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TRANSVERSE MOMENTUM

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## INCLUSIVE PRODUCTION AT LARGE MASS AND LARGE TRANSVERSE MOMENTUM

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**Abstract:** Production of massive lepton pairs in hadronic collisions is discussed in a model recently proposed by us (with E. Etim) which shows scaling for strong as well as e.m. processes and an increase in the average transverse momentum with the mass. Next we apply this model to obtain approximately a  $p_{\perp}^{-4}$  decrease with transverse momentum,  $p_{\perp}$ , in inclusive hadronic distribution functions.

### 1. Introduction

The production of leptons pairs in hadronic collisions at c.m. energy  $\sqrt{s}$ , and large dilepton mass  $\sqrt{Q^2}$ , were recently analyzed [1], using the idea of shrinkage in the transverse size of the photon with its mass. This led us to propose an intuitive picture for understanding the scaling properties of hadronic as well as non-hadronic processes through the requirement that all cross sections be determined by some transverse size characteristic of the process. In this scheme then, the hadron cross sections approach constants due to a fixed  $\langle p_{\perp} \rangle$ , while cross sections for  $e^+e^-$  annihilation into hadrons, virtual photon scattering of nucleons and massive photon production would all fall as  $1/Q^2$ .

The main conclusions of ref. [1] were (i) scaling for the process  $pp \rightarrow \ell\bar{\ell} + X$ ,

$$\frac{d\sigma^\gamma}{dQ^2} \xrightarrow[\tau = Q^2/s \text{ fixed}]{s, Q^2 \rightarrow \infty} \frac{1}{Q^4} F(\tau) , \quad (1)$$

(ii) Feynman scaling for single photon distribution function, and (iii) the average transverse momentum of photon grows as  $\langle q_{\perp}^2 \rangle \sim Q^2$ .

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An explicit realization of the above scheme was presented in the form of a model which couples the photon to a continuum of hadronic vector states with a linear mass spectrum of the Veneziano type. Such a model has been shown [2, 3] to describe successfully the salient features of high-energy processes involving real, time-like and space-like photons, including the observed scaling behaviour of the structure functions of deep inelastic ep scattering. An appealing feature of this approach is that it links together the large  $Q^2$  scaling properties in e.m. interactions to a similar scaling behaviour in the hadronic interactions, as  $M^2$  become large, where  $M$  is the mass of the hadronic continuum coupled to the photon.

Recent ISR experiments [4] on hadrons reveal a production cross section at large transverse momenta much in excess of the expectation from the sharp fall-off observed below  $\sim 1$  GeV/c. It is our claim that this large transverse momentum behaviour is indeed related to the above mentioned scaling properties of the hadronic interactions.

The purpose of this paper is twofold. We first extend the analysis done in ref. [1] and compare it with the BNL-Columbia experiment [5] on  $\mu\bar{\mu}$  production in pp collisions. Our model seems to describe the various aspects of the data including the rapid variation of the total cross section with incident energy. We then present our predictions for ISR energies. [As stated in ref. [1], an important test of the parton model will be provided by the growth (or lack of it) of  $\langle q_{\perp} \rangle$  with  $Q^2$ .]

In the second part of the paper we present an application of our model to hadronic production at large transverse momenta. We find that essentially the distribution function varies as an inverse fourth power of  $p_{\perp}$ , times a scaling function of  $(\sqrt{s}/2p_{\perp})$ , as could be expected from naive dimensional counting.

## 2. Lepton pair production

Consider the deep inelastic process

$$\text{proton } (p_1) + \text{proton } (p_2) \rightarrow \gamma(q) + X , \quad (2)$$

where  $p_i^2 = m^2$ ,  $q^2 = Q^2$  and  $X$  denote a system of hadrons. The kinematic variables are  $s = (p_1 + p_2)^2$ ;  $t = (p_1 - q)^2$ ;  $u = (p_2 - q)^2$ ,  $M_X^2 = (p_1 + p_2 - q)^2$ , along with Bjorken and Feynman scaling variables

$$x_j^B = 2(p_j \cdot q)/Q^2 , \quad j = 1, 2 ,$$

$$x_F = 2q^*/\sqrt{s} = \frac{Q^2}{s} \frac{x_1^B - x_2^B}{\sqrt{1 - 4m^2/s}} , \quad (3)$$

and a transverse scaling variable

$$x_T = \frac{q_T^2}{Q^2} \underset{\substack{Q^2, s \text{ large} \\ x_j^B \text{ fixed}}}{\sim} \frac{Q^2}{s} \left( x_1^B x_2^B - \frac{s}{Q^2} \right). \quad (4)$$

The single-particle distribution functions are defined as usual

$$f_{pp}^c(q, s) \equiv \frac{1}{\sigma_{pp}^{\text{tot}}(s)} \frac{d\sigma^c}{d^3 q / q_0}. \quad (5)$$

The Feynman scaling of  $f_{pp}^\gamma(q, s)$  was shown [1] to follow from the energy momentum sum rule. The large  $Q^2$  behaviour of  $f_{pp}^\gamma$  is related to the average transverse momentum dependence on  $Q^2$ . From eq. (4) it follows that in the Bjorken limit,  $s$  and  $Q^2$  large with  $\tau = Q^2/s$  fixed,  $q_T^2$  grows as  $Q^2$  unless an *ad hoc* cut-off in  $q_T$  is assumed. In the parton model [6] this is exhibit in the form of a  $\delta$  function in  $x_1^B x_2^B - 1/\tau$ . We take the opposite view of allowing  $q_T^2$  to increase with  $Q^2$  which leads us [1] to

$$f_{pp}^\gamma(Q^2, s, x_j^B, x_F, x_T) \xrightarrow[\tau, x_j^B, x_F, x_T \text{ fixed}]{Q^2 \text{ large}} \left( \frac{1}{Q^2} \right)^2 G_{pp}^\gamma(\tau, x_j^B, x_F, x_T), \quad (6)$$

which implies for the integrated cross section

$$\sigma_{pp}^\gamma(Q^2, s) \underset{\substack{Q^2 \text{ large} \\ \tau \text{ fixed}}}{\sim} \pi \left( \frac{1}{Q^2} \right) \sigma_{pp}^{\text{tot}}(s) \int \frac{dx_F dx_T}{\sqrt{x_F^2 + 4\tau(1+x_T)}} G_{pp}^\gamma(\tau, x_j^B, x_F, x_T). \quad (7)$$

The average multiplicity  $\langle n_{pp}^\gamma(Q^2, s) \rangle$ , defined as  $\sigma_{pp}^\gamma(Q^2/s) / \sigma_{pp}^{\text{tot}}(s)$ , grows as  $(1/Q^2) \ln[s/Q^2]$ . Eqs. (6) and (7) are our statements of scaling, to be contrasted with the parton model, which disagrees with our  $f_{pp}^\gamma$  [eq. (6)] but agrees with its integrated cross section.

To specify further the form of  $f_{pp}^\gamma$ , we use the model developed earlier [2, 3] which visualizes the process as the strong production of an infinite number of vector mesons which then decay into  $\ell\bar{\ell}$  pairs. Defining  $em_n^2/f_n$  as the coupling of the photon to the  $n$ th vector meson of mass  $m_n$  and width  $\Gamma_n$ , the total cross section for a given pair mass  $Q^2$  is

$$\frac{d\sigma_{pp}^{\ell\ell}}{dQ^2} = \frac{\alpha^2}{3\pi} \left( 1 + \frac{2\mu^2}{Q^2} \right) \left( 1 - \frac{4\mu^2}{Q^2} \right) \left( \frac{1}{Q^2} \right) \sum_n \frac{4\pi}{f_n^2} \frac{m_n^4}{(m_n^2 - Q^2)^2 + m_n^2 \Gamma_n^2} \sigma_{pp}^n(m_n^2, s), \quad (8)$$

where  $\mu$  is the lepton mass and  $\sigma_{\text{pp}}^n(m_n^2, s)$  refers to the inclusive cross section to produce a vector meson of mass  $m_n$ . In the same model, the total  $e^+e^-$  annihilation into hadrons is given by

$$\sigma_{e\bar{e}}^{\text{had}}(Q^2) = 4\pi\alpha^2 \sum_n \frac{4\pi}{f_n^2} \frac{m_n \Gamma_n}{(m_n^2 - Q^2)^2 + m_n^2 \Gamma_n^2}. \quad (9)$$

The scaling behaviour of eq. (9) is obtained [3] through  $m_n^2/f_n^2 \rightarrow \text{constant}$ ,  $\Gamma_n/m_n \rightarrow \text{constant}$  and the replacement

$$\sum_n \frac{1}{f_n^2} \frac{m_n \Gamma_n}{(m_n^2 - Q^2)^2 + m_n^2 \Gamma_n^2} \rightarrow \frac{2\pi}{3f_\rho^2} \frac{1}{Q^2}. \quad (10)$$

Comparison with eq. (8) then leads to the scaling behaviour of  $d\sigma_{\text{pp}}^{\text{tot}}/dQ^2$  if the strong cross sections  $\sigma_{\text{pp}}^n(m_n^2, s)$  scale similarly, i.e.,

$$\sigma_{\text{pp}}^n(m_n^2, s) \sim \frac{A}{m_n^2} \mathcal{F}(m_n^2/s), \quad (11)$$

consistent with naive dimensional analysis. Similar results were obtained [2, 3] in the framework of the same model in deep-inelastic electron-nucleon scattering. We are therefore led naturally to a unified scaling behaviour of both e.m. and strong interactions. Figuratively this says that cross sections are determined by some characteristic transverse size which shrinks as the mass increases.

Finally, eqs. (8), (10) and (11) give

$$\frac{d\sigma_{\text{pp}}^{\text{tot}}}{dQ^2} \sim \frac{4\pi\alpha^2}{3Q^4} \left(1 + \frac{2\mu^2}{Q^2}\right) \left(1 - \frac{4\mu^2}{Q^2}\right)^{\frac{1}{2}} \frac{2A}{3\gamma f_\rho^2} \mathcal{F}(Q^2/s), \quad (12)$$

where

$$\gamma = \lim_{n \rightarrow \infty} \frac{\Gamma_n}{m_n}.$$

The full dependence of the dilepton distribution function on the different kinematic variables is essentially the same in our approach, as that of the hadronic inclusive production of mass  $Q^2$  at c.m. energy  $\sqrt{s}$ . To discuss it, let us define  $f(Q^2, s)$  as

$$\sigma^{\text{strong}}(Q^2, s) = \sigma_{\text{pp}}^{\text{tot}}(s) \int \frac{d^3q}{q_0} f(Q^2, s). \quad (13)$$

For  $Q^2$  large, but  $s/Q^2, t/Q^2, u/Q^2$  also large we are in the so-called pionization region. Let us for simplicity restrict ourselves to this region only; the neglect of both fragmentation regions is of course not valid near the kinematical boundaries

( $t$  and/or  $u$  small). *À la* Mueller [7], and taking only the leading (double) pomeron exchanges, we obtain for the central region,

$$f(Q^2, s) \sim \left(\frac{1}{Q^2}\right)^2 h(q_T^2, Q^2) . \quad (14)$$

Eq. (14) follows by scaling both “photon” proton subenergies as  $(s_j/Q^2)^{\alpha_p}$ ,  $j = 1, 2$ , with constant residue functions. This is suggested by the successful deep-inelastic analysis [2, 3] using the same assumptions.

Comparing (14) with (6), (7) and (11), it is clear that our scaling results follow if  $h(q_T^2, Q^2)$  becomes a function of  $x_T = q_T^2/Q^2$  only, for large  $Q^2$ . This reinforces our earlier claim that for large variable mass  $\sqrt{Q^2}$ , the damping is not in the  $q_T^2$  variable but in  $x_T$ .

This is about all we can say on general grounds. To make comparison with experimental data we need a specific form for  $h(q_T^2, Q^2)$ , which reduces to the well-known  $q_T^2$  fall-off, when  $Q^2$  is fixed. So, in analogy with the mesonic distribution functions, we shall assume a factorized form, with a Gaussian in  $x_F$ , which will account for departures from the strictly  $x_F = 0$  central region. So, we have

$$\frac{d\sigma_{pp}^{q\bar{q}}}{dQ^2 dx_F dx_T} = \frac{\pi \alpha^2 B}{Q^4} \left(1 + \frac{2\mu^2}{Q^2}\right) \left(1 - \frac{4\mu^2}{Q^2}\right) \frac{e^{-C_1 x_F^2}}{\sqrt{x_F^2 + 4\tau(1+x_T)}} e^{-C_2 x_T} , \quad (15)$$

where  $B, C_1$  and  $C_2$  are free constants to be determined by comparison with experimental data.

### 3. BNL-Columbia experiment\*

This experiment observed muon pairs in collisions of protons with a uranium target at incident lab. energies of 22, 25, 28.5 and 29.5 GeV. The main results are the following.

- (i) The production cross section varies smoothly with mass as  $d\sigma/dQ \sim 1/Q^5$  for  $1 \leq Q \leq 6$  GeV.
- (ii) The total (integrated) cross section increases with energy by a factor of five over the range explored.
- (iii) The cross section *versus* lab dimuon momentum falls steeply by three to four orders of magnitude.
- (iv) The cross section shows a slow variation with  $\cos \theta$ , where  $\theta$  is the lab angle of the dimuon with respect to the beam direction.

Before comparison with our formula (15), a few remarks are in order regarding the experimental constraints which are essential in our opinion for an understanding of the above features of the data.

- (i) The lab angle  $\theta$  is constrained to be less than  $\bar{\theta} \approx 63$  mrad ( $\cos \theta \gtrsim 0.998$ ). As

\* Ref. [5].

we show below the  $1/Q^5$  fall-off of the dimuon mass spectrum due to this experimental restriction.

(ii) The lab momentum  $q^L$  of the  $\mu\bar{\mu}$  pair is constrained to be greater than  $q_{\min}^L = 12 \text{ GeV}/c$ . In the c.m. frame, this implies that  $\langle x_F \rangle$  changes at each incident proton energy. The sharp rise with  $s$  of the integrated cross section which is perhaps the most puzzling aspect of this experimental (and not satisfactorily explained by any of the other models) has to be attributed to this constraint.

Some kinematics on eq. (15) leads to obtain the following approximate formulae (*not* valid near the kinematic boundaries) for this experiment:

$$\frac{d\sigma}{dQ} \approx \left( \frac{\pi\alpha^2 B}{2C_1 m^2} \right) s^2 \exp\left[ \frac{4C_1 m^2 (q_{\min}^L)^2}{s^2} \right] \frac{1}{Q^5}, \quad (16)$$

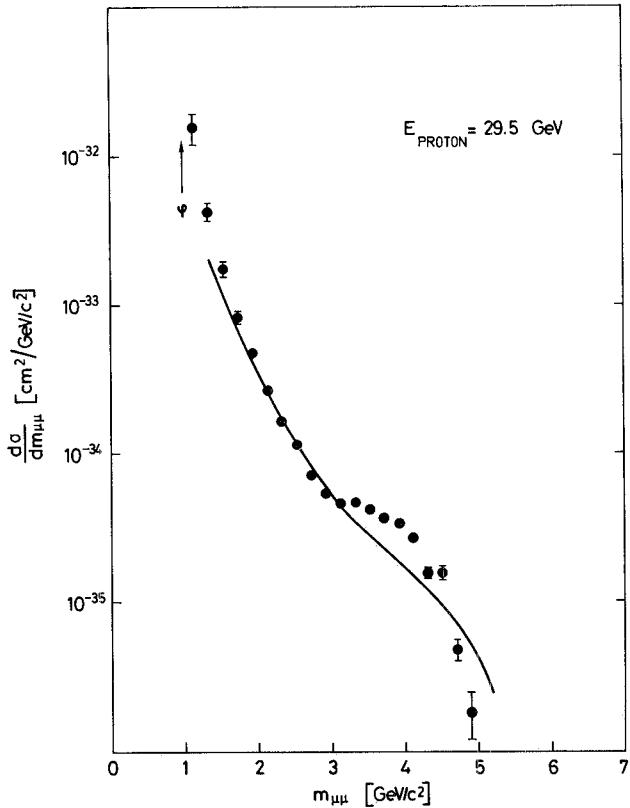


Fig. 1.  $d\sigma/dm_{\mu\mu}$  as a function of the dimuon mass  $m_{\mu\mu}$ . The data refer to the experiment of ref. [5].

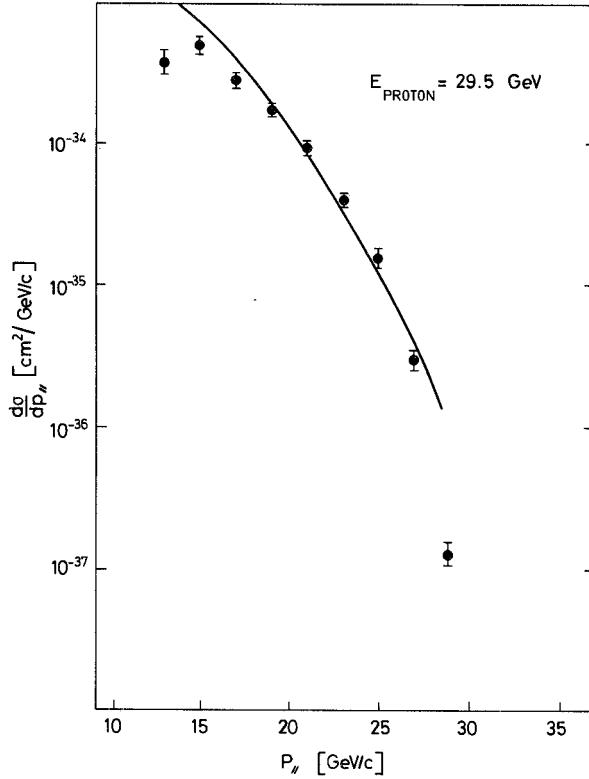


Fig. 2.  $d\sigma/dp_{\parallel}$  as a function of the longitudinal lab di-muon momentum  $p_{\parallel}$ .

$$\frac{d\sigma}{d\theta^2} \approx \left( \frac{\pi\alpha^2 B}{8C_1 m^2} \right) s^2 \exp \left[ -\frac{4C_1 m^2}{s^2} (q_{\min}^L)^2 \right] \exp \left[ -C_2 (q_{\min}^L)^2 \theta^2 \right], \quad (17)$$

$$\frac{d\sigma}{dq_L} \approx \frac{1}{2} \pi\alpha^2 B \bar{\theta}^2 \exp \left[ -\frac{4C_1 m^2}{s^2} q_L^2 \right]. \quad (18)$$

The  $1/Q^5$  behaviour of  $d\sigma/dQ$ , in agreement with the data, is obtained as follows:  $1/Q^3$  comes from the scaling part, an extra  $1/Q^2$  from the assumed  $x_T$  cut-off and the experimental restrictions to very small production angles. The  $s$ -dependence in eqs. (16) and (17) give rise to the sharp increase with energy of the observed total cross sections and as said before, arises experimentally due to the constraint on the pair lab momentum and theoretically due to the hadron-like behaviour of  $x_F$  distribution of the massive  $\gamma$  production.

Figs. 1,2,3 and 4 show that all the gross features of the data are satisfactorily reproduced, the agreement getting worse near the kinematic boundaries. The latter is due to our neglect of non-pomeron exchanges and also due to the dependence of the experi-

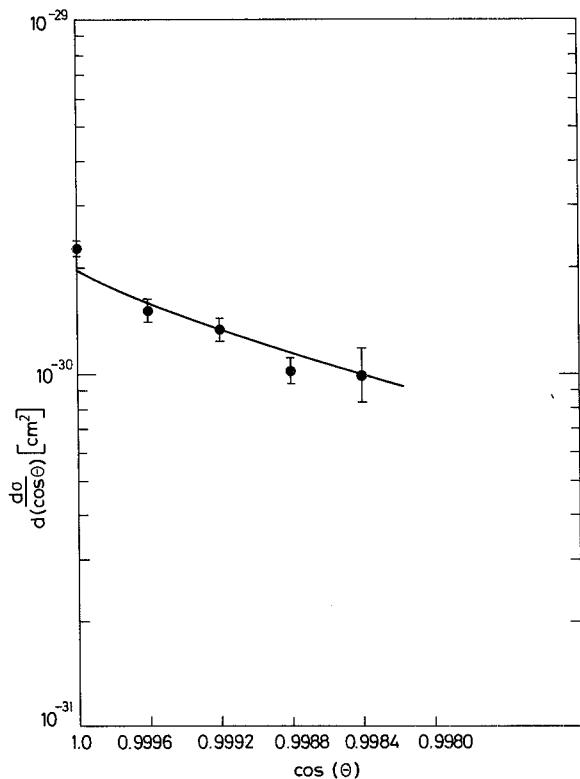


Fig. 3.  $d\sigma/d(\cos\theta)$  as a function of  $\cos\theta$ , where  $\theta$  is defined in the text.

mental detection efficiencies on the input models of production. Spectra at different beam energies are similarly well reproduced. The fitted values of the free parameters are  $C_1 \approx 8$  and  $C_2 \approx 2.3$ . It is amusing that the value  $C_1 \approx 8$  is strikingly close to the  $x$ -distribution of the pionic inclusive production. The normalization constants  $B \approx (8 \times 10^{-32})/\alpha^2 \text{ cm}^2 \text{ GeV}^{-2}$ .

#### 4. Predictions for ISR

Due to completely different experimental set-ups (large-angle production in c.m., etc.) the results will be sharply different from those discussed above. Here we expect a pure scaling behaviour of the mass distribution with roughly a logarithmic dependence on  $s$ . Of course, the most important test is the dependence of  $\langle q_T^2 \rangle$  on  $Q^2$ . Evidence in favour or against will put a serious mortgage on the future survival of our model of any parton model.

Eq. (15) is our basic formula. By integrating over the momenta with *no* kinematic restrictions, we get

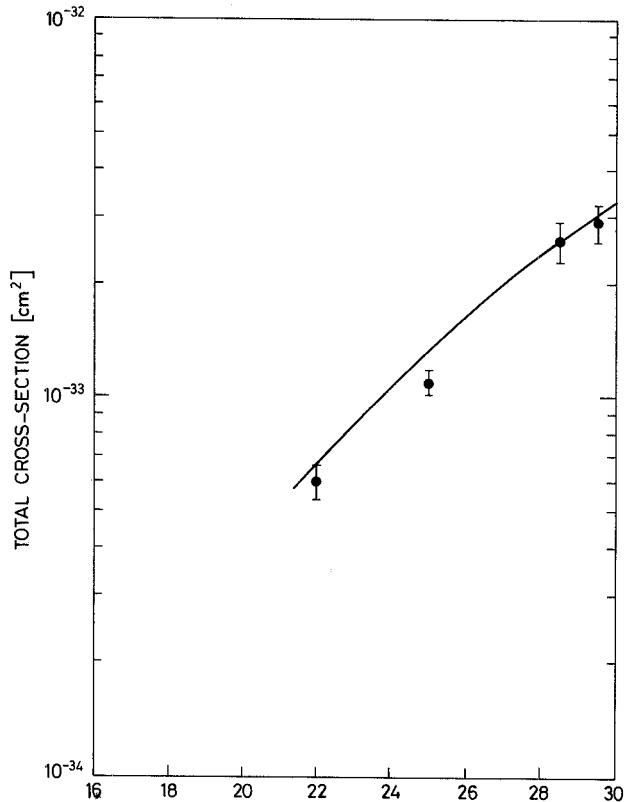
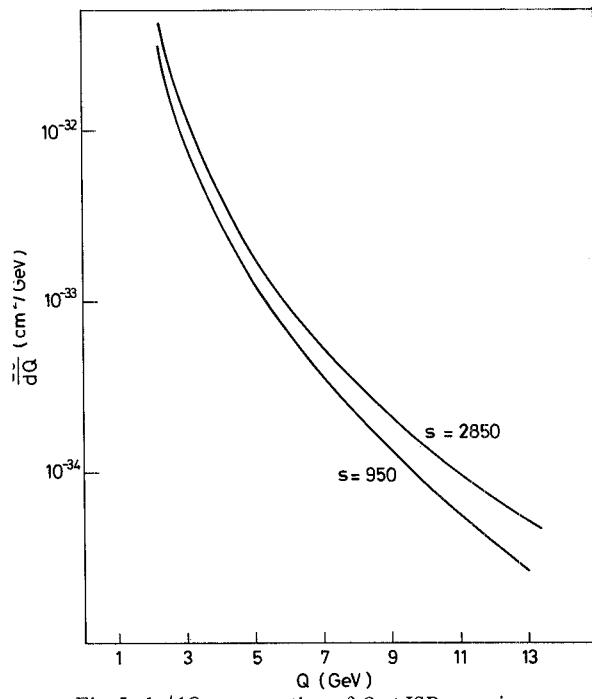
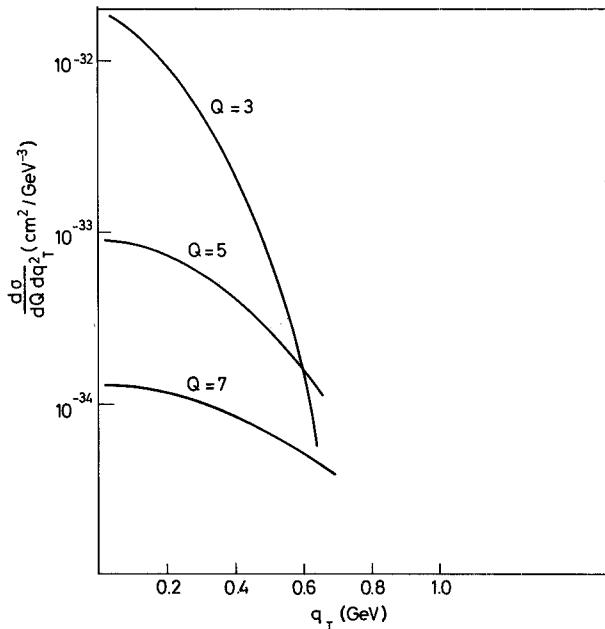


Fig. 4. Total cross section as a function of incident energy.

$$\frac{d\sigma}{dQ} \approx \frac{\pi\alpha^2 B}{C_2 Q^3} \ln \left[ \frac{s}{2C_1 Q^2} \right] \approx \frac{10^{-31}}{Q^3} \ln \left( \frac{s}{2C_1 Q^2} \right) \text{ cm}^2 \text{ GeV}^{-1} \quad (19)$$

in the approximation  $s/2C_1 Q^2 \gg 1$ . The numerically integrated results for two ISR energies are shown in fig. 5. Fig. 6 shows the effect of a transverse shrinkage for various  $Q^2$  values. This has to be compared with a  $Q^2$  independent  $\theta$  distribution (parton model results) strongly peaked around the beam direction. Needless to say that a measurement of the fall  $x_F$  and  $x_T$  dependence of the distribution function will provide a test of the hadron-like behaviour or not of the heavy photon production.

Fig. 5.  $d\sigma/dQ$  as a function of  $Q$  at ISR energies.Fig. 6.  $d\sigma/dQ dq_T^2$  as a function of  $q_T$ , for different values of  $Q$ , at  $s = 950 \text{ GeV}^2$ .

### 5. Hadronic production at large $p_\perp$

Since our previous arguments were based on the hadronic cross sections scaling in a specific way, it is natural to ask for some other consequences of such a hypothesis. Indeed, we find that due to the lack of an absolute transverse momentum cut-off, we would expect an abundant hadronic production at large angles with a rate of the order of  $(d\sigma/dm)_{\text{had}} \sim 1/\alpha^2 (d\sigma/dm)_{\text{em}}$ .

Recent ISR experiments [4] have shown rather large cross sections at high  $p_\perp$ . Theoretically, this had led to proposals [8] which try to obtain slowly decreasing distributions (with  $p_\perp$ ) with a parton-like view of the strong interactions. In our model, such a behaviour arises due to the above mentioned scaling properties. The physical picture is very simple. The hadronic continuum produced with a law similar to eq. (15), will decay, roughly isotropically in its rest frame, to the observed particle plus everything else. Consider for example pion production. In analogy to the  $e^+e^-$  case, let us define a distribution function of the simple form

$$\frac{dN}{d^3p/p_0} = \frac{12}{\pi} \langle n \rangle \frac{1}{Q^2} \left(1 - \frac{2p \cdot q}{Q^2}\right)^2, \quad (20)$$

where  $\langle n \rangle$  is the average multiplicity (assumed to be constant),  $p$  is the pion four-momentum and  $q (q^2 = Q^2)$  is the momentum of the hadronic state which is decaying. By definition

$$\langle n \rangle = \int \frac{d^3p}{p_0} \left( \frac{dN}{d^3p/p_0} \right). \quad (21)$$

The cross section for pion production is obtained as

$$\frac{d\sigma}{d^3p/p_0} = \frac{12}{\pi} \langle n \rangle \int dQ^2 \left( \frac{d^3q}{q_0} \right) \left( \frac{d\sigma^{\text{had}}}{dQ^2 d^3q/q_0} \right) \frac{1}{Q^2} \left(1 - \frac{2p \cdot q}{Q^2}\right)^2, \quad (22)$$

where  $d\sigma^{\text{had}}/dQ^2 d^3q/q_0$  is the invariant cross section for producing the hadronic state as mass  $\sqrt{Q^2}$  at c.m. energy  $\sqrt{s}$ , and we assume it to have a form similar to that given by eq. (15). The integral in (22) is constrained by the condition  $Q^2 - 2p \cdot q \geq 0$ , if we neglect the pion mass. This confines the three-momentum  $q$  to a paraboloid whose axis is along  $p$ , with an approximate constraint also on the magnitude of  $q$ . Writing

$$\frac{d\sigma^{\text{had}}}{dQ^2 d^3q/q_0} \sim \frac{B}{Q^6} \exp \left[ -C_1 \left( \frac{2q_L}{\sqrt{s}} \right)^2 \right] \exp \left[ -C_2 \left( \frac{q_\perp}{Q} \right)^2 \right] \quad (23)$$

and defining  $k_L = q_L/Q$ ,  $k_T = q_T/Q$ ,  $\alpha = \sqrt{1 + k_L^2 + k_T^2} - k_T \cdot \hat{n}$ ,  $y = 2p_\perp \alpha/Q$ , where  $n = p/|p|$ , we obtain

$$\begin{aligned} \frac{d\sigma}{d^3p/p_0} \Big|_{90^\circ} &\sim 2 \int \frac{d^3k}{\sqrt{1+k_L^2+k_T^2}} e^{-C_2 k_T^2} \frac{1}{(2p_\perp \alpha)^4} \\ &\times \int_{y_0}^1 y^3 dy \exp \left[ -4C_1 k_L^2 \left( \frac{2p_\perp \alpha}{\sqrt{s}y} \right)^2 \right] (1-y)^2 , \end{aligned} \quad (24)$$

with

$$y_0 = \frac{2p_\perp}{\sqrt{s}} \frac{\sqrt{1+k_L^2+k_T^2} - k_T \cdot \hat{n}}{\sqrt{1+k_L^2+k_T^2} - \sqrt{k_L^2+k_T^2}} .$$

In the limit of  $s, p_\perp$  large, it follows from eq. (24) that

$$\frac{d\sigma}{d^3p/p_0} \Big|_{90^\circ} \sim \left( \frac{1}{p_\perp} \right)^4 \left( 1 - \frac{2p_\perp}{\sqrt{s}} \right)^3 . \quad (25)$$

The final form of the scaling function reflects our initial choice in eq. (20). More generally, if we had started with a distribution function of the type  $(1 - (2p \cdot q/Q^2))^n$  for the  $Q$  decay, one would find

$$\mathcal{F}\left(\frac{\sqrt{s}}{2p_\perp}\right) \approx \left(1 - \frac{2p_\perp}{\sqrt{s}}\right)^{n+1}$$

for the produced particle at large  $p_\perp$ . Notice that the final form (25) depends crucially on the existence of our  $x_T$  cut-off in the  $q$  distribution function. The usual  $q_T$  cut-off will obtain a Gaussian but never a polynomial decrease with  $p$ . In our model, since the  $Q^2$  integral is roughly dominated by masses of the order of  $2p_\perp$ , an immediate and observable consequence is the associated production of a bunch of particles in the opposite direction of  $p_\perp$ .

The generalization of eq. (25) to angles other than  $90^\circ$  can be expressed in the form  $p_0 d\sigma/d^3p \sim (1/p_\perp)^4 F(x, p_\perp/\sqrt{s})$  where the function  $F$  saturates faster with increasing  $x$ , the asymptotic value sharply ( $\sim$  Gaussian) falling in  $x$ .

We do not make a detailed comparison with the experimental data since we are ignorant of the dynamics operative at small  $p_\perp$ , and thus of the actual form of the transverse distribution of the transverse distribution and its extrapolation the moderate values of  $p_\perp$ . Such an ambiguity is absent only at very large  $p_\perp$ . From the normalization obtained in the lepton pion analysis, we find however the right order of mag-

nitude for the cross sections. Regarding the nature of hadrons produced by our mechanism, it is reasonable to expect that  $\pi$  and K productions (at large  $p_{\perp}$ ) are comparable because of the similarity of our structure functions to that of  $e^+e^- \rightarrow$  hadrons. Of course, relative weights of different SU(3) multiplets is not determined.

## 6. Conclusions

In summary, we have compared in detail our model to the BNL-Columbia data and found reasonable agreement. Then, we have given our predictions for ISR sets ups to measure the di-lepton spectra. Finally, an explanation of the large hadronic cross-section at high  $p_{\perp}$  is also found to emerge naturally from our model.

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