Laboratori Nazionali di Frascati

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DEEP INELASTIC STRUCTURE FUNCTIONS
FROM COLLINEAR SU(1,1) SYMMETRY

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We exhibit a model of deep inelastic processes, based on conformal symmetry arguments, in which the structure functions for the scattering and annihilation channels both scale and are related by the Gribov-Lipatov relation. Analytic continuation, however, is not valid in general.

In this note we present a model in which the structure functions, upon diagonalization of the forward Compton amplitude with respect to representations of the conformal group [1], are given as generalized Born terms.

In the transition form factors $\langle p|J_\mu(0)|n\rangle$ figuring in the sum

$$W_{\mu\nu}(p, q) =$$

$$\sum_n (2\pi)^4 \delta^4(p+q-p_n)\, \langle p|J_\mu(0)|n\rangle\langle n|J_\nu(0)|p\rangle$$

the states $|p\rangle$ and $|n\rangle$ are classified according to zero and continuous mass representations respectively of the conformal group $SU(2, 2)$ [2]. The structure functions associated with a single irreducible representation in the set spanned by n turn out to be non-trivial as a continuous spectrum of resonances is contained in any such representation. Scaling of the structure functions follows from the requirement that the field associated with the target is massless while their functional form is fixed by covariance under $SU(1, 1)$ which is the relevant part of the conformal group in the scaling limit. In configuration space this corresponds to projective transformations along a light-like line [3].

Our classification of the intermediate states is in some sense analogous to a previous attempt by Del Giudice et al. [4] to assign partons to representations of $SU(1, 1)$. However, under the larger group $SU(2, 2)$ there is only one linear trajectory $l = 1 + |j|$ for zero mass states of helicity j and scale dimension l [5]. Many interesting consequences emerge from such a scheme.

i) The structure functions have a good threshold behaviour as $\omega \to 1$

$$F_2(\omega) \xrightarrow[\omega \to 1]{} A_\lambda(\omega-1)^{2\lambda+1} \tag{2}$$

where the exponent λ is the $SU(1, 1)$ quantum number of the continuum which builds up the inelastic processes.

ii) Their Regge behaviour is universal, i.e., independent of λ

$$F_2(\omega) \xrightarrow[\omega \to \infty]{} \omega^{-2} \log^2 \omega \tag{3}$$

Apart from the log this is the same as found in dual resonance models [6].

iii) The asymptotic behaviour of the excitation form factors is correlated with the threshold behaviour in the manner first suggested by Drell, Yan and West [7].

iv) The structure functions of scattering ($F_2(\omega)$) and annihilation ($\bar F_2$) are related by the Gribov-Lipatov reciprocity [8] relation

$$\bar F_2(1/\omega) = -\omega^3 F_2(\omega). \tag{4}$$

v) The analytic continuation

$$\bar F_2(\omega) = F_2(\omega) \tag{5}$$

does not hold in general although the threshold theorem is satisfied [9].

vi) $F_2(\omega)$ so calculated for $\lambda = 1$ fits the data points rather well in the interval $1 \leqslant \omega < 10$.

For simplicity we consider Compton scattering off a scalar target of four-momentum p as in fig. 1. The kinematical variables are standard
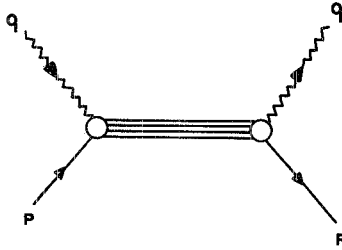
Fig. 1. A typical diagram contributing to the structure function.

$$\omega = 2pq/-q^2, \qquad S = -q^2(\omega - 1) > 0$$

with $1 \leqslant \omega < \infty$ for scattering and $0 \leqslant \omega \leqslant 1$ for annihilation and S the invariant mass of the resonances exchanged between the blobs in fig. 1. In the short distance limit the form of these latter ones is fixed by SU(1, 1) symmetry and using standard reduction techniques one gets as $x_- \to 0$ and $x^2 \to 0$

$$\text{Im} \int d^4x \, \exp(iqx)\langle p|(J_-(x_+, x_-, x_T)\psi^{(n)}(0))|0\rangle$$

$$= \text{Im} \int dx_+ dx_- \exp\,[(i/2)(q_+ x_- + q_- x_+)]$$

$$\times (x_-)^{-(d_n+1)/2}(x_+)^{-(\tau_n - 1)/2}$$

$$\times \int_0^1 du \, u^{(1-\tau_n)/2}(1-u)^{(\tau_n - 3)/2} \exp(iux_+ p_-/2)$$

$$(x_\pm = t \pm z)$$

$$= (q^2)^{(\tau_n - 3)/2}(q_+)^{1+\tau_n}\omega^{(1-\tau_n)/2}(\omega - 1)^{\tau_n - 2}$$

$$\times {}_2F_1(\tau_{n-1}/2, \tau_{n-1}/2, \tau_{n-1}, 1-1/\omega)\theta(\omega - 1)$$

where $\omega = p_-/q$. We have used the SU(1, 1) [10] light cone expansion to recover the vertex function in coordinate space. $\psi^{(n)}(x)$ is a conformal weight zero field [5] of spin $n = (d_n - \tau_n)/2$ and scale dimension $l_n = (d_n + \tau_n)/2$. Making use of (6) in eq. (1) one gets for a given conformal quantum number $\lambda = (\tau_n - 3)/2$

$$F_2^{(\lambda)}(\omega) = \frac{C_\lambda}{\omega(\omega - 1)} Q_\lambda^2 \frac{\omega + 1}{\omega - 1} \tag{7}$$

where $_2F_1(...)$ has been expressed in terms of the Legendre function $Q_\lambda(...)$ [11]. The physical meaning of λ is that it determines the asymptotic power growth of the spectrum of resonances [12]

$$\rho(S) \xrightarrow[S \to \infty]{} S^{2\lambda + 1}. \tag{8}$$

From eq. (6) the electromagnetic excitation form factor behaves for large q^2 as [10, 13]

$$G(q^2) \xrightarrow[q^2 \to \infty]{} (q^2)^{-(1+\lambda)} \tag{9}$$

which on comparison with eq. (8) reproduces the Drell, Yan and West relation [7]. From eq. (7) on the other hand one obtains for each λ, eqs. (2) and (3) together with

$$F_2^{(\lambda)}(1/\omega) = (-1)^{2\lambda + 1}\omega^3 F_2^{(\lambda)}(\omega) \tag{10}$$

while for the annihilation structure function one finds

$$\bar{F}_2^{(\lambda)}(\omega) = (-)^{2\lambda}F^{(\lambda)}(\omega). \tag{11}$$

Eqs. (9) and (10) together give

$$\bar{F}_2^{(\lambda)}(1/\omega) = -\omega^3 F_2^{(\lambda)}(\omega) \tag{12}$$

independently of λ whence eq. (4) upon summation over this latter. From the above results it follows that the analytic continuation in eq. (5) is not true in general so that the stronger reciprocity relation
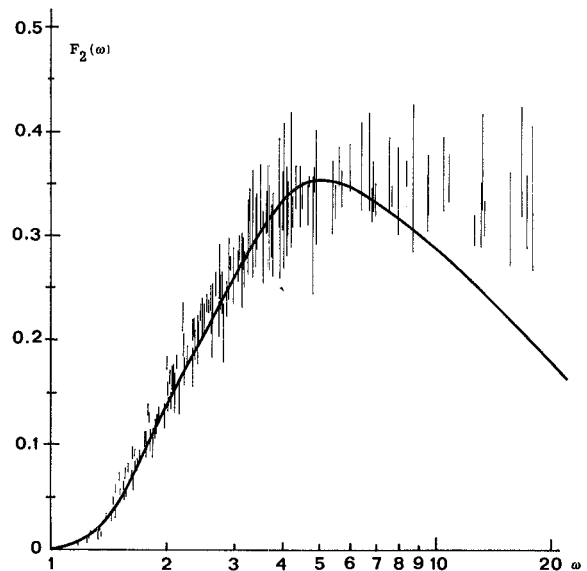

Fig. 2. A plot of the calculated structure function for $\lambda = 1$ (eq. (15)) as it compares with the data.

$$F_2(1/\omega) = -\omega^3 F_2(\omega) \tag{13}$$

does not hold. This would be so under the additional assumption that the set of contributing representations correspond to integer λ. It is interesting to point out that eq. (7) is a particular realization of the following formula of Gribov and Lipatov [8].

$$F_2(\omega) = \frac{\omega-1}{\omega^2} \, \varphi\left(\left[\frac{\omega-1}{\omega+1}\right]^2\right) \tag{14}$$

where $\varphi(x)$ is positive for $x > 0$ and regular at $x = 0$. The right hand side of eq. (7) becomes an elementary function for $\lambda = n > 0$ [11]. An interesting case is $n = 1$ which gives

$$F_2(\omega) = \frac{C}{\omega(\omega-1)} \left[\frac{\omega+1}{\omega-1} \ln \omega - 2\right]^2 . \tag{15}$$

A plot of eq. (15) is shown in fig. 2 together with the SLAC data.

References

[1] L. Castell, Phys. Rev. D2 (1970) 1161;
S. Ferrara, R. Gatto, A.F. Grillo and G. Parisi, Nucl. Phys. B49 (1972) 77;
A.A. Migdal, Landau Institute preprint, September 1972, to be published.

[2] L. Castell, Nuovo Cim. 46A (1966) 1;
H. Kastrup, Phys. Rev. 142 (1966) 1060.
[3] S. Ferrara, R. Gatto, A.F. Grillo and G. Parisi, University of Rome preprint n. 452 (January 1973), to be published.
[4] E. Del Giudice, P. Di Vecchia, S. Fubini and R. Musto, Nuovo Cim. 12A (1972) 813.
[5] S. Weinberg, Phys. Rev. 134B (1964) 882;
G. Mack and I.T. Todorov, J. Math. Phys. 10 (1969) 2078;
G. Mack and A. Salam, Ann. Phys. 53 (1969) 174.
[6] R. Casalbuoni and G. Gomis, Univ. of Florence Preprint (January 1973) and ref. [4].
[7] S.D. Drell and T.M. Yan, Phys. Rev. Lett. 24 (1970) 181;
G. West, Phys. Rev. Lett. 24 (1970) 1206.
[8] V.N. Gribov and L.N. Lipatov, Phys. Lett. 37B (1971) 78; Soviet J. Nucl. Phys. 15 (1972) 438, 675.
[9] R. Gatto, P. Menotti and T. Vendramin, Nuovo Cim. Lett. 5 (1972) 754.
[10] S. Ferrara, R. Gatto and A.F. Grillo, Nucl. Phys. 34B (1971) 349;
S. Ferrara, A.F. Grillo and G. Parisi, Nuovo Cim. 12A (1972) 952.
[11] I.S. Gradshteyn and I.M. Ryzhik, Table of integrals, series and products (Academic Press, New York 1965), eqs. (9) 134, 8 702 and 8 827.
[12] E. Etim, Phys. Lett. 41B (1972) 169; Nuovo Cim. 13A (1972) 427.
[13] A.A. Migdal, Phys. Lett. 37B (1971) 98, 386.
[14] E.D. Bloom et al., Phys. Rev. Lett. 23 (1969) 935.