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A. Bramon, E. Etim, S. Ferrara, M. Greco, A. F. Grillo, G. C. Rossi and Y. Srivastava: THEORETICAL ASPECTS OF HIGH ENERGY ELECTRON-POSITRON COLLISIONS.

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ABSTRACTS. -

In this paper we report on theoretical investigation carried out at Frascati on high-energy e^+e^- collisions.

CONTENTS. -

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1. - e^+e^- ANNIHILATION AND EVMD. -

In this section the e^+e^- annihilation into hadrons is considered from the point of view of a new approach to electromagnetic interactions which has been recently proposed[1,2] and which is based on the idea that the photon is coupled to an hadronic spectrum of vector states of Veneziano-like form $m_n^2 = m_\rho^2\,(1+2n)$. This approach attempts to unify two different descriptions for ligh-energy electromagnetic processes, namely the hadron-like behaviour of the photon, as clearly indicated from the study of photoproduction of hadrons and similar photon initiated hadronic processes at high energies, and the point-like scaling behaviour of the large q^2 processes.

A deeper discussion of the motivations, as well as the applications of this idea to the mostly known electromagnetic processes, can be found in the literature[2]. In the following we shall restrict ourselves to a discussion along this line of electron-positron annihilation into hadrons, both in the low-intermediate energy region of resonance production, and in the asymptotic regime.

Defining em_n^2/f_n as the coupling of the photon to the vector meson of mass m_n and width Γ_n, the total cross-section for $e^+e^- \to V_n \to f$ at a total energy $2E = \sqrt{s}$ near the cross m_n can be written as

$$\sigma_{e\bar{e}\to f}(s) \simeq \frac{16\pi^2\alpha^2}{f_n^2}\;\frac{m_n\Gamma_n^{\,e}\,\Gamma_n^{\,f}}{(s-m_n^2)^2+m_n^2\Gamma_n^2} \tag{1}$$

The coupling constant is assumed independent of the photon mass $\sqrt{s}$, and is measured on the vector meson mass shell.

In addition to the well-known ρ, ω and φ meson, whose properties have been extensively studied at the Orsay and Novosibirsk storage rings, a new vector state, the $\rho'(1600)$ has recently been observed at Frascati[3,4] and at SLAC[5] with $\Gamma_{\rho'} \sim 350$ MeV, $\rho\pi\pi$ as the main decay made, and $(f_{\rho'}/f_\rho)^2 \gtrsim 4$-$5$. The main properties of this vector state, and the corresponding sizeable production of inelastic final states in e^+e^- annihilation, were predicted[6] on the basis of a previous study of the radiative decays of mesons. The broadness of this state, and the experimental uncertainties, make its exact mass difficult to determine. However, for a world of linearly rising Regge trajectories of universal slope, we can identify it with the vector component of the first even doughter of the ρ and rename it $\rho''(1600)$.

In the framework of the model, with an infinity of vector mesons coupled to the photon, and with the assumption of a linear mass spectrum $m_n^2 = m_0^2(1+an)$, it is possible to give an answer to the problem of the density of the vector meson states, or, in other words, to dermine the spacing a. The result is a = 2, i.e. a pure Veneziano-like spectrum. The existence of a ϱ' (1250) is therefore implied in this approach. The experimental implications of this fact are extensively discussed in ref. (2) and can be summarized as follows.

(i) There exists evidence[(7)] in photoproduction of an $(\omega\pi)$ enhancement at masses around 1.25 GeV, with an energy-independent cross section of about 1 μb.

(ii) By identifying the above $\omega\pi$ enhancement with the $\varrho'(1250)$ one can predict[(2)]: $\sigma(e^+e^- \to \omega\pi)$ 40-50 nbs at ϱ' peak, consistently with the data from Novosibirsk and Adone, which also show evidence of a $(\pi^+\pi^-\pi^0\pi^0)$ enhancement in this energy region.

(iii) In the reaction $e^+e^- \to \pi^+\pi^-$ a clear deviation from the Gounaris-Sakurai expression for F_π is found[(8)] at Adone around $\sqrt{s} \simeq 1.25$. A very small elasticity, of the order of 10%, is also indicated.

(iv) The $p\bar{p}$ annihilation at rest in the $\omega\pi\pi$ final state also suggests a 1^{--} state with mass and width of 1.25 and 0.13 GeV respectively.

From the above considerations it follows that this model gives a satisfactory description of the low-intermediate energy phenomenology. It is therefore legitimate to ask wether the same model can account for an asymptotically scaling annihilation cross section. Assuming the total cross section completely dominated by a sequence of vector meson states, and ignoring interferences among the various vector mesons contributions, we can write:

$$\sigma_{tot}(e^+e^- \to had) \sim 16\pi^2\alpha^2 \sum_n \frac{1}{f_n^2} \frac{m_n \Gamma_n}{(s-m_n^2)^2 + m_n^2\Gamma_n^2} \qquad (2)$$

A simple solution to $\sigma_{tot} \sim \frac{1}{s}$ is given by

$$\frac{\Gamma_n}{\Gamma_\varrho} \simeq \frac{f_n}{f_\varrho} \simeq \frac{m_n}{m_\varrho}. \qquad (3)$$

which leads to:

(4) $$\sigma_{tot}(e^+e^- \to had) \simeq \frac{8\pi^2}{f_\rho^2} \times \frac{4\pi\alpha^2}{3s} .$$

By defining the asymptotic ratio R

$$R \equiv \lim_{s\to\infty} \frac{\sigma_{tot}(e^+e^- \to had)}{\sigma(e^+e^- \to \mu^+\mu^-)} ,$$

one finally predicts from (4)

(5) $$R \simeq \frac{8\pi^2}{f_\rho^2} = 2.5 - 3.$$

This result is consistent with the large cross sections recently observed(4) both at Frascati and CEA.

Equation (5) is quite interesting because it links together the asymptotic ratio R to a typical low-energy parameter f_ρ^2. This feature of the model is also common to what is found in photon and electro-production, where both the total photo absorption cross section and the deep inelastic structure functions can be explicitly expressed in terms of the photoproduction data for the ρ, ω and φ meson, in excellent agreement with experiments.

As it is clear, the point-like behaviour of $\sigma_{tot}(e^+e^- \to had)$ is provided in this picture by broader and broader higher mass vector mesons which add together to build up a smooth scaling continuum, in a very similar way to what happens in strong interactions, where an infinite number of resonances build up a smooth Regge-like behaviour of the scattering amplitude.

This analogy with strong interactions has been examined further by Sakurai(10), who has conjectured a new "duality" between vector meson formation and scaling behaviour of the total cross section. More explicitly the following finite-energy own rule has been suggested:

(6) $$\int_{4m_\pi^2}^{s_{max}} s ds \left[\sigma_{had}(s) - \sigma_{comp}(s)\right] = 0,$$

where $\sigma_{had}(s)$ is the total cross section for e^+e^- annihilation into hadrons and $\sigma_{comp}(s)$ is a "comparison" cross section which approaches $\sigma_{had}(S)$ as $s \to \infty$ and is extrapolated down at low energies with appropriate threshold factors.

Eq. (6) is then used to predict the actual numerical value of the asymptotic ratio R defined above, by integrating (6) up to $s_{max} \simeq 1.2\ GeV^2$, and including in $\sigma_{had}(s)$ contributions of the well known ρ , ω and φ mesons. By using for the comparison cross section the following forms:

(7a) $$\sigma_{comp}(s) = R\,\frac{4\pi\alpha^2}{3s}\left(1+\frac{2m_\mu^2}{s}\right)\left(1-\frac{4m_\mu^2}{s}\right)^{1/2},$$

(7b) $$\sigma_{comp}(s) = R\,\frac{4\pi\alpha^2}{3s}\left(1+\frac{2m_q^2}{s}\right)\left(1-\frac{4m_q^2}{s}\right)^{1/2},$$

(7c) $$\sigma_{comp}(s) = R\,\frac{4\pi\alpha^2}{3s}\left(1-\frac{4m_\pi^2}{s}\right)^{3/2},$$

with a "quark mass" $m_q = m_{N/3}$, the following values of R are found

(8a) $R = 2.9$,

(8b) $R = 5.0$,

(8c) $R = 3.9$.

These values of R are in satisfactory agreement with the experiments at high energies, but disagree with the data for $s \lesssim 5\ GeV^2$. On the other hand the range of values of R reflects the uncertainties arising from the extrapolation of $\sigma_{comp}(s)$ at low energies.

A different approach can be used to estimate R, in the same spirit of eq. (6), which is free from the above uncertainties. Defining $\sigma_{had}(s)$ as the hadronic part of the photon propagator,

$$D_{\mu\nu} \sim \frac{g_{\mu\nu}}{q^2} + \left(\frac{q_\mu q_\nu}{q^2} - g_{\mu\nu}\right)\frac{\Pi_{had}(q^2)}{q^2},$$

then one has, as well known,

(9) $$\sigma_{had}(s) = \frac{4\pi\alpha}{s}\,\mathrm{Im}\,\Pi_{had}(s)\ .$$

It is clear that if $\sigma_{had}(s) = R\,\sigma_{\mu\bar{\mu}}(s)$.

Then $Im\Pi_{had}(s)$ would approach asymptotically a constant value given by $\alpha R/3$. One can therefore write, instead of (6),

(10) $$\int Im\left[\Pi_{had}(s) - \frac{\alpha R}{3}\right] ds = 0,$$

and integrate over the known low energy resonances. The result is given by $R = 8\pi^2/f_\rho^2$, in agreement with eq. (5), and is also independent from the choice of vector state n, as long as the condition $m_n^2/f_n^2 = m_\rho^2/f_\rho^2$ is fulfilled. It follows therefore that the prediction of the model is rather unique, to be contrasted with the experimental situation which has not been settled yet.

Another possible estimate of the asymptotic ratio R could be obtained from the knowledge of the anomalous contribution to the muon anomalous magnetic moment. As well known in fact, it holds

(11) $$a_\mu(had) = \frac{1}{4\pi^2\alpha}\int_{thresh}^{\infty} ds\, k(s)\, \sigma_{e^+e^-\to had}(s),$$

where k(s) is given by

(12) $$k(s) = \int_0^1 dx \frac{x^2(1-x)}{x^2 + (s/m_\mu^2)(1-x)}$$

and $k(s) \sim 1/s$ as $s \to \infty$. The hadronic contribution to the muon anomalous magnetic moment has been recently evaluated(11) using the recent results from the colliding beam experiments. The result is

(13) $$a_\mu(had) = \left[66.5 + (R\times 1.5) \pm 9\right] \times 10^{-9}$$

where the contributions from the known ρ, ω, φ and $\rho''(1600)$ have been included in eq. (11), together with an asymptotic contribution above 2 GeV in the form $\sigma_{had}(s) = R\,\sigma_{\mu\bar{\mu}}$. Unfortunately, due to the large errors in eq. (13) which only reflects the uncertainties in the experimental cross sections, and the large error in the theoretical estimate of the pure Q.E.D. sixth order contributions(12) to a_μ , it follows that a very accurate measurement of the g-factor of the muon would not give any reasonable answer to the problem of R, unless new and really big phenomena will develop in very high energy e^+e^- collisions. It has to be noticed, in addition, that the existence of a $\rho'(1250)$ would imply $a_\mu(\rho') \simeq (5-10) \times 10^{-9}$, having used the estimate for the production cross section given earlier.

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2. - SUM RULES IN INCLUSIVE e^+e^- ANNIHILATION. -

In this section the e^+e^- annihilation into multihadrons is investigated through the energy-momentum and charge sum rules. By using these techniques one is able to shed some light on general grounds about the scaling properties of multi-hadron production processes, the question of multiplicities and correlations. A more extensive discussion of this argument is given in ref. 1.

Consider the inclusive e^+e^- annihilation into n identical hadrons (called π's), in the one-photon approximation:

$$(1) \qquad e^+(k_+) + e^-(k_-) \to \gamma(q) \to h(k_1) + h(k_2) + \ldots + h(k_n) + X.$$

Define the n-th inclusive cross section

$$(2) \qquad f^{(n)}_{e^+e^-} \equiv \frac{d\sigma^{(n)}}{\prod_{i=1}^{n}(d^3k_i/E_i)} = \frac{(2\pi)^2\alpha^2}{2s^3} L^{\mu\nu} H^{(n)}_{\mu\nu} ,$$

where

$$(3) \qquad L^{\mu\nu} = 4\left[k_+^\mu k_-^\nu + k_-^\mu k_+^\nu - (k_+ k_-) g^{\mu\nu}\right] ,$$

$$(4) \qquad H^{(n)}_{\mu\nu} = \frac{1}{2^n (2\pi)^{3n}} \sum_m \frac{1}{m!} (2\pi)^4 \delta^4(q - k_1 - k_2 - \ldots - k_n - P_m) \, d\mathcal{P}_m \langle 0 | J_\mu | k_1 \ldots k_n ; m \rangle \langle k_1 \ldots k_n ; m | J_\nu | 0 \rangle .$$

with

$$d\mathcal{P}_m = \prod_{i=1}^{m} \left[\frac{d^3 q_i}{(2\pi)^3 2 q_i^0} \right],$$

and $q^2 = s$.

This decomposition of $f^{(n)}$ into a pure leptonic tensor $L_{\mu\nu}$ and a pure hadronic tensor $H^{(n)}_{\mu\nu}$ is quite useful because it allows to concentrate on the study of the hadronic properties of the final state without any complication due to the presence of the initial leptons, as the decay of a particle of mass q^2 and spin 1. This makes a real difference with pure strong reactions, where the presence of two hadrons in the initial state affects the kinematical configuration and also the dynamics of the final state.

Defining the scaling variables $x_i = 2k_i\, q/s$, which in the e^+e^- CM frame is simply $2E_i/\sqrt{s}$, it is useful to give the explicit expression for the total cross-section, simple and double inclusive cross-sections in terms of scalar invariants. One has:

(5) $$H^{(o)}_{\mu\nu}(q) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{S}\right)\frac{s}{6\pi}\, R(s),$$

and

(6) $$\sigma_{tot}(s) = \frac{4\pi\alpha^2}{3s}\, R(s);$$

(7) $$H^{(1)}_{\mu\nu}(q,k_1) = \frac{1}{(2\pi)^2}\left\{\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right)\bar{F}_1(s,x_1) + \frac{2}{x_1 s}\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)\bar{F}_2(s,x_1)\right\}.$$

and

(8) $$\frac{d\sigma^{(1)}}{dx_1\, dz_1} \simeq \left(\frac{\pi\alpha^2}{2s}\right)\sqrt{x_1^2 - 4\mu^2/s}\left\{2\bar{F}_1 + \frac{1}{2x_1}\left(x_1^2 - 4\mu^2/s\right)\left(1-z_1^2\right)\bar{F}_2\right\};$$

(9) $$H^{(2)}_{\mu\nu}(q,k_1,k_2) = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{s}\right)G_1 + \frac{1}{\mu^2}\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)G_2$$
$$+ \frac{1}{\mu^2}\left(k_{2\mu} - \frac{k_2 q}{s} q_\mu\right)\left(k_{2\nu} - \frac{k_2 q}{s} q_\nu\right)G_3$$
$$+ \frac{1}{2\mu^2}\left[\left(k_{1\mu} - \frac{k_1 q}{s} q_\mu\right)\left(k_{2\nu} - \frac{k_2 q}{s} q_\nu\right) + \left(k_{2\mu} - \frac{k_2 q}{s} q_\mu\right)\left(k_{1\nu} - \frac{k_1 q}{s} q_\nu\right)\right]G_4,$$

and

$$(10)\quad \frac{d\sigma^{(2)}}{dx_1 dx_2 dz} = \frac{(2\pi)\alpha^2}{3s}\sqrt{x_1^2-4\mu^2/s}\,\sqrt{x_2^2-4\mu^2/s}\left\{3Y_1 + Y_2 + Y_3 + zY_4\right\}$$

In the above eqs. μ is the pion mass, z_1 is the cosine of the angle made by the pion with respect to the e^+e^- beam, the invariants G_1, G_2, G_3 and G_4 depend upon 4 Lorentz scalars which may be chosen to be s, x_1, x_2 and the missing mass variable $M_X^2 = (q-k_1-k_2)^2$, $z = \cos(\hat{k}_1 \cdot \hat{k}_2)$ and the Y_i's are defined as follows:

$$(11)\quad G_1 = \frac{1}{(2\pi)^3}\left(\frac{4}{s}\right)Y_1 \;;\qquad G_i = \frac{1}{(2\pi)^3}\left(\frac{16\mu^2}{s^2}\right)(x_i^2 - 4\mu^2/s)^{-1}Y_i \;;\qquad (i = 2, 3)$$

$$G_4 = \frac{1}{(2\pi)^3}\left(\frac{16\mu^2}{s^2}\right)\frac{1}{\sqrt{x_1^2-4\mu^2/s}\,\sqrt{x_2^2-4\mu^2/s}}\,Y_4 \;.$$

The energy-momentum sum rules, for the general case of n-particle inclusive production, are given by:

$$(12)\quad (q-k_1-k_2..k_n)\; H^{(n)}(q;k_1..k_n) = \int\left(\frac{d^3k_{n+1}}{E_{n+1}}\right)k_{n+1}H^{(n+1)}(q;k_1..k_n,k_{n+1}).$$

For many types of particles, one simply sums over all types on the right hand side of (12). Let us explore this equation for n = 0 and in same detail.

(i) <u>n = 0:</u>

Eq. (12) leads to only one non trivial condition:

$$(13)\quad R^{(s)} = \frac{3}{4}\int_{2\mu/\sqrt{s}}^{1} dx_1 \cdot \sqrt{x_1^2-4\mu^2/s}\left\{\overline{F}_1 + \frac{1}{6x_1}(x_1^2-4\mu^2/s)\overline{F}_2\right\},$$

which fixes as $\overline{F}_1 \sim x_1^{-2-\alpha}$ and $\overline{F}_2 \sim x_1^{-3-\alpha}$ ($\alpha < 1$) the highest (power) singularities allowed as $x_1 \to 0$. Correspondingly the hadronic multiplicity, defined as

$$(14)\quad \langle n(s)\rangle = \frac{3}{4R(s)}\int_{2\mu/\sqrt{s}}^{1} dx_1\sqrt{x_1^2-4\mu^2/s}\left\{2\overline{F}_1 + \frac{1}{3x_1}\left(x_1^2-\frac{4\mu^2}{s}\right)\overline{F}_2\right\},$$

behaves as $(\sqrt{s})$ $(\langle n\rangle \sim \ln s$ for $\alpha = 0)$ as s approaches infinity.

(ii) n = 1:

Now we obtain 5 independent relations between $\overline{F}_1$, $\overline{F}_2$ and the Y_i's belonging to two - particle production cross-section. The most interesting one is the following:

(15)
$$-\sqrt{x_1^2-4\mu^2/s}\left[3\overline{F}_1+\frac{x_1^2-4\mu^2/s}{2x_1}\overline{F}_2\right]=\int dx_2(x_2^2-4\mu^2/s)\int dz\; z\; x$$
$$x\left\{3Y_1+Y_2+Y_3+zY_4\right\},$$

which, together with eq. (14) gives:

(16)
$$\langle n(s)\rangle=\frac{3s}{(4\pi\alpha^2)\,R(s)}\int\frac{dx_1}{\sqrt{x_1^2-4\mu^2/s}}\;\frac{dx_2}{\sqrt{x_2^2-4\mu^2/s}}\Big[(x_1^2-4\mu^2/s)+$$
$$+(x_2^2-4\mu^2/s)\Big]\left(\frac{-1}{2}\right)\int dz\, z\left(\frac{d\sigma^{(2)}}{dx_1\,dx_2\,dz}\right)$$

Actually in eq. (16) only the correlated part of the cross-section ($\sigma^{(2)}_{corr}$) is left upon integration over z. Thus, the behaviour of $\langle n(s)\rangle$ is directly tied to the behaviour of $(d\sigma^{(2)}_{corr})/(dx_1 dx_2 dz)$.

For purely hadronic case it is found phenomenologically (as well as in the Regge model) that the normalized correlation functions $\varrho^{12}(y_1, y_2)$ behaves as

$$\varrho^{12}(y_1,y_2)\sim e^{-\frac{|y_1-y_2|}{\zeta}}\sim\left(\frac{1}{s_{12}}\right)^{1/\zeta}$$

where the correlation length ζ is approximately two. While this result is an asymptotic one valid for large rapidity differences it seems howe ver to work well also for small rapidity differences[2].

In our case we can catalog the behaviour of $\langle n(s)\rangle$ by assuming for the correlation functions a similar form

$$\left(\frac{d\sigma^{(2)}}{\frac{d^3k_1}{E_1}\;\frac{d^3k_2}{E_2}}\right)_{corr}\sim\frac{1}{\left(x_1x_2-z\sqrt{x_1^2-4\mu^2/s}\sqrt{x_2^2-4\mu^2/s}\right)^{\alpha}}$$

where is an undetermined constant.

One easily finds that for

$\alpha = 2$ $\langle n(s) \rangle \sim \sqrt{s}$

$\alpha = 1$ $\langle n(s) \rangle \sim \ln s$

$\alpha < 1$ $\langle n(s) \rangle$ finite.

Thus if an analogy with strong interactions holds then α is equal to $1/\zeta = 1/2$, in which case the multiplicities will be finite. Conversely to obtain growing $\langle n(s) \rangle$ one has to invoke correlations which behave much more violently than in hadronic interactions.

Relations analogous to (12) hold for any additive quantum number, e.g. charge, strangeness, beryon number. We have

$$(0-e_1-e_2-\dots-e_n)H^{(n)}_{\mu\nu}(q,k_1,\dots,k_n) = \sum_{e_{n+1}} e_{n+1} \int \left(\frac{d^3k_{n+1}}{E_{n+1}}\right) \times H^{(n+1)}_{\mu\nu}(q;k_1,\dots,k_{n+1}), \tag{17}$$

where a stands for the generalized "charge". Let us consider in detail eq. (17) for n = 0 and n = 1.

(i) <u>n = 0</u>:

This is almost trivial, since it simply states that

$$H^{(h)}_{\mu\nu}(q;k) = H^{(\bar{h})}_{\mu\nu}(q;k),$$

where $\bar{h}$ denotes the antiparticle of h.

(ii) <u>n = 1</u>:

This sector leads us to an interesting sum rule on particle production. First, we notice that any two-particle production function, $H^{(h1,h2)}(q;k_1,k_2)$, satisfies the relation

$$H^{(h_1,h_2)}_{\mu\nu}(q;k_1,k_2) = H^{(h_1,h_2^*)}_{\mu\nu}(q;k_1,k_2), \tag{18}$$

where h_1 is "neutral" and h_2^* is the "charge conjugate" to h_2.

Examples:

$$H_{\mu\nu}^{(\pi^o, \pi^+)} = H_{\mu\nu}^{(\pi^o, \pi^-)}; \qquad H_{\mu\nu}^{(\pi^+, k^+)} = H_{\mu\nu}^{(\pi^+, k^-)};$$

$$H_{\mu\nu}^{(k^+, p)} = H_{\mu\nu}^{(k^+, \bar{p})} = H_{\mu\nu}^{(k^-, p)}, \text{ etc..}$$

Consider now π^+ production. Eq. (17) reads:

(19) $$-H_{\mu\nu}^{(\pi^+)}(q;k) = \sum_{h'} e'_h \int \left(\frac{d^3k'}{E'}\right) H_{\mu\nu}^{(\pi^+, h')}(q; k, k').$$

The sum on the right simplifies considerably, since the contribution from strangeness and baryon number carrying h's cancel in pair upon using eq. (18). Thus, (19) reduces to:

(20) $$H_{\mu\nu}^{(\pi^+)}(q;k) = \int \left(\frac{d^3k'}{E'}\right) \left[H_{\mu\nu}^{(\pi^+\pi^-)}(q;k,k') - H_{\mu\nu}^{(\pi^+\pi^+)}(q;k,k') \right]$$

This is the sum rule, which for the general case reads:

(21) $$H_{\mu\nu}^{(h)}(q;k) = \int \left(\frac{d^3k'}{E'}\right) \left[H_{\mu\nu}^{(h\bar{h})}(q;k,k') - H_{\mu\nu}^{(hh)}(q;k,k') \right]$$

provided h is not completely neutral (e.g. π^o or η^o).

Eq. (20) states the interesting fact that the inclusive single pion distribution function is only related to the double inclusive pion-pion distribution function and more precisely to the correlated part of it. From an experimental point of view this is a property very useful to solve background problems, because it allows to measure the single inclusive properties, by triggering the experimental set-ups on pion pairs.

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3. - GRIBOV-LIPATOV RELATIONS AND CONNECTION BETWEEN DEEP INELASTIC SCATTERING AND ANNIHILATION. -

In this paragraph we study a possibility of connecting experimental results in deep inelastic scattering and annihilation.

This can be achieved by the use of a relation between one particle inclusive structure functions:

(1)
$$F^{ep}(1/\omega, q^2) = \mp \omega \, F_1^{e^+e^-}(\omega, q^2) \qquad \text{(-refers to fermions)}$$
$$F_2^{ep}(1/\omega, q^2) = \mp \, \omega^3 F_2^{e^+e^-}(\omega, q^2)$$

firstly proved by Gribov and Lipatov(1).

It is not easy to verify this relation simply by matching two functions (of the experimental results): rather it is physically more appealing to try to investigate possible consequences.

Let consider for example the reaction

(2)
$$e^+e^- \to m(j) + \text{anything}$$

where m(j) is an hadron of mass m and spin j.

If there is scaling we can define the average multiplicity for the production of the particle m, with the help of eq. 1:

(3)
$$\langle n(s) \rangle = \frac{(2j+1)}{2R} \int_1^{s/2m} \frac{d\omega}{\omega} F_2(\omega)$$

where $R = \sigma_{tot}(s) / \sigma_{\mu^+\mu^-}(s)$.

Eq. (3) is interesting by itself, since a quantity measured in e^+e^- collisions ($\langle n(s) \rangle$) is related to the structure function of the d.i.s.

It is well known that $\langle n(s) \rangle$ may diverge: and in fact, using a simple Regge parametrization, (assuming that one is allowed to do so in the Bjorken limit)

(4)
$$F_2(\omega) \underset{\omega \to \infty}{\longrightarrow} \sum b_i (\omega)^{\alpha_i - 1}$$

we have that the Pomeron part (b_o) gives actually the divergent contribution.

For example, if one uses $b_o = 0.22$(2) for the proton and $b'_o = 2/3\,(b_o)$ for the pion (as in the quark model), and R = 2 one has

(5)
$$\langle n_p(s) \rangle = 0.15 \ln s + \ldots.$$
$$\langle n(s) \rangle = 0.05 \ln s + \ldots..$$

(dots indicate non-diverging terms).

Here a major difference between π and proton multiplicities is evident, namely that the experimental values are almost saturated by the Pomeron contribution ($\langle n_p \rangle_{exp} \sim 0.32$) for protons[3], while the non-diverging terms are of leading importance for the pion at the Adone energy ($\sim$ 1.5 GeV): in fact the measured number of charged particles (mostly pions and kaons) is about 3.

This suggest, through the G.L. relation, a quite different form for the π d.i.s. structure function than for the proton.

We remark that this is not at all an accademic problem, since data on the pion structure function may be forth coming from Serpukhov.

To investigate further this problem another constraint can be imposed to the structure function, namely the energy conservation

(6) $$2\sigma_{TOT}(s) = \sum_i \int_{2m/\sqrt{s}}^{1} d\omega\, \omega \left(\frac{d\sigma^i}{d\omega}\right)$$

where the summation extends over all stable hadrons.

Extending the summation in eq. (6) over stable baryons and pseudoscalar mesons and assuming the contribution from each baryon to be approximately the mean of the proton and neutron contributions one gets an interesting bound

(7) $$R > 1$$

on the ratio R which seems to be respected by experiments[3]. Simple quark models with $R < 1$ seem thus to be excluded[5].

Experimentally a good fit to the proton and neutron structure functions has been given in terms of three adjustable parameters[6]. Assuming the same can be done for the pion we consider two simple alternatives

(8.1) $$F_2(\omega) = C_1\left(1-\frac{1}{\omega}\right)^2 + C_2\left(1-\frac{1}{\omega}\right)^3 + C_3\left(1-\frac{1}{\omega}\right)^4$$

(8.2) $$F_2(\omega) = C_1\left(1-\frac{1}{\omega}\right) + C_2\left(1-\frac{1}{\omega}\right)^2 + C_3\left(1-\frac{1}{\omega}\right)^3$$

The coefficients C_i are determined from the constant asymptotic value of $F_2(\omega)$(2/3 that of the proton), the constant value of the ratio R (the energy sum rule eq. (6)) and the constant value of the pion multiplicity as discussed above. Treating the baryons as done previously one has sufficient equations to determine the coefficients C_i in terms of the annihilation parameters R and $\langle n_\pi \rangle$ (*), values of which must be so assigned as to make $F_2(\omega)$ always positive. In Fig. 1 we have plotted the structure functions so determined for $\langle n_\pi \rangle = 0.8$, $R = 4$ (curve (a)) $\langle n_\pi \rangle = 0.4$, $R = 2$ (c), using eq. (8.1) and for

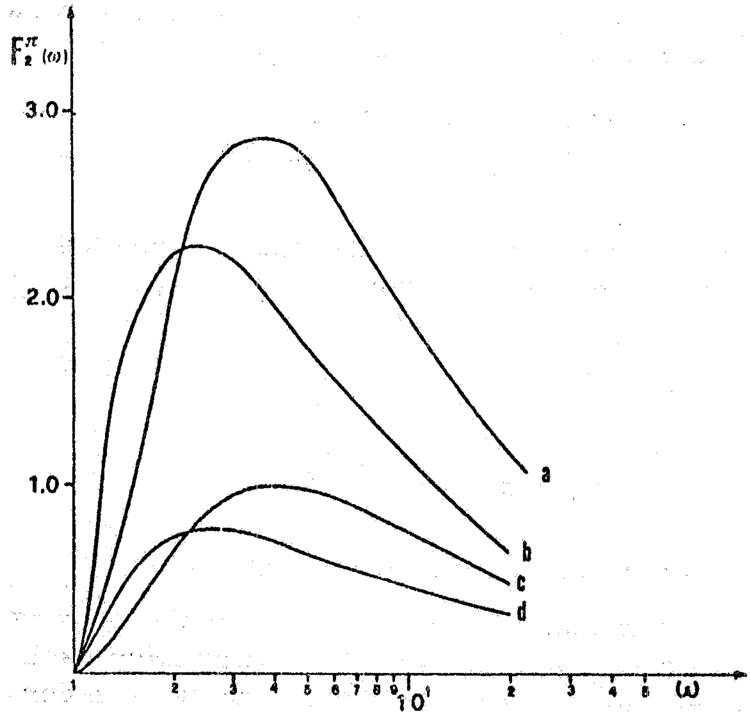

FIG. 1 - Plot of the pion structure functions for various values of the parameters $\langle n_\pi \rangle$ and R.

$\langle n_\pi \rangle = 0.6$, $R = 4$ (b) and $\langle n_\pi \rangle = 0.5$, $R = 2$ (d), using eq. (8.2). Compared to the proton structure function one notes the following distinguishing features:

(i) the overall large values of the structure functions, in particular their large peak values,
(ii) the maxima occur approximately within the same range of ω between 2.5 and 4, as for the proton and neutron,
(iii) the relatively narrow spread,
(iv) the decreasing behaviour (except for curve (d)) of the structure functions at values of ω at which the constant (maximum) value is about reached for the proton.

These features are evidently very closely related; taken together they have a direct bearing on the structure of the pion. Clearly the most remarkable aspect is the overall large values; the question thus arises as to how the structure functions of other hadrons would compare with those of the nucleon and the pions.

(*) - Here $\langle n_\pi \rangle$ is the non-diverging contribution.

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4. - MULTIPLICITIES AND MULTIPARTICLE PRODUCTIONS IN AN INDEPENDENT EMISSION MODEL FOR e^+e^- - ANNIHILATION. -

In this section we shall consider some general features of multiparticle production in high-energy electron-positron annihilation, based on an independent emission model. For details see ref. (1).

The basic idea is an old one. The pioneering work of Kastrup is recommended[(2)]. The e.m form factor of an hadron measures, in some sense, the probability that the hadron, scattered by the e.m field, does not "radiates" other hadrons. This probability is assumed to have a bremsstrahlung-like behaviour, i.e. $e^{-\bar{n}}$, where $\bar{n}$ is the mean measured multiplicity. $\bar{n}(s)$ is assumed to behave logarithmically for large s[(3)], like in the "central" or "non-diffractive" region in p-p multiproduction:

(1) $\bar{n}(s) \sim \log s$, for large s.

This behaviour does not seem to be in contradiction with the existing data[(4)].

For the production of n identical particles the basic formula

(2) $$\sigma(e^+e^- \to n) = \sigma_{tot} P(n) = \sigma_{tot} \frac{\bar{n}^n}{n!} e^{-\bar{n}}$$

follows from the model. Eq. (2) holds in a world with one kind of particles. If different kinds of particles are present, they are assumed to be independently emitted and eq. (2) is generalized as follows:

(3) $$\sigma(e^+e^- \to n_1+n_2+n_3+\ldots) = \sigma_{tot} P(n_1)P(n_2)P(n_3)\ldots P_{others}(0),$$

where $P_{others}(0) = e^{-\bar{n}_{others}}$ is a factor associated to particles not produced in the reactions.

Let us now consider some quantitative consequences of the model. We have to make some preliminary assumptions. Supported by strong interaction data, we assume that pseudoscalar particles are predominantly produced (as compared to barions). The emission of a particle-antiparticle pair is considered as the elementary creation process[(5)], so that in eq. (3) n_i refers to the number of pairs of the kind i produced. Self-conjugate particles are also assumed to be independently emitted. In order to be as much predictive as possible we use in a rather crude way asymptotic SU(3) symmetry. If the multiplicity of a given particle p is for large s

(4) $$\bar{n}_p(s) \sim C_p \log s,$$

then asymptotic SU(3)[6] implies

(5) $$C_{\pi^+} = C_{\pi^-} = C_{\pi^o} = C_{k^+} = C_{k^-} = C.$$

Moreover in the present model the relation[7]

(6) $$\sigma(e^+e^- \to K_o\bar{K}_o) = 0$$

gives

(7) $$C_{K^o} = C_{\bar{K}_o} = 0$$

($\sigma(e^+e^- \to K_o\bar{K}_o)$ is proportional to C_{K^o}).

From this equation and the asymptotic SU(3) result[6] $C_{K^+} + 2C_{K^o} = 3C_\eta$, we get $C_\eta = (1/3)C$; therefore

(8) $$\bar{n}_{tot}(\text{pseudoscalars}) \underset{\text{large } s}{\sim} C_{tot} \log s \sim \frac{16}{3} \times \bar{n}_{\pi^+}(s).$$

Let us now consider some peculiar features of the model. Given

(9) $$\bar{n}_{tot}(s) = C_{tot} \log s + \gamma_{tot} \qquad (s \text{ measured in GeV}^2)$$

and

(10) $$\bar{n}_p(s) = C_p \log s + \gamma_p \qquad (\text{multiplicity of the particle "p"}),$$

we get

$$(11) \qquad \frac{\sigma\,(e^+e^- \to \text{n particles of type "p"})}{\sigma_{tot}} = \frac{(\bar{n}_p(s))^n}{n!}\, e^{-\bar{n}_{tot}(s)} \, .$$

This curve has a maximum at

$$(12) \qquad s_{MAX} = \exp\left(\frac{n}{C_{tot}} - \frac{\gamma_p}{C_p} \right) \text{GeV}^2$$

and its value at the maximum is

$$(13) \qquad \left. \frac{\sigma(e^+e^- \to \text{n particles of type "p"})}{\sigma_{tot}} \right|_{s = s_{MAX}} = \left(\frac{nC_p}{C_{tot}}\right)^n \times$$

$$\times \exp\left[-n + \gamma_p \frac{C_p}{C_{tot}} - \gamma_{tot} \right]$$

The maximum is an (experimentally) increasing function of n, while the cross section at the maximum is a decreasing function of n.

These characteristic shapes, expecially for kaons(8), should be very easily distinguishible in a high energy e^+e^- storage ring, like Super Adone, giving a definite answer whether the independent emission model is tenable or not.

Cross sections are power behaved (up to logs) and the power in the exclusive cases is related to C_{tot}. In particular the pion elastic form factor is given by(1)

$$(14) \qquad \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma_{tot}} \sim \text{logs} \cdot s^{-C_{tot}} \quad \text{for large s.}$$

The same formula holds for Kaons.

In figs. 1 and 2 we report

$$\frac{\sigma(e^+e^- \to n\,(\pi^+\pi^-)}{\sigma_{tot}}$$

for n = 1, 2, 3 and C_{tot} = 1, 2. Various cross sections for multiple pion and Kaon productions are plotted in figs. 3 and 4. In fig. 5 we give the ratios

$$\frac{\sigma(e^+e^- \to n(\pi^+\pi^-))}{(e^+e^- \to n(k^+k^-))}$$

for n = 1, 2, 3. These ratios are C_{tot} - independent.

Note that in this model

$$(15) \qquad \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to k^+k^-)} = \frac{\bar{n}_\pi(s)}{\bar{n}_k(s)} .$$

As the asymptotic SU(3) symmetry is certainly broken by particle masses, formula (15) (which includes particle masses as threshold factors in the multiplicities) implies that, if $C_{tot} \simeq 1, 2$, SU(3) symmetry comes in at extremely high energy. For instance, in our oversimplified model, at Adone energy ($s \sim 9\ \mathrm{GeV}^2$)

$$(16) \qquad \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to k^+k^-)} \simeq 2,$$

while at Super Adone energy ($s \sim 400\ \mathrm{GeV}^2$)

$$(17) \qquad \frac{\sigma(e^+e^- \to \pi^+\pi^-)}{\sigma(e^+e^- \to k^+k^-)} \simeq 1.5.$$

We conclude noticing that

$$(18) \qquad \frac{\sigma(e^+e^- \to n(\pi^+\pi^-))}{\sigma(e^+e^- \to n(\pi^+\pi^-)+\text{neutrals})} = \exp\left[-\bar{n}_{\text{neutr.}}(s)\right] .$$

independently on n. Eq. (18) suggests a nice way to estimate the average multiplicity of the neutral particles, which is usually a rather diffucult quantity to measure.

In fig. 6 we report the ratio (18) in the approximation $\bar{n}_{\text{neutr.}}(s) \simeq \bar{n}_{\pi^0}(s)$.

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(3) - $s = 4E^2$, where E is the energy per beam in the center of mass system.

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(5) - This partially takes into account charge and strangeness conservation.

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(8) - For pions this maximum appears at too low values of s, unless n is very large ($n \sim 4$).

FIGURE CAPTIONS -

FIG. 1 - $$\frac{\sigma\,(e^+e^- \to n(\pi^+\pi^-))}{\sigma_{tot}}$$ for n = 1, 2, 3 and $C_{tot} = 1$

FIG. 2 - $$\frac{\sigma(e^+e^- \to n(\pi^+\pi^-))}{\sigma_{tot}}$$ for n = 1, 2, 3 and $C_{tot} = 2$

FIG. 3 - $$\frac{\sigma\,(e^+e^- \to \pi^+\pi^-k^+k^-)}{\sigma_{tot}}, \quad \frac{\sigma(e^+e^- \to 2(\pi^+.\pi^-)k^+k^-)}{\sigma_{tot}},$$

$$\frac{\sigma(e^+e^- \to \quad {}^{+}\quad{}^{-}2(k^+k^-))}{\sigma_{tot}}$$ for $C_{tot} = 1$,

FIG. 4 - $$\frac{\sigma\,(e^+e^- \to \pi^+\pi^-k^+k^-)}{\sigma_{tot}}, \quad \frac{\sigma\,(e^+e^- \to 2(\pi^+\pi^-)k^+k^-)}{\sigma_{tot}},$$

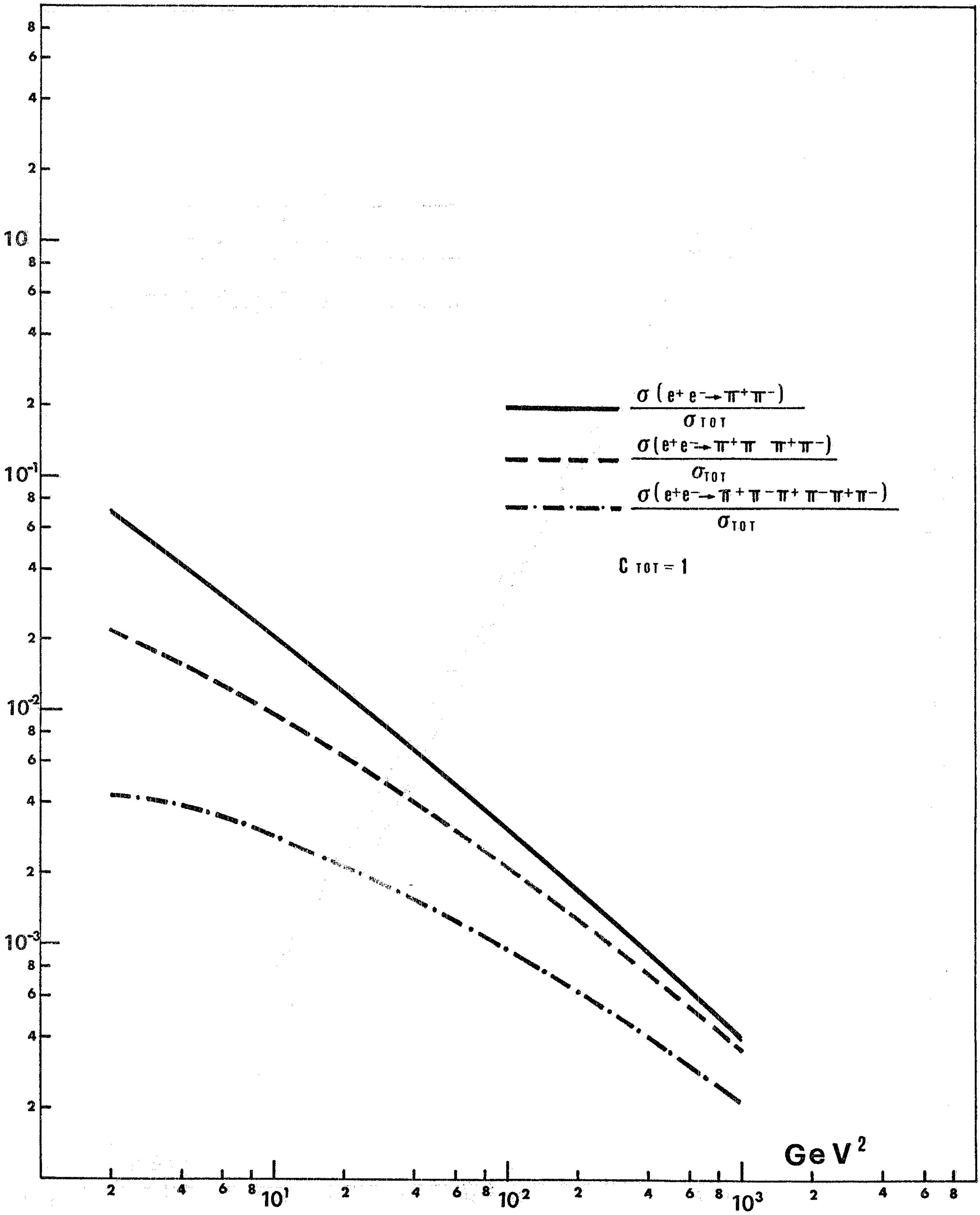
$$\frac{\sigma\,(e^+e^- \to \pi^+\pi^-.2(k^+k^-))}{\sigma_{tot}}$$ for $C_{tot} = 2$.

FIG. 5 - $$\frac{\sigma\,(e^+e^- \to n(\pi^+\pi^-))}{\sigma\,(e^+e^- \to n(k^+k^-))}$$ for n = 1, 2, 3. This ratio is C_{tot} independent.

FIG. 6 - $$\frac{\sigma\,(e^+e^- \to n(\pi^+\pi^-))}{\sigma\,(e^+e^- \to n(\pi^+\pi^-)\text{+neutrals})}$$ for $C_{tot} = 1, 2$.

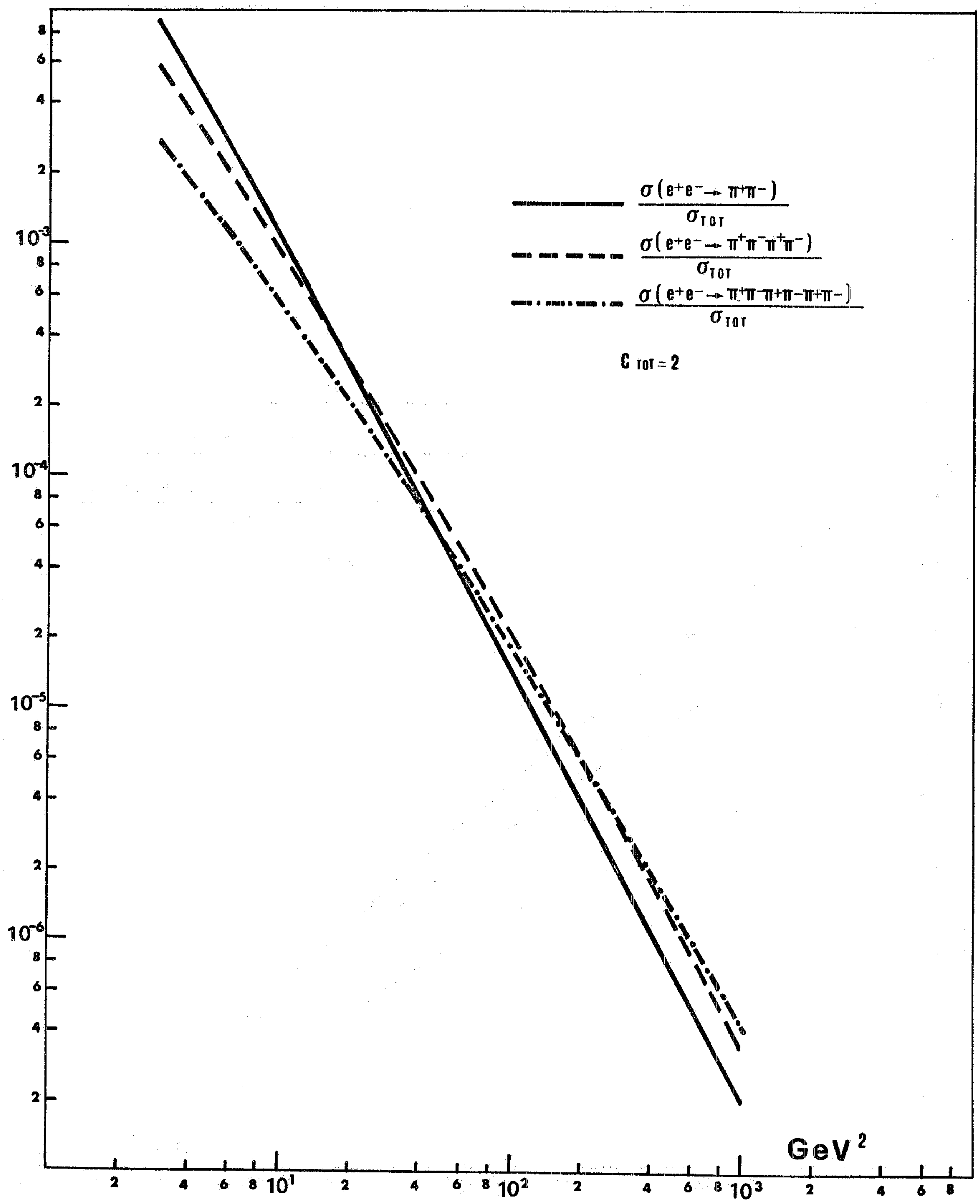
This ratio is n - independent.

All the curves are plotted for $2\ GeV^2 \leqslant s \leqslant 10^3\ GeV^2$.



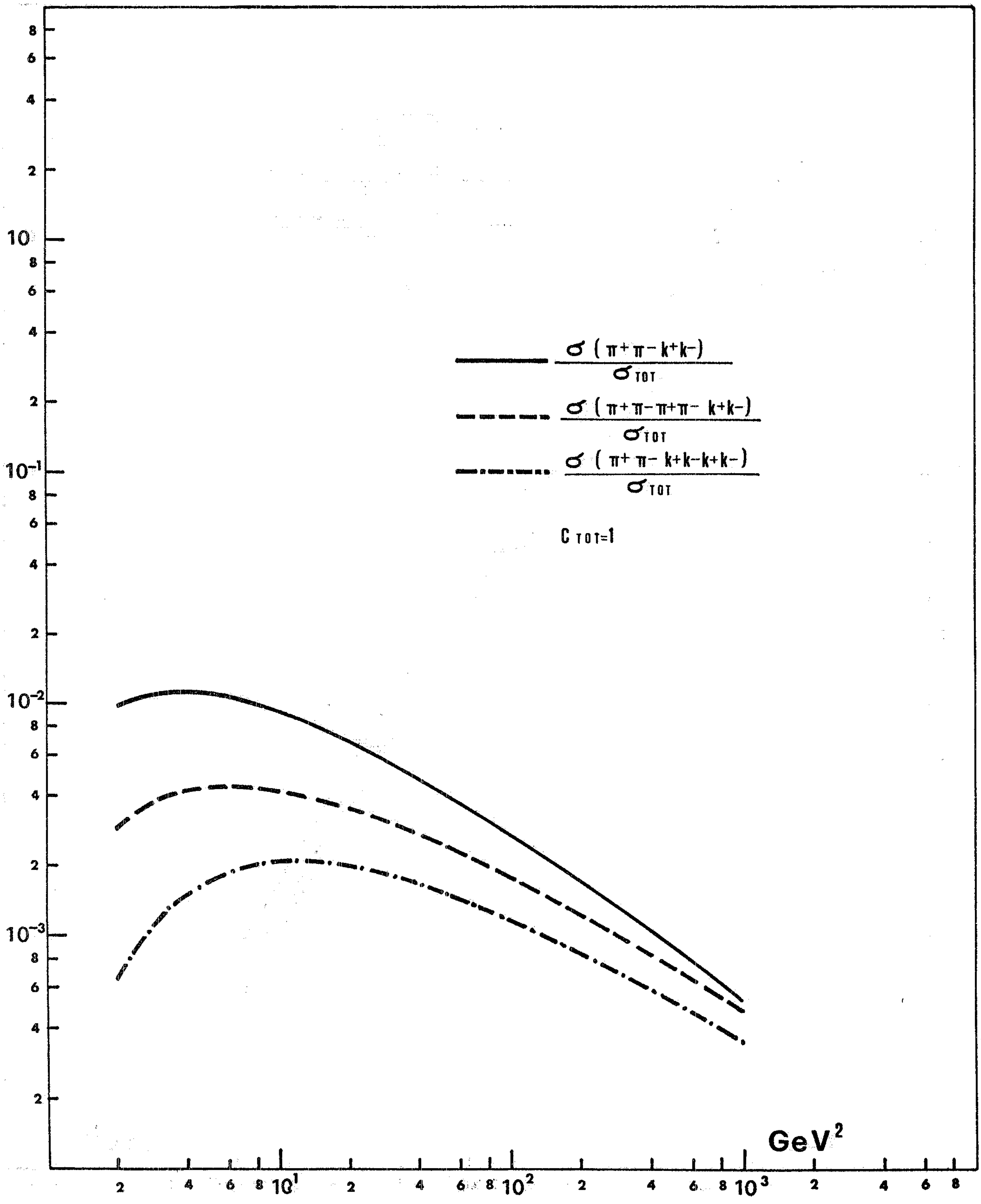
10
10^{-1}
10^{-2}
10^{-3}
10^{1}
10^{2}
10^{3}
GeV²
$\sigma(e^+e^- \to \pi^+\pi^-) / \sigma_{TOT}$
$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-) / \sigma_{TOT}$
$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-) / \sigma_{TOT}$
C TOT = 1

FIG.1



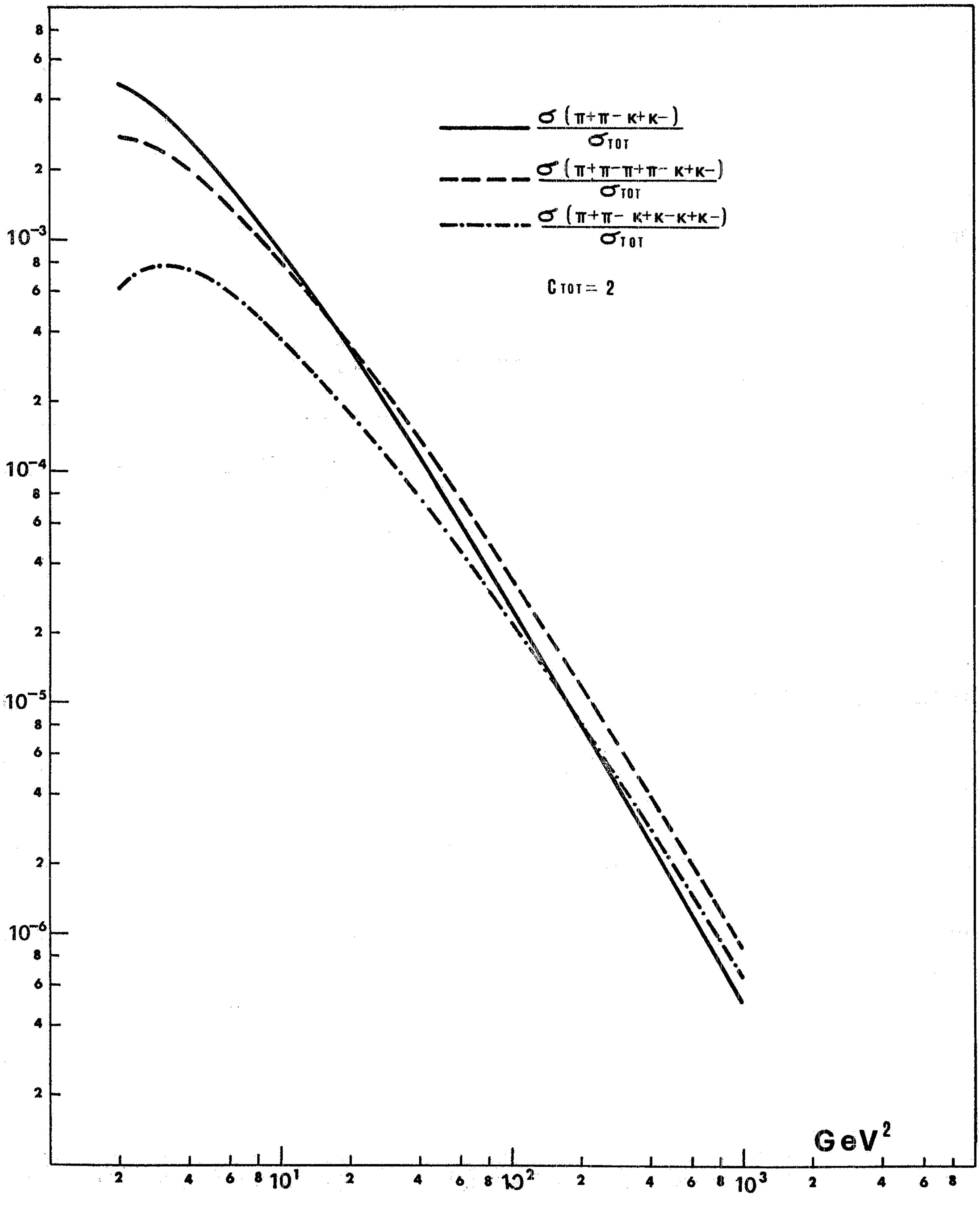
$\sigma(e^+e^- \to \pi^+\pi^-)$ / σ_{TOT}
$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-)$ / σ_{TOT}
$\sigma(e^+e^- \to \pi^+\pi^-\pi^+\pi^-\pi^+\pi^-)$ / σ_{TOT}
$C_{TOT} = 2$
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^1
10^2
10^3
GeV^2

F I G. 2

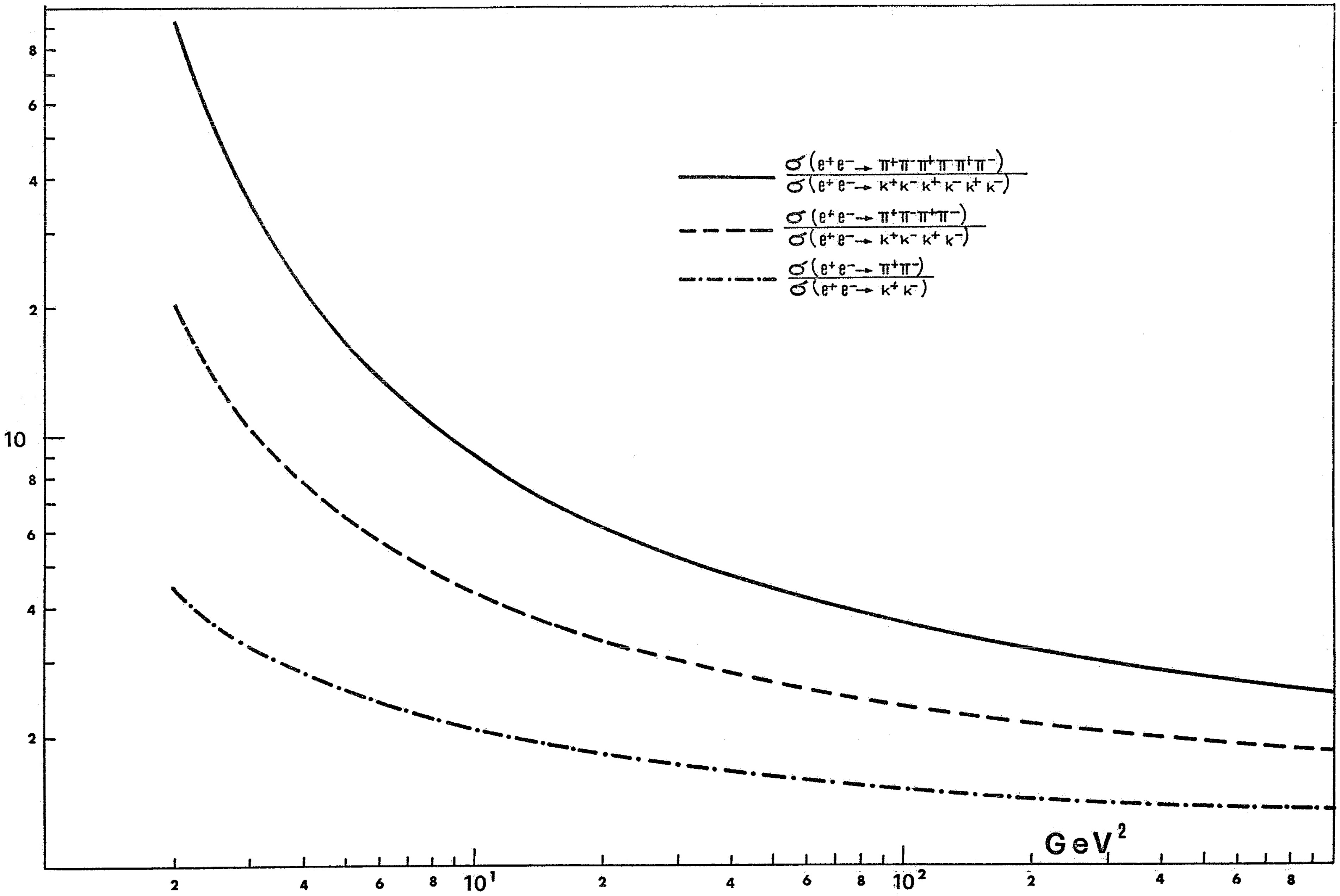


σ (π+π− k+k−) / σ TOT
σ (π+π−π+π− k+k−) / σ TOT
σ (π+π− k+k−k+k−) / σ TOT
C TOT=1
GeV²
10
10⁻¹
10⁻²
10⁻³
10¹
10²
10³

F I G. 3


$\frac{\sigma(\pi^+\pi^-K^+K^-)}{\sigma_{TOT}}$
$\frac{\sigma(\pi^+\pi^-\pi^+\pi^-K^+K^-)}{\sigma_{TOT}}$
$\frac{\sigma(\pi^+\pi^-K^+K^-K^+K^-)}{\sigma_{TOT}}$
$C_{TOT} = 2$
GeV^2
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^1
10^2
10^3

F I G. 4


σ(e+e− → π+π−π+π−π+π−) / σ(e+e− → K+K−K+K−K+K−)
σ(e+e− → π+π−π+π−) / σ(e+e− → K+K−K+K−)
σ(e+e− → π+π−) / σ(e+e− → K+K−)
10
8
6
4
2
10¹
10²
GeV²

F I G. 5

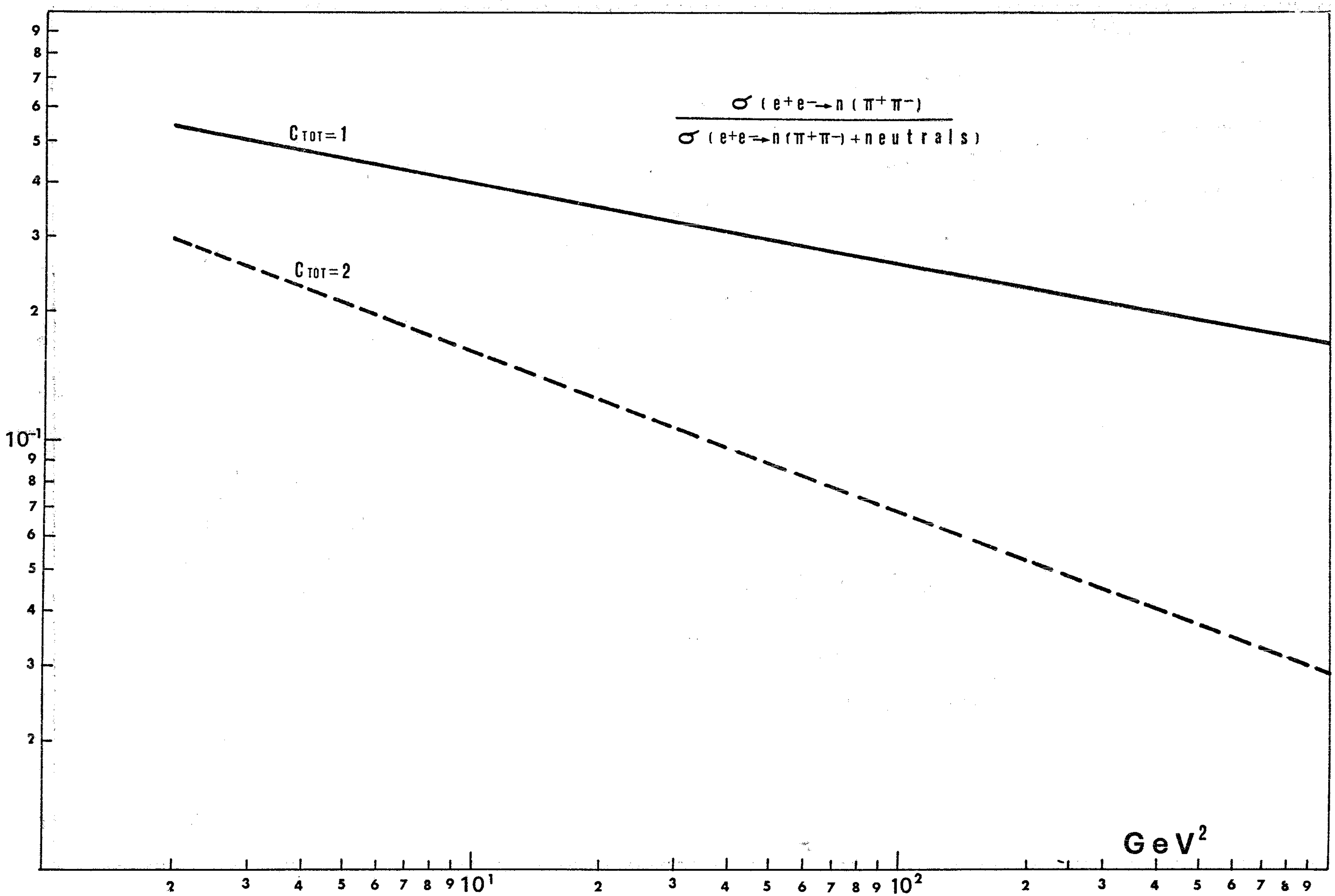


FIG. 6