. \tag{5}$$

We work out now the consequences of the above assumptions by considering first the radiative decays of mesons with only one photon in the final state. The quark model predictions can be simply stated in the following relations [3]:

$$g_{\rho\pi^\circ\gamma} = -\tfrac{1}{3} g_{\omega\pi^\circ\gamma} \equiv \tfrac{1}{3} g, \quad g_{\rho\eta\gamma} = -g_{\rho\eta'\gamma} = \frac{1}{\sqrt{2}} g,$$
$$g_{\omega\eta\gamma} = g_{\omega\eta'\gamma} = \frac{1}{3\sqrt{2}} g, \quad g_{\varphi\eta\gamma} = g_{\varphi\eta'\gamma} = \frac{\sqrt{2}}{3} g. \tag{6}$$

The corresponding predictions for the decay widths are shown in table 1 and compared with the experimental data [4].

The two body decay widths have been obtained by using the well known expressions:

$$\Gamma(V \to P\gamma) = \frac{1}{96\pi} g^2_{VP\gamma} (m_V^2 - m_P^2)^3/m_V^3,$$
$$\Gamma(P \to V\gamma) = \frac{1}{32\pi} g^2_{PV\gamma} (m_P^2 - m_V^2)^3/m_P^2. \tag{7}$$

The $\eta \to \pi^+\pi^-\gamma$ decay width has been evaluated by using finite dispersion relations implemented by duality and finite energy sum rules, as discussed in detail in refs. [3] and [10]. This method gives an important modification to the simple pole model $\eta \to \rho\gamma$ by about a factor of three. The large error reflects both the uncertainties arising from the experimental inputs and those associated with the choice of the cut-off parameter in the use of the finite dispersion relations. From the known branching ratio [4] $\Gamma(\eta \to \pi^+\pi^-\gamma)/\Gamma(\eta \to \gamma\gamma) \approx 0.13$ we can predict $\Gamma(\eta \to \gamma\gamma) = (310 \pm 120)$ eV, which strongly favours the new determination [5] of the $\eta \to \gamma\gamma$ partial width.

The same method of finite dispersion relations has been applied to the $\eta' \to \pi^+\pi^-\gamma$ decay. In this case, however, the corrections to the simple pole model for $\eta' \to \rho\gamma$ are negligible, due to the fact that the invariant mass of the two pions is essentially equal to m_ρ^2. From the absolute value $\Gamma(\eta' \to \pi^+\pi^-\gamma) \approx 118$ keV and the experimental knowledge of the branching ratio $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \text{all})$ we can predict $\Gamma(\eta' \to \text{all}) = (450 \pm 70)$ keV.

We consider now the radiative decays of mesons with two photons in the final state, namely $P \to \gamma\gamma$. The quark model predictions for the coupling constants are given by [3]:

$$g_{\eta\gamma\gamma} = g_{\pi^\circ\gamma\gamma} \frac{1}{\sqrt{3}} (\cos\theta_p - 2\sqrt{2}\sin\theta_p),$$
$$g_{\eta'\gamma\gamma} = -g_{\pi^\circ\gamma\gamma} \frac{1}{\sqrt{3}} (\sin\theta_p + 2\sqrt{2}\cos\theta_p). \tag{8}$$

Table 1
The theoretical errors quoted come from the experimental inputs alone. For the process $\eta \to \pi^+\pi^-\gamma$ see the main text.

Process		Predictions	Experimental data
$\Gamma(\omega \to \pi^\circ\gamma)$	(MeV)	0.89 ± 0.07 (Input)	0.89 ± 0.07
$\Gamma(\omega \to \eta\gamma)$	(keV)	7.3 ± 0.5	8.9 ± 40 [7]
$\Gamma(\rho \to \pi\gamma)$	(keV)	83 ± 6	< 730
$\Gamma(\rho \to \eta\gamma)$	(keV)	55 ± 4	–
$\Gamma(\varphi \to \eta\gamma)$	(keV)	175 ± 13	306 ± 80 [8] 110 ± 30 [9]
$\Gamma(\eta \to \pi^+\pi^-\gamma)$	(eV)	41 ± 16	49 ± 8 [5] 130 ± 29 [4]
$\Gamma(\eta' \to \pi^+\pi^-\gamma)$	(keV)	118 ± 9	$\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \text{all})$ = 26.2 ± 3.5
$\Gamma(\eta' \to \omega\gamma)$	(keV)	11 ± 1	–

Table 2
Theoretical errors come from the experimental inputs alone.

Process		Predictions	Experimental data
$\Gamma(\pi^\circ \to \gamma\gamma)$	(eV)	7.8 ± 0.9 (input)	7.8 ± 0.9
$\Gamma(\eta \to \gamma\gamma)$	(keV)	0.375 ± 0.042	0.374 ± 0.060 [5] 1.00 ± 0.22 [4]
$\Gamma(\eta' \to \gamma\gamma)$	(keV)	6.35 ± 0.73	$\Gamma(\eta' \to \gamma\gamma)/\Gamma(\eta' \to \text{all}) = (1.9 \pm 0.3\%)$

These relations have been derived by expressing the quark structure of the photon in terms of the usual mixture of ρ, ω and φ-like mesons.

From eqs. (8) and using the expression:

$$\Gamma(\mathrm{P} \to \gamma\gamma) = g^2_{\mathrm{P}\gamma\gamma} \frac{m^3_{\mathrm{P}}}{64\pi}, \tag{9}$$

we can easily evaluate the 2γ decay widths of the pseudoscalar mesons. The results are given in table 2.

The new prediction of the $\eta \to \gamma\gamma$ decay width is consistent with that obtained above and agrees astonishingly well with the recent experimental determination. This fact gives strong support to the quadratic $\eta-\eta'$ mass formula.

All the results obtained above are based on a straightforward correlation among the processes $\mathrm{V(P)} \to \mathrm{P(V)}\gamma$ and $\mathrm{P} \to \gamma\gamma$ alone, via quark model and the additional assumption of a quadratic $\eta-\eta'$ mixing. They give a quite satisfactory description of the radiative meson decays.

It should be noted that a value $\Gamma(\eta \to \gamma\gamma) \approx$ 0.350 keV rules practically out the $\eta-\eta'$ linear mixing. In fact, using the value $\theta^{\mathrm{lin}}_{\mathrm{p}} \approx -23°$, and allowing the ratio singlet to octet to be a free parameter fitted by the $\eta \to \gamma\gamma$ partial width, one gets a value for $\Gamma(\varphi \to \eta\gamma)$ which is an order of magnitude smaller than the Orsay result [9]. The ratio $\Gamma(\eta' \to \pi^+\pi^-\gamma)/\Gamma(\eta' \to \gamma\gamma)$, obtained through the same procedure, is also about a factor of two smaller than that observed experimentally.

As is well known, one can relate together all the processes discussed above using VMD. The chain $\omega \to 3\pi$, $\omega \to \pi^\circ\gamma$ and $\pi^\circ \to \gamma\gamma$ is a typical example. There is, however, only qualitative agreement between theory and experiments. The situation is sensibly improved by extending the VMD hypothesis to include additional hadronic vector states coupled to the photon, as discussed in detail in ref. [3] †.

Under this assumption, the main result is represented by the following relations among the coupling constants:

$$g_{\omega\pi^\circ\gamma} = g_{\rho\omega\pi} \frac{e}{f_\rho}(1+\lambda),$$
$$g_{\pi^\circ\gamma\gamma} = g_{\rho\omega\pi} \frac{2e^2}{3f^2_\rho}(1+2\lambda), \tag{10}$$

where λ accounts for the small correction induced by the additional continuum to the simple VMD. Using as input the value $\Gamma(\omega \to 3\pi) = 9.4$ MeV and fitting $\lambda \approx -0.15$ one gets $\Gamma(\omega \to \pi^\circ\gamma) \approx 1$ MeV and $\Gamma(\pi^\circ \to \gamma\gamma) \approx$ 8 eV in excellent agreement with experiments. It is needless to say that the entire set of radiative meson decays we have so far considered can be related altogether by this procedure and deduced by the strong $\omega \to 3\pi$ decay alone.

† The applications of these ideas to e^+e^--annihilation, photoproduction and electron-nucleon scattering are discussed in ref. [11].

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