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Abstract: Electron-positron annihilation into hadrons, photoproduction, total and singly inclusive electro-production are discussed in the framework of a scaling model of electromagnetic interactions in which the photon is coupled to a continuum of hadronic states whose properties are established consistently. The model makes use of the concept of the shrinkage of the size of the particle with its mass. The exhibition of scaling properties in e.m. interactions demands a similar scaling behaviour of strong interactions. Predictions are made which are in good agreement with experiments.

1. Introduction

Over the past few years, considerable efforts have been devoted to the understanding of the electromagnetic interactions of hadrons. The measurements of inelastic electron scattering performed at SLAC [1] have given evidence of a scaling behaviour of the nucleon structure functions and suggested an intuitive picture of the hadrons as composite systems of point-like constituents with simple properties. On the other hand, the study of the photoproduction of hadrons and similar photon initiated hadronic processes at high energies has indicated strong similarities between these processes and ordinary hadronic reactions. This hadron-like behaviour of the photon is quantitatively stated in the conjecture that the electromagnetic current is saturated by a few vector mesons poles, say, ρ, ω and φ. This statement leads to a set of relations [2] between the cross sections for photon absorption by nucleons, photoproduction of vector mesons, Compton scattering, and vector meson production through electron-positron annihilation. This set of relation is only qualitatively satisfied. Furthermore, the results from e^+e^- annihilation suggest that higher mass vector mesons can be coupled to the photon, the broad $\rho'(1600)$ [3] being one typical example. The high energy data from Frascati and CEA indicate [4] in addition a strong hadronic production with total cross sections also larger than $\sigma_{\mu\bar{\mu}}$, the point-like muon pairs production cross section. The ultimate goal would be to reach a unified understanding of the electro-magnetic properties of the hadrons

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as the photon mass is continuously varied from the $q^2 \simeq 0$ limit, where hadron-like behaviour is clearly indicated, to the large q^2 (q^2 both positive and negative) deep inelastic region, where point-like behaviour seems to be suggested.

Some attempts along this line have been reported recently [5, 6], suggesting a simple and unique scheme of electromagnetic interactions. The underlying physical idea is that the photon is coupled to a continuum of hadronic vector states with a linear mass spectrum of the Veneziano type, the same which can be excited in e^+e^- annihilation. As long as the q^2 values are rather small, the effect of the continuum is only to "renormalize" the ρ dominance results, the corrections being however, essential to a quantitative understanding of the links among different processes. Typical examples are the improvements found in total photon absorption and Compton scattering on nucleons [5, 7, 8], and in radiative meson decays [9].

In e^+e^- annihilation, of course, this is no longer true; the cross sections are dominated locally by the nearest pole, for example as indicated by the recent Frascati data [4] around the $\rho'(1600)$. On the other hand, when q^2 gets large higher mass contributions become more and more important, and the continuum gives rise, under suitable conditions, to a smooth scaling behaviour of the cross sections, which simulates a point-like interaction. Several applications of this idea have been successful in deriving scaling results in e^+e^- annihilation into hadrons [5, 10, 11], deep inelastic electron-nucleon scattering [5, 8, 11], and lepton pair production in hadron collisions [6].

The aim of the present paper is to provide a unified picture for the main high energy processes involving real, space-like and time-like photons, by using this idea of hadronic constituents of the photon, and determining their properties from consistent links among different processes. Some of the results reported here have already been published [5, 6].

We first study the e^+e^- annihilation into hadrons, both in the low-intermediate energy region of resonance production, and in the asymptotic regime. In particular, we discuss the implications of the existence of a $\rho'(1250)$ meson, suggesting its identification with the $\omega\pi$ enhancement diffractively produced in the reaction $\gamma p \rightarrow \omega\pi^0 p$ [2]. In the asymptotic scaling region we determine the vector-meson photon coupling constants and the total widths of the hadronic components of the photon, giving also a prediction for the total e^+e^- annihilation cross section. The pion form factor F_π is also studied along the same lines.

Photo and electroproduction are then discussed in the framework of two-component duality (diffraction + resonances). By extending to the whole set of vector states the well-known VMD relation between the photon nucleon total cross section and that for photoproduction of vector mesons, we are able to determine the density of the mass spectrum, which turns out to be of the form $m_n^2 = m_\rho^2 (1 + 2n)$, in other words a pure Veneziano-like spectrum. The nucleon structure functions are then nicely predicted, in good agreement with experiments. An appealing feature of this approach is that the scaling properties in e.m. interactions as $Q^2 \rightarrow \infty$ requires a similar scaling behaviour of the hadronic interactions as m^2 approaches infinity, m being the mass of the hadronic continuum coupled to the photon.

By employing the general fremawork proposed by Mueller [12], we extend our results to simple particle inclusive electroproduction. Particular emphasis is placed on the Q^2 dependence of the distribution functions. The main feature of our approach is that the average transverse momentum of produced hadrons in the photon fragmentation region grows as $\sqrt{Q^2}$. A similar dependence is also found in large mass photon production in hadronic collisions [6]. All the above results can be easily understood in an intuitive picture of hadronic as well as non-hadronic scaling which has been recently proposed [6], and which makes use of the hypothesis of shrinkage of the size of a particle with its mass. As also stated earlier, all our scaling results coincide with those of the parton model [13] as far as the integrated cross sections are concerned, but the inclusive distribution functions have a quite different Q^2 dependence.

Finally, we also report for completeness the results recently obtained [14] in the framework of the same picture of hadronic scaling, concerning inclusive production at large transverse momentum.

2. e^+e^- annihilation into hadrons

We consider electron-positron annihilation into an arbitrary final hadron state f in the one-photon exchange approximation, and on the hypothesis of dominance by an intermediate vector meson state. Define em_n^2/f_n as the coupling of the photon to the vector meson of mass m_n and width Γ_n. Then the total cross section for $e^+e^- \to$ $V_n \to$ f at a total energy $2E = \sqrt{s}$ near the mass m_n can be written as

$$\sigma_{e\bar{e}\to f}(s) \simeq \frac{16\pi^2\alpha^2}{f_n^2} \frac{m_n\Gamma_{n\to f}}{(s-m_n^2)^2 + m_n^2\Gamma_n^2} . \tag{1}$$

The coupling constant f_n is assumed independent of the photon mass $\sqrt{s}$, and is measured on the vector meson mass shell.

In addition to the well-known ρ, ω and φ mesons, whose properties have been extensively studied at the Orsay and Novosibirsk storage rings, a new vector state, the $\rho'(1600)$ has recently been observed at Frascati [3, 4] and at SLAC [15] with $\Gamma_{\rho'} \sim$ 350 MeV, $\rho\pi\pi$ as the main decay mode, and $(f_{\rho'}/f_\rho)^2 \approx 4{-}5$. The main properties of this vector state, and the corresponding sizeable production of inelastic final states in e^+e^- annihilation, were predicted [7, 9] on the basis of a previous study of the radiative decays of mesons, together with the use of FESR techniques. The broadness of this state, and the present experimental uncertainties, make its exact mass difficult to determine. However for a world of linearly rising Regge trajectories of universal slope, we can identify it with the vector component of the first even daughter of the ρ, and rename it $\rho''(1600)$.

What about the odd daughters, and in particular the $\rho'(1250)$? In the framework of our model, with its infinity of vector mesons coupled to the photon, and with the assumption of a linear mass spectrum $m_n^2 = m_0^2(1 + an)$, we are able to determine the

spacing a, as shown below in discussing the $q^2 = 0$ processes. The result will be $a = 2$, in other words, a pure Veneziano-like spectrum. It is therefore interesting to look at the experimental situation concerning the $\rho'(1250)$ in photoproduction and e^+e^- annihilation. There exists evidence [2] in photoproduction in both bubble chamber and counter experiments of an $\omega\pi$ enhancement at masses around 1.25 GeV with compatible results for the cross section, which is observed to be of about 1 μb and roughly energy independent, suggesting a pomeron exchange mechanism. The absence of a strong $\pi^+\pi^-$ signal at the same mass, and the lack of a spin parity analysis, has led to the suggestion that this enhancement be identified with the $J^P = 1^+$ B meson, leading to the first clear counter example to Morrison's [16] rule. If, however, for some dynamical reason the mode $\rho' \to \pi^+\pi^-$ is depressed, the observed enhancement could be identified with the $\rho'(1250)$. In the following we shall show that this hypothesis is consistent with the current experimental situation. Assuming the approximate equality of the ρ and ρ' elastic cross-sections, and neglecting non-diagonal terms $\rho p \to \rho' p$, one can deduce

$$\left(\frac{f_\rho}{f_{\rho'}}\right)^2 \frac{\Gamma_{\rho' \to \omega\pi}}{\Gamma_{\rho'}} \approx 0.06, \tag{2}$$

having used for ρ photoproduction the value $\sigma_\rho = 16$ μb at 4 GeV.

In the reaction $e^+e^- \to \pi^+\pi^-$ [17], on the other hand, one observes a clear deviation from the Gounaris-Sakurai expression for F_π around $\sqrt{s} \sim 1.25$ GeV, in a region in which the data are not contaminated by possible K^+K^- pairs. Assuming a ρ' presence in the data one finds [18] $g_{\rho'\pi\pi}/f_{\rho'} \approx -0.05$ and, therefore

$$\left(\frac{f_\rho}{f_{\rho'}}\right)^2 \frac{\Gamma_{\rho' \to \pi\pi}}{\Gamma_{\rho'}} \approx 0.5 \times 10^{-2}, \tag{3}$$

where we have assumed $\Gamma_{\rho'} \approx 130$ MeV [19]. From (2) and (3) there follows $\Gamma_{\rho' \to \pi\pi}/\Gamma_{\rho' \to \omega\pi} \approx 0.08$, showing consistence in identifying with the $\rho'(1250)$ the $\omega\pi$ enhancement found in photoprodluction. Another suggestion of a 1^- state with mass and width of 1.25 and 0.13 GeV respectively comes from $p\bar{p}$ annihilation at rest in the $\omega\pi\pi$ final state [19]. Eq. (2) implies for $e^+e^- \to \rho' \to \omega\pi$ a cross section of about 40 nbs at the ρ' peak, which is again consistent with the storage rign results [4]. Finally, a strong $\omega\pi$ coupling is also suggested from a previous study of radiative meson decays [9] which also predicts $g_{\rho'\omega\pi}/g_{\rho\omega\pi} < 0$. Clearly, further and more accurate data are needed to decide definitely upon this vector state which, from the above rough estimates, appears to be quite weakly coupled to the photon.

We would like to discuss now the scaling implications for our model in the asymptotic region. Let us observe first that the total annihilation into hadrons can be written in terms of the decay width of a virtual time-like photon of mass $\sqrt{s}$ as

$$\sigma_{e\bar{e} \to h}(s) = \frac{4\pi\alpha}{s^{3/2}} \Gamma_{\gamma \to h}(s). \tag{4}$$

The scaling hypothsis requires that $\Gamma_{\gamma \to h}(s) \sim \sqrt{s}$, which is consistent with naive dimensional analysis.

Assuming an infinite set of vector states coupled to the photon, we have from (1)

$$\sigma_{e\bar{e}\to h}(s) \approx 16\,\pi^2\alpha^2 \sum_n \frac{1}{f_n^2} \frac{m_n\Gamma_n}{(s-m_n^2)^2 + m_n^2\Gamma_n^2}\,,\tag{5}$$

where interference terms have been neglected. This approximation will be discussed in detail in the case of a simple final state, namely $e^+e^- \to \pi^+\pi^-$.

A simple solution for an asymptotically scaling cross section is given by

$$\frac{\Gamma_n}{m_n} \equiv \gamma = \text{const}, \qquad \frac{m_n^2}{f_n^2} \equiv b^2 = \text{const},\tag{6}$$

consistent with $\Gamma_{\gamma\to h}(s) \sim \sqrt{s}$. From eq. (5) we obtain then

$$\sigma_{e\bar{e}\to h}(s) \approx \frac{64\pi^2\alpha^2}{3}\,\frac{\gamma b^2}{am_\rho^2}\,\frac{1}{s}\int_{m_{\bar{n}}^2/s}^{\infty} \frac{\mathrm{d}y}{(y-1)^2 + \gamma^2 y^2}\,,\tag{7}$$

and, therefore,

$$\sigma_{e\bar{e}\to h}(s) \approx \frac{64\pi^2\alpha^2}{3s}\,\frac{b^2}{am_\rho^2}\,[\tfrac{1}{2}\pi + \text{arctg}(1/\gamma)]\,,\tag{8}$$

where a factor $\tfrac{4}{3}$ accounts for both the $I = 1$ and $I = 0$ components, and as usual $m_n^2 = m_0^2(1 + an)$. Using the value $b^2 = m_\rho^2/f_\rho^2$, which seems to be suggested already by the results on the $\rho''(1600)$, we predict

$$\sigma_{e\bar{e}\to h}(s) \approx \frac{8\pi^2}{f_\rho^2}\,\sigma_{\mu\bar{\mu}}(s) \approx (2.5 - 3)\,\sigma_{\mu\bar{\mu}}(s)\,,\tag{9}$$

where the value $a = 2$ has been used. This result is consistent with the large cross sections ($\sigma_{e\bar{e}\to h}/\sigma_{\mu\bar{\mu}} \gtrsim 2$) recently observed [4] both at Frascati and CEA ($9 \leqslant s \leqslant 16 \text{ GeV}^2$).

As it is clear, the point-like behaviour of $\sigma_{e\bar{e}\to h}(s)$ is provided in our picture by broader and broader higher mass vector mesons which add together to build up a smooth scaling continuum, in a very similar way to what happens in strong interactions, where an infinite number of resonances build up a smooth Regge-like behaviour of the scattering amplitude. As an immediate consequence of this picture we predict a quite isotropic distribution of the observed final particles, and therefore the absence of jets which, in the parton model [13], arise from the existence of a transverse momentum cut-off.

We discuss now, within the framework of our model, a particular simple final state, i.e., $\pi^+\pi^-$. Our aim is to provide some control of the approximation in (5) of neglecting possible interferences among different intermediate vector mesons. The main results concerning the pion form factor $F_\pi(s)$, together with a discussion of the energy dependence of the final multiplicities in e^+e^- annihilation will be reported elsewhere [18]. As they bear on this question of interference, for the reader's convenience they are summarized in the following. Given $F_\pi(s)$ in the form

$$F_\pi(s) = \sum_{n=0}^{\infty} \frac{m_n^2}{f_n} \frac{1}{s-m_n^2 + im_n \Gamma_n} g_{n\pi\pi} \,,$$ (10)

the sum is then separated into two pieces,

$$\Sigma \equiv \sum_{n=0}^{N} \,, \qquad \Sigma' \equiv \sum_{n=N+1}^{\infty} \,,$$

where asymptotic considerations are applied to Σ'. The following two working hypothises are made.

(i) The term having $1/s$ behaviour coming from Σ is asymptotically cancelled by the continuum, all components of which are in phase. This condition is realized in all Veneziano-like models for the pion form factor [20].

(ii) The asymptotic behaviour of $F_\pi(s)$ is $s^{-3/2}$, being fixed by the threshold behaviour of the inclusive single pion distribution function which is assumed to have the form $F(x) \sim (1 - x)^2$.

Using (6) one then finds that asymptotically

$$g_{n\pi\pi} \sim g n^{-2}, \qquad \frac{\Gamma_{n\pi\pi}}{\Gamma_n} \sim n^{-4} \sim \left(\frac{m_\rho^2}{m_n^2}\right)^4,$$ (11)

$$F_\pi(s) \sim \frac{1}{s} \left\{ \sum_{n=0}^{N} \frac{m_n^2}{f_n} g_{n\pi\pi} + \frac{4m_\rho^3 g}{m_{N+1}f_\rho} \right\} - \left(\frac{m_\rho^2}{s}\right)^{3/2} \left\{ \frac{2g}{f_\rho}\pi(1 + \gamma^2)^{1/4}(\text{siu } \theta + i \cos \theta) \right\} \,,$$

with (12)

$$\cos \theta \equiv \left\{\frac{1}{2}\left(1 + \frac{1}{\sqrt{1 + \gamma^2}}\right)\right\}^{1/2} \approx 1, \quad \sin \theta \equiv \left\{\frac{1}{2}\left(1 - \frac{1}{\sqrt{1 + \gamma^2}}\right)\right\}^{1/2} \approx 0 \,.$$

We consider now the case of narrow resonances, which leads to no interferences in the final state. By writing

$$\Sigma' = \sum_{n=N+1}^{\infty} \frac{m_n^2}{f_n} \frac{1}{s-m_n^2 + i\epsilon} g_{n\pi\pi} \,,$$ (13)

and performing at the end the limit $\epsilon \to 0$, we find, with the above assumptions (i) and (ii):

$$F_\pi(s) \sim -\frac{2g}{f_\rho}\pi i \left(\frac{m_\rho^2}{s}\right)^{3/2} \,.$$ (14)

The only difference with eq. (12) consists in the factor $(1 + \gamma^2)^{\frac{1}{4}}(\sin \theta + i \cos \theta) \approx i$. This result supports therefore the approximate validity of (5) in the general case of the total cross sections.

Let us briefly comment on eqs. (11) and (12). Eq. (11) states the interesting result of a very sharp decrease of the elasticities with increasing n, in agreement to what is already observed for very low values of n. The cancellation of the first term of the

right-hand side of (12) determines g from the knowledge of the lowest poles. The fit to the colliding beam results up to energies $\sqrt{s} \sim 1.8$ GeV leads to

$$F_\pi(s) \underset{s \to \infty}{\approx} 3i \left(\frac{m_\rho^2}{s} \right)^{3/2} .$$

(15)

3. Photoproduction

The failure of the vector dominance model in relating the cross section for photoproduction of vector meson to the total hadronic photon nucleon cross section is well known. More explicitly, using the relation [2]

$$\sigma_{\text{tot}}(\gamma p) = \sum_{v=\rho,\omega,\varphi} \sqrt{\frac{16\pi^2 e^2}{f_V^2} \frac{1}{1 + \eta_V^2} \frac{d\sigma^0}{dt}(\gamma p \to V p)} ,$$

(16)

where $d\sigma^0/dt$ denotes the forward cross section, and η_V the ratio of the real to the imaginary parts, the value of $f_\rho^2/4\pi$ obtained through eq. (16) from photoproduction data is smaller by a factor of about two than the results from e^+e^- colliding beams. The inclusion of additive vector states has been suggested [5, 7, 8] as a possible way out of this difficulty. We will show in detail that eq. (16) can be exactly satisfied in the framework of our model, determining in addition the exact density of the vector mass spectrum.

We shall assume the well-known two-component duality framework in which the ordinary Regge exchanges and the pomeron exchange average the resonances and the background respectively. The total photoabsorption cross-section can then be written in terms of a diffractive and resonance contribution, and is parametrized as follows [2]:

$$\sigma_{\gamma p}(\nu) \equiv \sigma_{\gamma p}^D + \sigma_{\gamma p}^R(\nu) = 98.7 + \frac{64.9}{\sqrt{\nu}} \ \mu\text{b}.$$

(17)

Defining

$$\sigma_{\rho p}(\nu) = \sqrt{\frac{16\pi}{e^2} f_\rho^2 \frac{1}{1 + \eta_\rho^2} \frac{d\sigma^0}{dt}(\gamma p \to \rho p)} ,$$

(18)

with an analogous separation into diffractive and resonant terms, the inclusion of the set of vector states leads to the following extensions of (16) for both diffractive and resonant components:

$$\sigma_{\gamma p}^D = \frac{4\pi\alpha}{f_\rho^2} \sigma_{\rho p}^D \sum_n \frac{1}{1 + an} \frac{\sigma_{np}^D}{\sigma_{\rho p}^D} + \text{isoscalars} ,$$

(19a)

$$\sigma_{\gamma p}^R(\nu) = \frac{4\pi\alpha}{f_\rho^2} \sigma_{\rho p}^R(\nu) \sum_n \frac{1}{1 + an} \frac{\sigma_{np}^R(\nu)}{\sigma_{\rho p}^R(\nu)} + \text{isoscalars},$$

(19b)

where we have used for all n the previous result $m_n^2/f_n^2 \simeq m_\rho^2/f_\rho^2$, and the isoscalars account for both ω-like and φ-like production. In writing eqs. (19) we have explicitly assumed the validity of a diagonal approximation for the forward imaginary part of the vector dominated Compton amplitude. For the diffractive part this is supported by the dominance of diagonal pomeron couplings over off-diagonal ones in two-body and quasi-two-body hadronic processes. In addition, the value of $f_\rho^2/4\pi$ obtained in this approximation from forward ρ-photoproduction is in good agreement with the storage rings value. As far as the non-diffractive part is concerned, we will take it as a working hypothesis and check its validity a posteriori.

Let us consider eq. (19a). To proceed further we need the purely strong interaction cross sections σ_{np}^D. We will fix them by naive dimensional analysis, i.e., $\sigma_n^D \sim 1/m_n^2$. In Regge language, this implies for the forward elastic $(V_n p)$ amplitude A_n:

$$A_n \sim \Sigma_i \left(\frac{s}{m_n^2} \right)^{\alpha_i} \beta_i \, , \tag{20}$$

with the same β_i for all n. Such a scaling behaviour of strong interactions has been shown by Rittenberg and Rubinstein [21] in the framework of dual resonance models. We finally obtain:

$$\sigma_{\gamma p}^D = \frac{11}{9} \frac{4\pi\alpha}{f_\rho^2} \sigma_{\rho p}^D \frac{1}{a^2} \zeta\left(2, \frac{1}{a} \right) \, , \tag{21}$$

where a factor $\frac{2}{9}$ accounts for the isoscalars production and $\zeta(2, 1/a)$ is the generalized Riemann's zeta function defined as

$$\zeta(s, v) = \sum_{n=0} (v + n)^{-s} \, .$$

Using photoproduction data [2] eq. (21) holds with $a = 2$, the correction to the pure VDM result being 23%. The mass spectrum of the vector states coupled to the photon is therefore pure Veneziano-like, as stated earlier. Eq. (19b), augmented by eq. (20) is now a prediction of the model and provides a useful check for both the diagonal approximation and for our statements on the m_n^2 dependence of the Regge residues. We obtain

$$\sigma_{\gamma p}^R(\nu) = \frac{11}{9} \frac{4\pi\alpha}{f_\rho^2} \sigma_{\rho p}^R(\nu) \frac{1}{2^{3/2}} \zeta(\tfrac{3}{2}, \tfrac{1}{2}) \, , \tag{22}$$

which gives a 69% correction to the VDM, and is in excellent agreement with the data [both sides of (20) agree within $\sim 5\%$[.

We have shown how the actual situation existing in photoproduction can be sensible improved within the framework of our picture, providing also agreement with the colliding beam results. As seen above, moderate but significant corrections arise from the continuum of higher mass states at $q^2 = 0$. In deep inelastic scattering, as discussed below, the continuum will be responsible for the observed scaling behaviour of the structure functions, giving also a nice description of the data.

4. Inelastic electron-nucleon scattering

Let us now discuss electron-proton scattering, particularly in the deep inelastic region[*]. Recall the definition of the structure functions $W_1(\nu, q^2)$ and $W_2(\nu, q^2)$ in terms of the transverse and longitudinal virtual photon cross section $\sigma_T(\nu, q^2)$ and $\sigma_L(\nu, q^2)$:

$$W_1(\nu, q^2) = \frac{1}{4\pi^2\alpha} \left(\nu - \frac{q^2}{2M} \right) \sigma_T(\nu, q^2) , \tag{23a}$$

$$W_2(\nu, q^2) = \frac{1}{4\pi^2\alpha} \left(\nu - \frac{q^2}{2M} \right) \frac{q^2}{q^2 + \nu^2} \left[\sigma_T(\nu, q^2) + \sigma_L(\nu, q^2) \right] , \tag{23b}$$

where M is the proton mass. Let us separate the transverse cross section $\sigma_T(\nu, q^2)$ into diffractive and resonant part, as previously done at $q^2 = 0$. The obvious generalization of eqs. (19) gives

$$\sigma_T^D(\nu, q^2) = \frac{4\pi\alpha}{f_\rho^2} \left(1 - \frac{1}{\omega} \right) \sigma_{\rho p}^D \sum_n \frac{1}{\left(\frac{q^2}{m_\rho^2} + 1 + 2n \right)^2} + \text{isoscalars}, \tag{24a}$$

$$\sigma_T^R(\nu, q^2) = \frac{4\pi\alpha}{f_\rho^2} \left(1 - \frac{1}{\omega} \right) \sigma_{\rho p}^R(\nu) \sum_n \frac{(1 + 2n)^{1/2}}{\left(\frac{q^2}{m_\rho^2} + 1 + 2n \right)^2} + \text{isoscalars}, \tag{24b}$$

where as usually $\omega = 2M\nu/q^2$ ($q^2 > 0$). Let us discuss separately the two contributions. Eq. (24a) can be rewritten as

$$\sigma_T^D(\nu, q^2) = \frac{\pi\alpha}{f_\rho^2} \left(1 - \frac{1}{\omega} \right) \sigma_{\rho p}^D \zeta\left(2, \frac{q^2}{2m_\rho^2} + \frac{1}{2} \right) + \text{isoscalars}, \tag{25}$$

which asymptotically behaves as

$$\sigma_T^D(\nu, q^2) \underset{q^2 \to \infty}{\sim} \frac{2\pi\alpha}{f_\rho^2} \left(1 - \frac{1}{\omega} \right) \sigma_{\rho p}^D \frac{m_\rho^2}{m_\rho^2 + q^2} + \text{isoscalars}. \tag{26}$$

From (24b) we similarly obtain

$$\sigma_T^R(\nu, q^2) \underset{q^2 \to \infty}{\sim} \frac{\pi^2\alpha}{f_\rho^2} \left(1 - \frac{1}{\omega} \right)^{1/2} \sigma_{\rho p}^R(\nu) \sqrt{\frac{m_\rho^2}{m_\rho^2 + q^2}} + \text{isoscalars}. \tag{27}$$

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[*] The argument goes along the same lines as of ref. [5].

The transverse parts of the structure function is finally given by

$$\nu W_{2T}^{D}(\nu, q^2) \to 0.22\left(1 - \frac{1}{\omega}\right)^2,$$

(28a)

$$\nu W_{2T}^{R}(\nu, q^2) \to 0.3 \frac{1}{\sqrt{\omega}}\left(1 - \frac{1}{\omega}\right)^{3/2}.$$

(28b)

We would like to emphasize that our results are pure predictions of the model, with no use of adjustable parameters. Let us discuss them in detail. In the framework of two-component duality we have shown that starting from Regge behaved meson proton cross sections, scaling results for νW_2 naturally arise once a similar scaling behaviour is also shared by purely strong forward amplitudes, namely $A_n \sim \Sigma_i s^{\alpha_i} \beta_i(m_n^2)$ with $\beta_i(m_n^2) \sim (1/m_n^2)^{\alpha_i}$ [eq. (20)]. We recall that this follows from the n dependence of the couplings f_n as inferred from e^+e^- scaling. Furthermore we have been able to construct the structure functions *explicitly*, starting only with a knowledge of the photoproduction data for the ρ, ω and φ mesons. The precocious onset of scaling is evident from eqs. (26) and (27), since m_ρ^2 fixes the scale of the momentum transfer. On the other hand, as follows from eqs. (28), the threshold behaviour is quite poor, the reason being simply due to the lack of any low energy ($s \sim m_n^2$) resonant behaviour in the Regge-like expressions of σ_n.

Eqs. (28) are plotted in fig. 1 and compared with experiments [1]. The presence of a longitudinal contribution is accounted for by multiplying $\nu W_{2T}^{D,R}(\omega)$ by a fac-
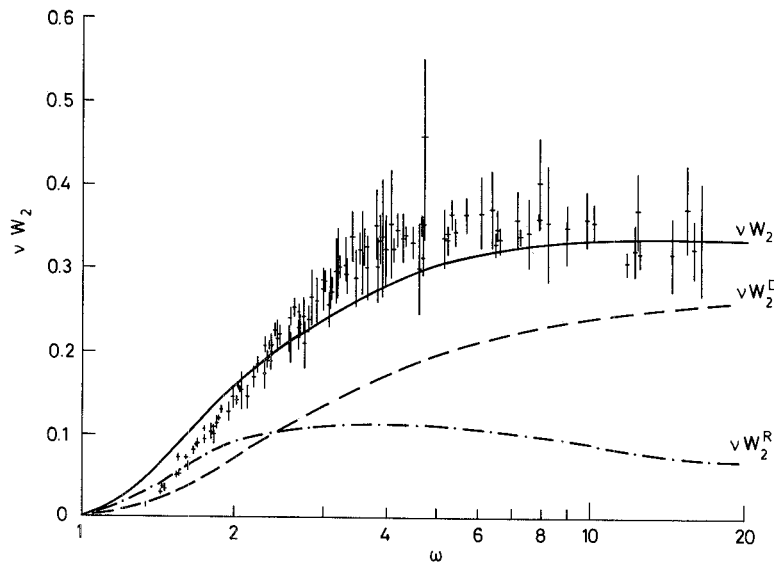

Fig. 1. Diffractive (D) and resonent (R) contributions to the structure function $\nu W_2(\omega)$. See the text for details.

tor 1.2. As one sees the agreement is exceptionally good, especially if one bears in mind that eqs. (28) are absolute predictions of the model.

A few words about the longitudinal cross sections $\sigma_L(\nu, q^2)$ are in order here. The naive extrapolation in q^2 from $q^2 = 0$ leads, as is well known, to a logarithmic growth in q^2, and therefore to a breakdown of scaling. On the other hand, it is a basic assumption of vector dominance that certain matrix elements are smooth, which is actually quite ambiguous as the choice of the particular set of extrapolation functions is not a priori obvious. In fact, it has been shown [22] that on the basis of only gauge invariance and analyticity the commonly accepted statement $\sigma_L^V/\sigma_T^V \sim \zeta q^2/m_V^2$ is not a general feature of the model; other forms for σ_L^V/σ_T^V are also possible which preserve Bjorken's scaling. For example, if, just for fun, we assume $\sigma_L^V/\sigma_T^V \sim \zeta q^2/(m_V^2 + q^2)$ then scaling is easily obtained in analogy to (26) and (27). Thus in the absence of a dynamical statement concerning σ_L^V, we have simply taken it into account by using the experimental ratio $R = \sigma_L/\sigma_T \simeq 0.2$.

Insofar as the neutron structure functions are concerned, one can proceed [5] analogously to the proton case, once the separation of the $I = 0$ and 1 contributions to the photoproduction cross sections are known. The results are

$$\nu W_{2n}^D = \nu W_{2p}^D, \qquad \nu W_{2n}^R = \frac{5.2}{11.2} \nu W_{2p}^R, \tag{29}$$

which extrapolated at threshold give 0.46 for the ratio $\nu W_{2n}/\nu W_{2p}$, consistent with the present experimental indication [23].

5. Inclusive deep inelastic processes

In the previous sections we have discussed a picture of electromagnetic interactions which in addition to giving consistent relations between different processes at low values of q^2, provides also a nice description of scaling for q^2 large, both positive and negative. If, however, in e^+e^- annihilation scaling is essentially built up from the electromagnetic properties of the hadronic constituents of the photon, in deep inelastic scattering on the contrary we need in addition that the strong interactions themselves have to share the same scaling properties. This feature is also found in the production of massive lepton pairs in hadronic collisions, which has been analyzed [6, 14] along the same lines. All these results can be visualized in a simple picture [6] for the scaling properties of hadronic as well as non-hadronic processes which suggests that the cross sections are determined by some transverse "size" characteristic of the process which shrinks with the mass of the particle.

The idea that the average transverse size of virtual photons of mass Q^2 may decrease as $1/\sqrt{Q^2}$ has emerged from Cheng and Wu's [24] calculations in Q.E.D. A growth with Q^2 of the average transverse momentum of produced hadrons correlated to a virtual photon, has been also investigated by Abarbanel and Kogut [25] in the framework of multiperipheral models. Simple kinematical analysis generally shows that in the appropriate regions one is naturally led to $q_T^2 \sim Q^2$, unless an *ad hoc* cut-off in q_T itself is assumed, as in the parton model [13].

By using this concept of an average transverse momentum increasing with Q^2, we give a number of specific predictions concerning some inclusive processes like massive photon production, single particle electroproduction, and pure hadronic production at large transverse momenta, which can be easily tested, providing equally crucial tests for the parton model [13].

The production of massive lepton pairs in hadronic collisions has been extensively studied in refs. [6, 14]. However, in view of the great similarity in our framework with the single particle inclusive electroproduction, we only quote here the main results:

$$\frac{d\sigma^\gamma}{dQ^2} \underset{\substack{Q^2, s \to \infty \\ \tau \equiv Q^2/s \text{ fixed}}}{\sim} \frac{1}{Q^4} F(\tau) , \tag{30}$$

$$\frac{d\sigma^\gamma}{dQ^2 d^3q/q_0} \underset{\substack{Q^2, s \to \infty \\ \tau \text{ fixed}}}{\sim} \frac{1}{Q^6} G(x_F, x_T, x_j^B, \tau) , \tag{31}$$

where $x_F = 2q_\parallel^*/\sqrt{s} \approx \tau(x_1^B - x_2^B)$, $x_j^B = 2p_j \cdot q/Q^2$, $j = 1, 2$ are the usual Feynman and Bjorken variables, and $x_T \equiv q_T^2/Q^2$. The momenta are defined as $p_1 + p_2 \to q + X$. Eq. (30) is the usual statement of scaling while eq. (31) contains in addition the statement $\langle q_T^2 \rangle \sim Q^2$. Eq. (31) is a consequence of the Mueller [12] formalism; for example, in the pionization region (s/Q^2, t/Q^2 and u/Q^2 all large), it is obtained by scaling both photon-proton subenergies as $(s_j/Q^2)^\alpha$, $j = 1,2$. The function G has a hadron-like dependence on the longitudinal and transverse variables, with the exception of the broadening of the transverse distribution with Q^2. Eq. (31) predicts a $1/Q^6$ dependence of $d\sigma/dQ^2$ as long as the photon's transverse momentum is restricted to a small region around $q_T \approx 0$. The BNL-Columbia experiment [26] has indeed shown exactly that behaviour in this special kinematical regime.

In the case of a process of the type $p + p \to \gamma(Q^2) + h + X$, one obtains a similar Q^2 dependence of $\langle h_T^2 \rangle$ as long as the hadron h is correlated to the photon, and the two-particle distribution function scales in this kinematical region similarly to (31), but with an extra $1/Q^2$ dependence. This follows from the energy momentum sum rule on the two-particle correlation functions. Similar results have also been found in the framework of the multiperipheral model [25].

We would like to discuss now the implications of our picture in inclusive electroproduction with particular emphasis on the Q^2 dependence of the single particle distribution functions which, in the photon fragmentation region, turn out to be similar to that found above. Let us define

$$f(s, q^2, k_L, k_T) \equiv \frac{1}{\sigma_{tot}(s, q^2)} \frac{d\sigma}{d^3k/k_0} , \tag{32}$$

where k_L and k_T denote the longitudinal and transverse momenta of the detected hadron. In our picture of a hadron-like photon we can apply a Mueller-Regge analysis of the process, similarly to what we have done in the production of a massive photon. Similar results have also been obtained by Cleymans [27]. Both in the nucleon fragmentation region and in the central region, by virtue of the factorization of Regge residues, one obtains for f results similar to any other hadronic process, photoproduction being of course the most natural process to compare with. In going to the photon fragmentation region (s, q^2 large with ω fixed, $(q \cdot k)/q^2$ fixed and $x = k_L/k_{L\ max} > 0$), we find new, interesting, and quite different results concerning the q^2 dependence of f, essentially due to the increase of $\langle k_T^2 \rangle$ with q^2.

In the lab frame, denoting by z the longitudinal fraction of the momentum taken by k, i.e., $k_L = z\nu$, mere kinematics tells us that $k_T^2 \simeq q^2 z(z + 2(q \cdot k)/q^2)$ for $\nu^2 \gg q^2$. Therefore, unless an *a priori* q^2 independent cut-off in k_T is assumed, we are naturally led to $\langle k_T^2 \rangle \sim q^2$, as in the processes previously discussed. This implies

$$f(s, q^2, k_L, k_T) \underset{\substack{s, q^2 \to \infty \\ \omega, x, x_T \text{ fixed}}}{\overbrace{\hspace{2cm}}} \frac{1}{q^2} G(\omega, x, x_T), \tag{33}$$

where we have defined $x_T = k_T^2/q^2$. Using Mueller-Regge arguments we finally obtain:

$$\sigma_{tot}(\nu, q^2) f(\nu, q^2, k_L, k_T) \underset{\substack{\nu, q^2 \to \infty \\ \omega, x, x_T \text{ fixed}}}{\xrightarrow{\hspace{2cm}}} \frac{1}{\nu} \frac{1}{q^2} \Sigma_i \omega^{\alpha_i} \beta_i(x, x_T), \tag{34}$$

where the sum is extended to both pomeron and ordinary Regge trajectories and the functions β_i are damped in x_T. The extra factor $1/q^2$ in (34) has to be compared with the analogous factor present in eq. (31). From the general form of (34) one can deduce the following general features:

(i) The transverse momentum distributions in the γ fragmentation region should broaden with increasing q^2. This q^2 variation would not manifest itself in the proton fragmentation nor central regions.

(ii) For $k_T \approx 0$ the longitudinal momentum distributions will decrease with increasing q^2. Again this drop will not be present in the other two regions.

(iii) The extra q^2 dependence in the denominator of the right-hand side of (34) will lead to a faster decrease of the diffractive contributions with respect to the contributions coming from the non-leading Regge trajectories. This will favour therefore the $I = 1$ exchanges with the proton, leading to asymmetries in the particle-antiparticle yields. In this sense the photon does "discriminate" between the charges. The composition of the observed final states will consequently also be affected.

All these characteristics are present in the main features of the present data [28]. It will be of course interesting to see if the actual trend continues to higher energies and larger q^2. It should be again emphasized that the observation of the increase of the average transverse momentum with q^2, together with the associated effects we have just discussed, provides also a crucial test for the parton model.

We finally report the main results on the inclusive hadronic productions at large transverse momentum discussed along the same lines [14]. They essentially follow from the strong scaling properties of the hadronic continuum which couples to the photon. The physical picture is the following. In high energy hadronic collisions large masses can be produced with a law similar to (31), with no absolute cut-off in q_T, which subsequantly decay into the particle finally detected with the transverse momentum p_T, plus everything else. By integration on the intermediate momentum and mass spectrum one easily finds:

$$\frac{d\sigma}{d^3p/p_0} \underset{\text{large } p_\perp}{\sim} \left(\frac{1}{p_\perp}\right)^4 \mathcal{F}\left(\frac{2p_\perp}{\sqrt{s}}\right),$$

(35)

as guessed from dimensional analysis, where $\mathcal{F}(2p_\perp/\sqrt{s})$ is a scaling functions of the argument which depends on the threshold behaviour of the decay mechanism. Similar slowly decreasing distributions are also obtained with a parton-like view of strong interactions [29].

6. Conclusion

We have discussed a simple model of electromagnetic interactions which successfully describes the main features of high energy processes, involving real, space-like photons, relating different processes with a high degree of predictive power. The exhibited scaling properties, and the consequent point-like behaviour of the cross sections, do not follow however from the existence of any elementary constituents, but rather from the sharing of similar scaling properties of strong and electromagnetic interactions. Although the results of scaling for the integrated cross sections are the same as in the parton model, the absence of an absolute transverse momentum cut-off leads to quite different and easily testable predictions for the inclusive distributions. Future experiments will therefore be of great relevance to discriminate between the two different pictures.

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