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M. Greco and Y. N. Srivastava: VECTOR MESONS AND (VIRTUAL)  
NUCLEAR OPTICS. -

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ABSTRACT. -

Nuclear photoproduction and inelastic electron-nucleus scattering at high energies are investigated in the framework of a scaling model of electromagnetic interactions which has been recently proposed. No shadowing is predicted to take place for large  $q^2$ . Our results are in agreement with the available experimental data.

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In a series of papers<sup>(1)</sup>,  $e^+e^-$  annihilation into hadrons, photoproduction, total and singly inclusive electroproduction have been discussed in the framework of a scaling model of electromagnetic interactions in which the photon is coupled to a continuum of hadronic states whose properties are established consistently. A similar extended vector meson dominance (EVMD) model has been developed by Sakurai and Schildknecht<sup>(2)</sup> for inelastic electron-proton scattering. Unfortunately, the deep inelastic data on  $\gamma p$  scattering are not able to choose between the two approaches which basically differ in their choice of coupling the  $n$ -th vector meson to the photon and the vector meson-hadron cross sections.

In addition to the different behaviour in  $e^+e^-$  annihilation into hadrons, where a scaling cross section  $\sigma(e^+e^- \rightarrow \text{hadrons}) = R \sigma(e^+e^- \rightarrow \mu^+\mu^-)$ <sup>(1)</sup>, with  $R \approx 2.5$ , has to be contrasted with  $\sigma(e^+e^- \rightarrow \text{hadrons}) \sim 1/s^2$ , obtained in the second approach<sup>(2)</sup>, there does appear to be an important set of reactions - about which reasonable experimental data already exist - which can serve to delineate between the two approaches. We have in mind the problem of shadowing in total cross section of (off-mass shell) high energy photons on complex nuclei. This reaction has been a source of some embarrassment to the traditional vector dominance people in the following sense. The vector dominance model was shown in ref. 3 and more fully in ref. 4, to lead to considerable shadowing at sufficiently high energies. This is indeed true experimentally for real photons ( $q^2 = 0$ ) and energies  $\nu \gtrsim 2-3$  GeV, although pure VMD predicts considerably more shadowing than observed<sup>(5)(x)</sup>. But the shadowing effect seems to disappear dramatically with  $q^2$ . For example, we quote a typical

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(x) - The disagreement is particularly pronounced in the case of Carbon, where the experimental data are more accurate<sup>(5, 8)</sup>.

figure<sup>(5)</sup>: for gold ( $A = 197$ ), at  $\nu \simeq 3$  to 8 GeV, the relevant ratio  $S = \sigma \gamma^A / A \sigma \gamma^N \simeq 0.65$  at  $q^2 = 0$ , and is  $\simeq 1$  for  $0.25 \leq q^2 \leq 0.75$  GeV/c<sup>2</sup>. Such a sharp change in  $S$  with  $q^2$  is difficult to bring about in VMD and this has been hailed by some to provide the coup de grace to the (old) vector dominance model.

In EVMD the situation is much better. In fact, the same mechanism which helps to obtain Bjorken scaling in " $\gamma p$ " scattering also alleviated this failing in " $\gamma$ "-nucleus scattering. Qualitatively, the criterion for shadowing is given by

$$(1) \quad \frac{M_V^2 + q^2}{\nu} l_\nu \ll 1,$$

where  $l_\nu$  is the mean free path of the vector meson  $V$  in nuclear matter ( $l_\nu \propto 1/\sigma_\nu N$ ). Now Bjorken-scaling in EVMD is obtained by having the higher mass vector states become more and more important as  $q^2$  increases.

This, through the above equation, leads us immediately to the conclusion that shadowing is shifted to higher energies (larger  $\nu$ ) as  $q^2$  increases. So far both ours<sup>(1)</sup>, as well as the other scheme<sup>(2, 6)</sup>, are in accord. The difference arises in  $l_\nu$ . In our approach  $l_\nu$  becomes unbounded for the  $n$ -th vector meson as  $n \rightarrow \infty$ , whereas in ref. 6 it is constant. Thus, we predict that for  $q^2$  large no shadowing takes place, whereas the other model predicts<sup>(2, 6)</sup> that shadowing comes back, for large fixed  $q^2$  as  $\nu$  is sufficiently large.

This is the main conclusion of this paper, where we study quantitatively the problem of shadowing for realistic nuclei and compare our predictions with those of ref. 6 and the experimental data.

Following the work of Brodsky and Pumplin<sup>(4)</sup> we have

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$$(2) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A \sigma_{\gamma N}(\nu, q^2)} = 1 - \frac{\sum_n \text{Im} \left\{ F(x_n) V_{\gamma n}^2 / (m_n^2 + q^2 + V_{nn}) \right\}}{\text{Im} V_{\gamma\gamma}}$$

where the sum is restricted to the vector states only which can be actually produced at the energy  $\nu$ ,

$$(2a) \quad F(x_n) = 1 - \frac{3}{x_n} \left\{ e^{-x_n} (1+x_n)^{-1} + \frac{1}{2} x_n^2 \right\},$$

$$(2b) \quad x_n = 2iR \left\{ |\vec{q}| - |\vec{k}_n| + \frac{1}{2|\vec{k}_n|} V_{nn} \right\},$$

$$(2c) \quad V_{\gamma n} \simeq \frac{2\sqrt{a\pi}}{f_n} \frac{m_n^2 V_{nn}}{q^2 + m_n^2},$$

$$(2d) \quad V_{nn} = -i d |\vec{k}_n| (1-i\eta) \sigma_{nN}(\nu),$$

$$(2e) \quad \text{Im} V_{\gamma\gamma} = -d |\vec{q}| \sigma_{\gamma N}(\nu, q^2),$$

$d$  ( $d \simeq 0.16 \text{ Fermi}^{-3}$ ) is the constant density of the nucleus with radius  $R = r_0 A^{1/3}$  ( $r_0 = 1.3 \text{ Fermi}$ ),  $|\vec{q}|^2 = \nu^2 + q^2$  ( $q^2 \geq 0$ ),  $|\vec{k}_n|^2 = \nu^2 - m_n^2$  and  $\eta$  is the ratio of the real to imaginary part of the vector meson-nucleon forward scattering amplitude, supposed to be constant for all states  $n$ . We have neglected  $t_{\min}$  effects (eq. 2c) and used the diagonal approximation ( $V_{nN} \rightarrow V_{n'N}$ , with  $n \neq n'$ ).

Using the results of the previous work<sup>(1)</sup> in photo- and electro-production, where we have found excellent agreement with the experimental data, we can now express the photon-nucleon cross section  $\sigma_{\gamma N}(\nu, q^2)$  (eq. 2e) in terms of the vector meson-nucleon

cross sections  $\sigma_{nN}(\nu)$ . We have

$$(3) \quad \sigma_{\gamma N}(\nu, q^2) = \frac{4\alpha\pi}{f_\rho^2} \sum_n \frac{(1+2n)}{(1+2n+q^2/m_\rho^2)^2} \sigma_{nN}(\nu) + \text{isoscalars},$$

where we have made use of the fact that in our model the mass spectrum of the vector states is of the form  $m_n^2 = m_\rho^2 (1+2n)$  and the photon-meson couplings satisfy the recurrence relations  $f_n^2 = (m_n^2/m_\rho^2) f_\rho^2$ . Furthermore, following ref. 1, we write  $\sigma_{nN}(\nu)$  in terms of a diffractive and a resonance contribution, which satisfy the following relation:

$$(4) \quad \sigma_{nN}(\nu) = \sigma_{nN}^D + \sigma_{nN}^R(\nu) = \frac{1}{(1+2n)} (\sigma_{\rho N}^D + \sigma_{\rho N}^R(\nu) \sqrt{1+2n}),$$

and similarly for the isoscalar parts. Eq. (4) is a consequence of the scaling behaviour of the strong forward amplitudes  $A_n \sim \sum_i \beta_i (s/m_n^2)^{\alpha_i}$ .

We finally obtain through (2), (3) and (4), with some straightforward algebra,

$$(5) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A \sigma_{\gamma N}(\nu, q^2)} = 1 - \frac{1}{\sqrt{1+q^2/\nu^2}} \frac{\sum(\nu, q^2)}{\sum'(\nu, q^2)}$$

where:

$$(5a) \quad \sum(\nu, q^2) = \sum_n \frac{\left\{ \sigma_{\gamma N}^D + \sigma_{\gamma N}^R(\nu) \sqrt{1+2n} \right\} \sqrt{1 - \frac{m_\rho^2(1+2n)}{\nu^2}}}{\left(1 + 2n + \frac{q^2}{m_\rho^2}\right)^2} A_n +$$

+ isoscalars,

$$(5b) \quad \sum'(\nu, q^2) = \sum_n \frac{\left\{ \sigma_{\rho N}^D + \sigma_{\rho N}^R(\nu) \sqrt{1+2n} \right\}}{\left(1 + 2n + \frac{q^2}{m_\rho^2}\right)^2} + \text{isoscalars},$$

6.

$$(5c) \quad A_n = \left\{ \text{Im} U_n \right\} \left\{ \text{Re} F(x_n) \right\} + \left\{ \text{Re} U_n \right\} \left\{ \text{Im} F(x_n) \right\},$$

$$(5d) \quad U_n = \frac{(1-i\eta)^2 \left( \frac{k_n}{l_n} \right)}{\left[ (m_n^2 + q^2 - \eta \frac{k_n}{l_n}) - i \frac{k_n}{l_n} \right]}$$

and  $l_n = 1/d\sigma_{nN}$  is the mean free path of the n-th vector meson in nuclear matter. Because of eq. (4),  $l_n$  becomes unbounded for  $n \rightarrow \infty$ , as previously stated. In deriving eqs. (5) we have restricted ourselves to the transverse part of  $\sigma_{\gamma N}$ .

The small contribution from  $\sigma_L$  would affect both numerator and denominator of eq. (5) with roughly the same amount and would therefore be negligible.

Despite the apparent complexity of (5), the results are very simple in the high energy limit ( $\nu \rightarrow \infty$ ,  $q^2/\nu^2 \rightarrow 0$ ) and for  $\eta = 0$ . In these limits in fact  $U_n = i$ ,  $\text{Re} x_n = R/l_n$ ,  $\text{Im} x_n = 0$ ,  $A_n = \text{Re} F(x_n) = 1 - G(x_n)$  and eq. (5) becomes

$$(6) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A\sigma_{\gamma N}(\nu, q^2)} = \frac{\tilde{\Sigma}(\nu, q^2)}{\Sigma'(\nu, q^2)},$$

with

$$(7) \quad \tilde{\Sigma}(\nu, q^2) = \frac{\sum_n \left\{ \sigma_{\rho N}^D + \sigma_{\rho N}^R(\nu) \sqrt{1+2n} \right\} G(x_n)}{\left( 1+2n + \frac{q^2}{m_\rho^2} \right)^2} + \text{isoscalars.}$$

In the case of all  $\sigma_{nN}$  equal, in other words if the mean free path in nuclear matter would be the same for all the hadronic constituents of the photon, then one would have  $\tilde{\Sigma}(\nu, q^2) \simeq G(x_\rho)$   $\Sigma'(\nu, q^2)$  and therefore

$$(8) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A \sigma_{\gamma N}(\nu, q^2)} \simeq G(x_\rho) \approx \frac{3}{2} \frac{l_\rho}{R},$$

where the last equality follows from having used the asymptotic form  $G(x) \approx 3/2x$  for  $x$  large, and is valid for large nuclei ( $l_\rho \simeq 3$  fermi). The result (8), which coincides with a pure  $\rho$ -dominance prediction, is physically very clear, because the amount of shadowing depends only on the mean free path in matter, and not upon the intrinsic properties of the hadronic constituents of the photon. Therefore, in the approach of refs. 2 and 6, one predicts that, despite a reduction in shadowing at moderate energies due only to a partial fulfillment of (1), the shadowing effect comes back at sufficiently high energies, both at  $q^2 = 0$  and  $q^2$  large ( $q^2/\nu$  fixed).

The situation is quite different in the case of  $\sigma_{nN} \sim \sim 1/n$  (eq. 4). As long as  $n$  increases, in fact, the mean free path  $l_n$  also increases and therefore the shadowing effect decreases for the large mass components of the hadronic spectrum. This effect implies a different behaviour of the nuclear absorption cross sections in the case of real or highly virtual photons.

At  $q^2 = 0$  the photon dissociates predominantly into a meson and therefore the reduction of shadowing is small. More explicitly we find from (6) and (7) ( $\nu \rightarrow \infty$ ,  $A$  large)

$$(9) \quad \frac{\sigma_{\gamma A}(\nu)}{A \sigma_{\gamma N}(\nu)} \approx \frac{3}{2} \frac{l_\rho}{R} \quad (1-20\%),$$

having used the approximate form  $G(x_n) \approx 1 - \frac{3}{8} x_n$  for  $n$  large.

For  $q^2$  large on the other hand the cross section  $\sigma_{\gamma N}(\nu, q^2)$  is dominated<sup>(1)</sup> by large masses ( $\langle m^2 \rangle \approx q^2$ ) which have a large mean free path in nuclear matter and therefore no shadowing takes place. More in detail from eqs. (6) and (7), taking into account for



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large  $\nu$  only the diffractive part of  $\sigma_{nN}$ , we have

$$(10) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A \sigma_{\gamma N}(\nu, q^2)} \xrightarrow[\nu/q^2 \text{ fixed}]{\text{large } \nu, q^2} 1 - \frac{2}{(1 + \frac{q^2}{m_\rho^2})} \left\{ 1 - G(x_\rho) \right\} -$$

$$- 2 \left( 1 + \frac{q^2}{m_\rho^2} \right) \sum_{n=1}^{\infty} \frac{[1 - G(x_n)]}{(1 + 2n + q^2/m_\rho^2)^2}$$

Using the approximate form  $F(x_n) \approx 1 - \frac{3}{8} x_n$ , and, converting the last sum into an integral, we finally get:

$$(11) \quad \frac{\sigma_{\gamma A}(\nu, q^2)}{A \sigma_{\gamma N}(\nu, q^2)} \xrightarrow[\nu/q^2 \text{ fixed}]{\text{large } \nu, q^2} 1 - H(q^2),$$

where  $H(q^2)$  is given by

$$(12) \quad H(q^2) \approx \frac{2}{(1 + q^2/m_\rho^2)} \left( 1 - \frac{3l_\rho}{2R} \right) + \frac{3R}{8l_\rho} \left( 1 + \frac{m_\rho^2}{q^2} \right) \left\{ \frac{m_\rho^2}{q^2} x \right.$$

$$\left. x \ln \left( 1 + \frac{q^2}{3m_\rho^2} \right) - \frac{m_\rho^2}{q^2 + 3m_\rho^2} \right\}.$$

The shadowing disappears therefore as  $1/q^2$ , for large  $q^2$ . The accuracy of eq. (12) is still good at  $q^2 \gtrsim 1 \text{ GeV}^2$ , provided the energy is large enough ( $\nu \gtrsim 100 \text{ GeV}$ ). At lower energies one has both to take into account the non-diffractive contributions to the cross sections, and use the exact eqs. (5), instead of (6) and (7). This can be worked out only numerically and will be discussed below. These modifications however go in the direction of a further decrease of the shadowing

effect.

For realistic nuclei, and as well as for moderate energies, we have evaluated numerically eqs. (5) using  $\sigma_{\rho N} = \sigma_{\omega N} = 2\sigma_{\rho N}$ ,  $\sigma_{\rho N} = (\sigma_{\rho p} + \sigma_{\rho n})/2$ ,  $\sigma_{\rho p} = (23 + 11.2/\sqrt{\nu})$ mb and  $\sigma_{\rho n} = (23 + 5.2/\sqrt{\nu})$ mb. We have assumed for  $\eta$  the same value and energy dependence as observed in forward Compton scattering<sup>(7)</sup>. Our results are plotted in Figs. 1, 2 and 3 and compared with the experimental data. At  $q^2 = 0$  the agreement with experiments is still true in the case of Carbon, where the data are more accurate and the disagreement with the old VMD was particularly pronounced. As long as  $q^2$  increases (Fig. 2), the shadowing decreases quite rapidly, although the data are not accurate enough to test exactly our predictions. Finally in Fig. 3 we compare our results with those of ref. (6), where a strong  $q^2$ -independent shadowing effect is predicted for sufficiently high energies.

We emphasize that our results are pure predictions of the model, with no use of adjustable parameters, depending only upon the  $\rho$ -nucleon cross sections which, on the other hand, lead to a nice description<sup>(1)</sup> of photo- and electro-production data on nucleon targets.

To conclude, we have shown that our model provides a nice description of the observed features of the  $\nu$ ,  $q^2$  and  $A$  dependence of the photon-nuclei cross sections. We predict that for large  $q^2$  no shadowing takes place, in contrast with the predictions made in ref. (6). Of course further and more accurate experimental data on the  $q^2$  dependence would be highly desirable, particularly at large energies.

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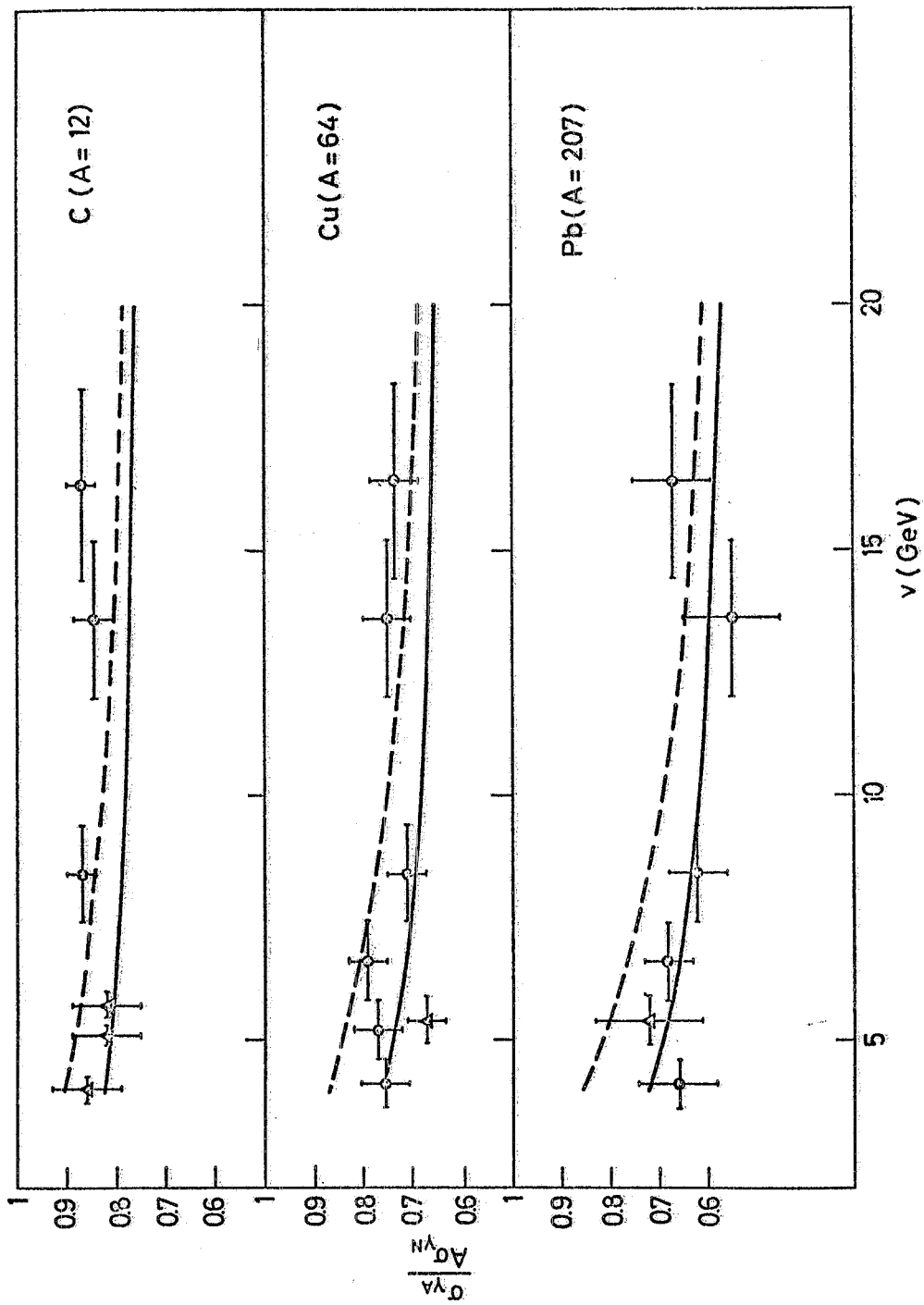


FIG. 1 - Predictions for  $\sigma_{\gamma A}(\nu)/A \sigma_{\text{Compton}}(\nu)$  in photoproduction for various nuclei. The dashed line corresponds to  $\eta = 0$ , the full line to  $\eta = \eta_{\text{Compton}}$  (see the main text). The data are taken from refs. 5 and 8.

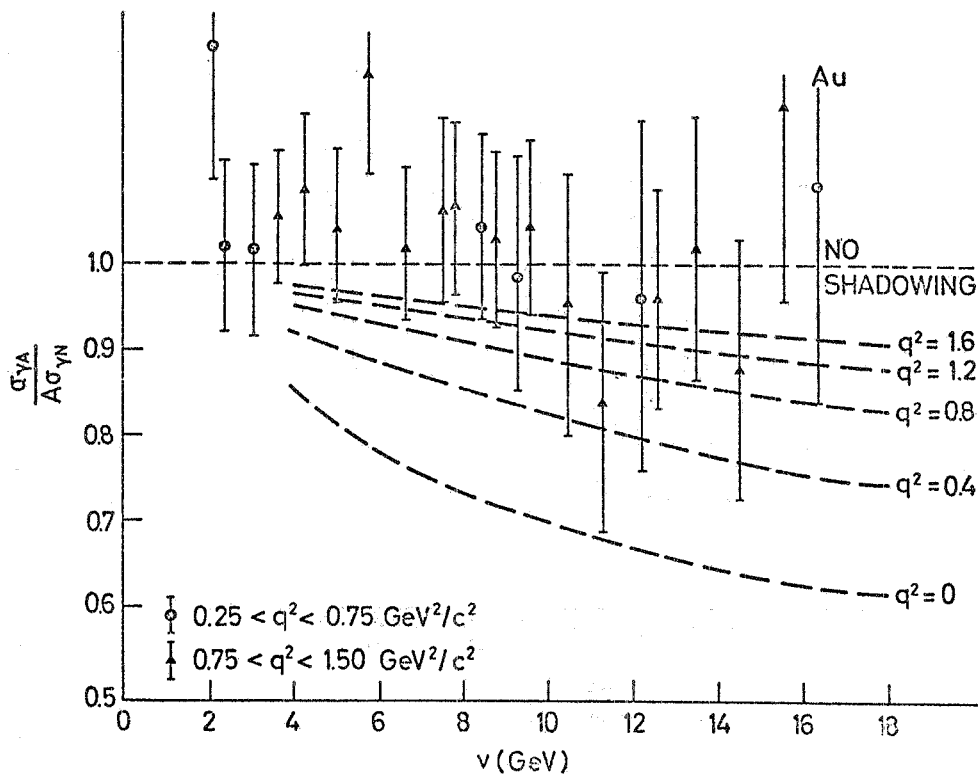
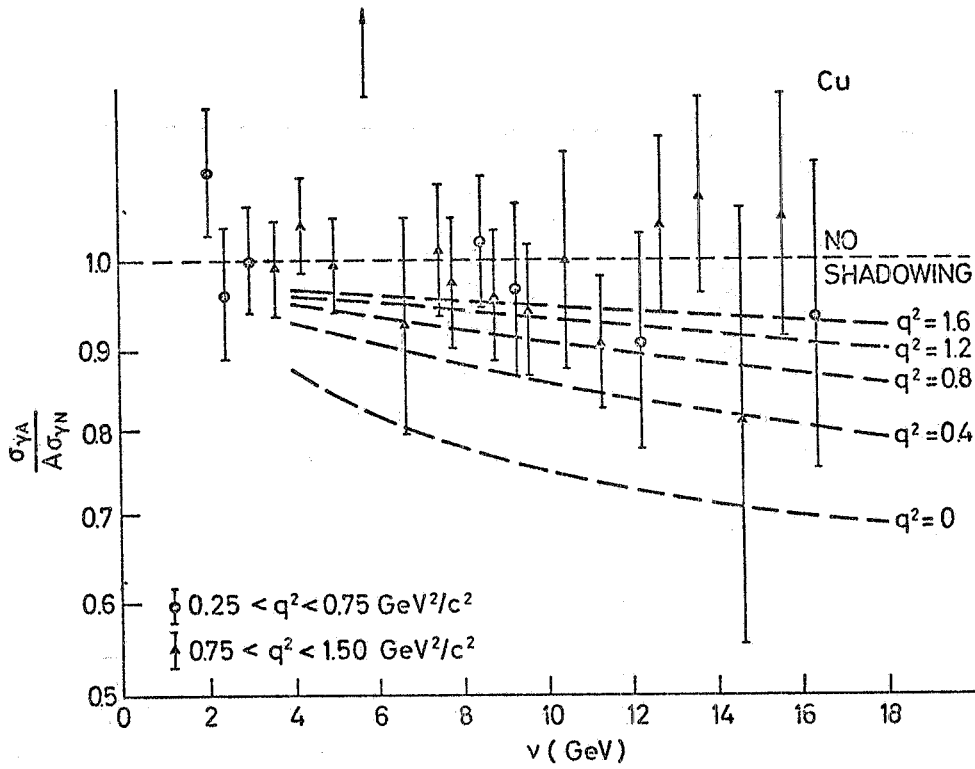


FIG. 2 - Predictions for  $\sigma_{\gamma A}(\nu, q^2)/A \sigma_{\gamma N}(\nu, q^2)$  as a function of the photon energy  $\nu$  and photon mass squared  $-q^2$ . The data are taken from refs. 5 and 8.

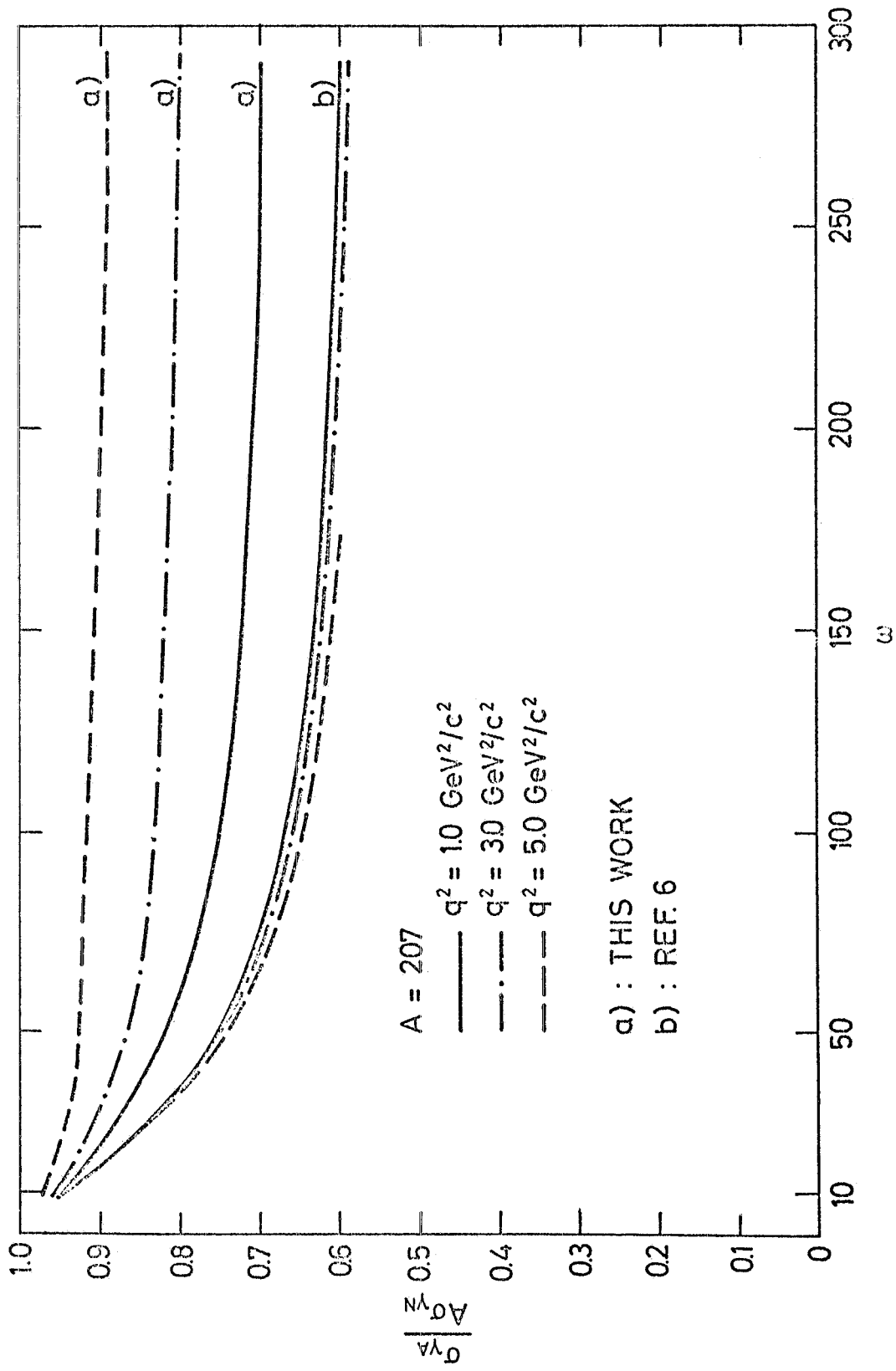


FIG. 3 - Predictions for  $\sigma_{\gamma A}(\nu, q^2)/A \sigma_{\gamma N}(\nu, q^2)$  as a function of  $\omega = 2M\nu/q^2$  and  $q^2$ .